



# An Improved Target Location Algorithm of MIMO Radar Based on Fuzzy C Clustering

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**Abstract.** This paper deals with multi-target localization in statistical MIMO radar. An improved target locating algorithm is proposed which combines Kalman filtering with fuzzy C clustering. The Kalman filter is utilized to acquire the information of target location and fuzzy C clustering is used for data fusion as there are multiple receivers in radar. For target locating in MIMO radar, we first utilize the maximum likelihood estimation algorithm to estimate the parameters of targets. To eliminate the influence of noise on the parameter estimation, we take advantage of the gliding property of Kalman filter to process the result of parameter estimation. All these processing data from different receivers is fused by fuzzy C cluster to obtain the parameters estimation of all targets. We give scenarios including MIMO radar and targets to analyze the performance of this target location algorithm. With considering the effects of noise, the position of receivers and transmitters and the moving of targets, the analysis is carried out by evaluating the location accuracy of the algorithm. The simulation result shows that the proposed method can locate multiply targets effectively and improves the location accuracy.

**Keywords:** Statistical MIMO radar · Multi-target localization · Kalman filtering · Fuzzy C clustering

## 1 Introduction

MIMO (Multiple-input multiple-output) radar is a new kind of system radar. It is superior to traditional radar in target detection and parameter estimation. Target positioning is one of the core issues in the fields of radar, sonar and UAV interaction and it is performed by processing the echo. Typical methods include the Maximum Likelihood method and the Best Linear Unbiased Estimation method.

A series of solutions are proposed for the MIMO radar which take the distance equation on target localization into consideration. Wang et al. achieved target localization by hyperbolas without considering the effect of transmit and receive antennas [2]. Yang et al. proposed a hyperbolic localization algorithm based on the measurement of the arrival time of the transmitting/receiving antenna. This algorithm solves the local convergence problem in the traditional algorithm and analyzes the influence of transmitting/receiving antenna pairs on the positioning accuracy. But it requires an

initial estimate close to the target position [3]. These algorithms require transmit and receive antenna to meet time synchronization strictly.

For the research of processing echo information in statistical MIMO radar, the proposed method achieves target positioning directly. Two approximate ML algorithms are proposed with the influence of noise [4]. Taking phase error correction into consideration, the author solved the problem of multi-target localization under the circumstances of phase mismatch [5]. The method achieves positioning of multiple targets by estimating alternately the reserve reflection coefficient and phase difference. A method that paired target distances with Doppler frequency estimates in different observation channels to estimate the target position is proposed [6]. The research on the accuracy of the target position estimation needs to be further developed.

In order to enhance the performance multiple targets location, an improved localization algorithm based on fuzzy C clustering and Kalman filtering is proposed. Fuzzy C clustering can form new clusters from data that have similarity to existing categories and process position parameter estimation on information fusion. Kalman filtering estimates the state of a dynamic system from a series of incomplete and noise measurement. The algorithm achieves multi-target positioning and solves the situation where two adjacent targets appear in the air.

## 2 Radar System Model

The signal model focuses on multiple targets  $(x_q, y_q, z_q)$  ( $q = 1, 2, \dots, N$ ) in the far field of transmitting and receiving antenna. Figure 1 illustrates the model. We express the received signals which originate from the transmitter and are reflected by the target:

$$r_n^{(q)}(t) = \sqrt{E/M \sum_{m=1}^M \zeta \rho_{mn}(X_q) s_m(t - \tau_{mn}(X_q))} + w_n(t) \quad (1)$$

Where,  $\zeta$  represents coincident reflectivity,  $w_n(t)$  is Gaussian white noise, and  $f_c$  is carrier frequency.

The distance between the transmitting antenna and the target and the distance between the receiving antenna and the target can be written as follows:

$$\begin{aligned} d(T_m, X_q) &= \sqrt{(x_{tm} - x_q)^2 + (y_{tm} - y_q)^2 + (z_{tm} - z_q)^2} \\ d(R_n, X_q) &= \sqrt{(x_{rn} - x_q)^2 + (y_{rn} - y_q)^2 + (z_{rn} - z_q)^2} \end{aligned} \quad (2)$$

Where,  $\tau_{tm}(x_q)$  represents propagation delay of the signal from transmitting antenna to target,  $\tau_{rn}(X_q)$  is the delay of signal from target to receive antenna, as follows:

$$\tau_{mn}(X_q) = \tau_{tm}(X_q) + \tau_{rn}(X_q) = d(R_n, X_q)/C + d(T_m, X_q)/C$$

$$\rho_{mn}(X_q) = \exp(-j2\pi f_c \tau_{mn}(X_q)) \tag{3}$$

### 3 Target Location Method

In the proposed method, it estimates the position of target using maximum likelihood. Where,  $X_q = [x_q, y_q, z_q]^T$  represents the position of target and we can puts the unknown parameters into the vector  $\theta = [x, y, z, \zeta_{q1}, \zeta_{q2}]^T$ . The Joint Probability Density Function of observation is determined by the vector, as follows:

$$p(r; \theta) = \exp\left\{-\frac{1}{\sigma^2} \sum_{n=1}^N \int_T |r_n(t) - \zeta_q E/M \sum_{m=1}^M \rho_{mn}(X_q) s_m(t - \tau_{mn}(X_q))|^2 dt\right\}$$

$$\hat{\theta}_{ML} = \arg\{max[lgp(r; \theta)]\} = \arg\{max\{max[lgp(r; X_q, \zeta_{q1}, \zeta_{q2})]\}\} \tag{4}$$

Where,  $(.)^*$  represents conjugate, maximum likelihood estimation is expressed as:

$$\hat{X}_{qML} = \arg\left\{max\left[\sum_{m=1}^M \sum_{n=1}^N \rho_{mn}^*(X_q) \int r_n(t) s_m^*(t - \tau_{mn}(X_q)) dt\right]\right\} \tag{5}$$

Where, the coordinate  $X_c = (\hat{x}_c, \hat{y}_c, \hat{z}_c)$  of the target position parameter is obtained.

The target position estimation parameters obtained are placed in the set  $C_k^{Q \times G}$ . The number of targets is used as the number of clusters  $c$ , and the state prediction value of target is a center of the class, and the other collections under the same condition.

The state equation to find the classification center at the time of  $k$  is:

$$X_q(k) = A_q X_q(k-1) + B_q u_q(k-1) + w_q(k) \tag{6}$$

The measurement equation of the moment by the formula is written as:

$$z_q(k) = H_q X_q(k) + v_q(k) \tag{7}$$

Where,  $X_q(k)$  represents the state of target  $q$  at the moment of  $k$ ,  $A_q$  is the state transition matrix,  $w_q(k)$  represents the system noise vector,  $H_q$  is the observation matrix,  $I_q$  represents the identity matrix,  $u_q(k), v_q(k)$  represent the process and measurement noise respectively. The covariance  $Q_q(k)$  and  $R_q(k)$  are Gaussian white noise. Where,  $K_q(k)$  is the Kalman gain matrix, and  $\hat{X}_q(k+1)$  is the best filter value.

The state prediction equation of target is obtained by Kalman filtering as follows:

$$\hat{X}_q(k+1) = \hat{X}_q(\bar{k}) + K_q(k) (z_q(k) - H_q \hat{X}_q(\bar{k}))$$

$$v_q(k) = X_q(k) - H_q \hat{X}_q(\bar{k})$$

$$\begin{aligned}
 P_q(k) &= (I_q - K_q(k)H_q)P_q(\bar{k}) \\
 S_q(k+1) &= H_qP_q(\bar{k})H_q^T + R_q(k)
 \end{aligned}
 \tag{8}$$

Where,  $v_q(k)$  is innovation and  $P_q(k)$  represents filter covariance,  $S_q(k+1)$  is innovation of covariance matrix. The state prediction of target is the classification center.

The Mahalanobis distance formula calculate the distance between all target position parameter estimations and the classification center. The formula is given by:

$$d_q^2 = v_q(k)'S_q(k)v_q(k) \tag{9}$$

If the distance between the position parameter and target state predicted value is less than or equal to a threshold, then it is the effective measured value. Otherwise, it was rounded off. Effective position parameter estimation  $Y_l(l = 1, 2, \dots, L)$  is judged by the threshold. We can put it into the other set and divide the data into class  $C(C = Q)$ . Where,  $P = \{P_1, \dots, P_c, \dots, P_C\}$  is defined as cluster center. The data belongs to a class and has a membership degree of  $u_{cl}$  we find that the objective function is:

$$J(W, P) = \sum_{l=1}^L \sum_{c=1}^C (u_{cl})^{m_2} \|Y_l - P_c\|^2 \tag{10}$$

Where,  $m_2$  is the fuzzy weighted index. When  $J(W, P)$  taking the minimum, using the Lagrange multipliers method can be written in the following form as:

$$u_{cl} = \left[ \sum_{c_2=1}^C \left( \|Y_l - P_{c_1}\|^2 / \|Y_l - P_{c_2}\|^2 \right)^{\frac{2}{m_2-1}} \right]^{-1} \tag{11}$$

Iteration stops until the threshold reached, and the range is  $1.5 \leq m_2 \leq 2.5$ . We put target location parameters into a collection as  $Q = \{Z_1, Z_2, \dots, Z_Q\}$ , they are averaged:

$$z_q^n = 1/N_1 \sum_{n_1=1}^{N_1} z_{qn_1}^n (n = 1, 2, \dots, N) \tag{12}$$

Where,  $z_{qn_1}^n$  represents the estimation of all position parameters of the receiving antenna for target, and  $N_1$  is the number of position parameter estimation.

Data fusion processes for  $Z_1, Z_2, \dots, Z_Q$ . Assume that the observation error is Gaussian white noise. The position parameters estimated of linear fusion are expressed by:

$$Z_{qf} = f_1Z_q^1 + f_2Z_q^2 + \dots + f_RZ_q^R \left( f_j = \frac{1/\sigma_n^2}{\sum_{n=1}^N 1/\sigma_n^2} \right) \tag{13}$$

Kalman filtering filtered it for the target by means of  $Z_{qf}$ , and the state value based on the fusion measurement is obtained. Where the state is estimated as:

$$Z_{qf}(k) = AZ_{qf}(k - 1) + Bu_{qf}(k - 1) + w_{qf}(k) = H_{qf}Z_{qf}(k) + v_{qf}(k) \quad (14)$$

Where,  $A_{qf}$  is the state transition matrix of the system,  $w_{qf}(k)$  represents the system noise vector, and  $H_{qf}$  is the observation matrix,  $u_{qf}(k)$  is the process noise, and  $v_{qf}(k)$  is the measurement noise,  $Q_{qf}(k)$  and  $R_{qf}(k)$  are the Gaussian white noise.

Where,  $P_{qf}(k)$  is Filter covariance,  $K_{qf}(k)$  represents Kalman gain,  $P_{qf}(\bar{k})$  is the prediction error covariance,  $\hat{Z}_{qf}(k + 1)$  is Optimal Filter, the vectors are expressed as:

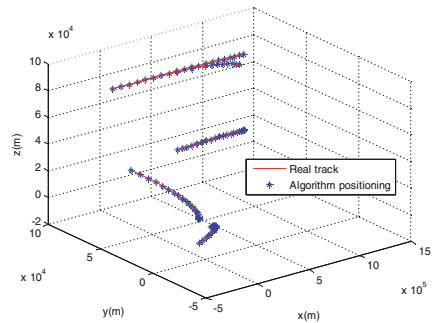
$$\begin{aligned}
 P_{qf}(k) &= (I_{qf} - K_{qf}(k)H_{qf})P_{qf}(\bar{k}) \\
 K_{qf}(k) &= P_{qf}(\bar{k})H_{qf}^T \left[ H_{qf}P_{qf}(\bar{k})H_{qf}^T + \sigma_f^2 I_{qf} \right]^{-1} \\
 v_{qf}(k) &= Z_{qf}(k) - H_{qf}\hat{Z}_{qf}(\bar{k}) \\
 P_{qf}(\bar{k}) &= A_{qf}P_{qf}(k - 1)A_{qf}^T + Q_{qf}(k) \\
 \hat{Z}_{qf}(k + 1) &= \hat{Z}_{qf}(k) + K_{qf}(k) \left[ \left( z_{qf}(k) - H_{qf}\hat{Z}_{qf}(k) \right) \right]
 \end{aligned} \quad (15)$$

### 4 Simulation and Analysis

This section presents simulations to verify the accuracy and resolution performance of the proposed approach. It is assumed that there are three pairs of transmit/receive antennas that they are both time and space aligned. The sampling period is 1 s. There are 5 targets with different initial positions, speeds, and accelerations. The process noise and measurement noise are Gaussian white noise, the signal-to-noise ratio is 10 dB, and the fuzzy weight is two.

The target real motion trajectory and algorithm estimation points after Monte Carlo experiments are shown in Fig. 1. The trace is the target estimate of the improved algorithm. The position estimation point of the algorithm has a high coincidence degree with the target motion curve, so the positioning of maneuvering target can be achieved. It can be seen that there is no disorder in the algorithm which solves the problem of positioning of adjacent target.

Assuming that the signal-to-noise ratio of the simulation environment is 10 dB, in the 0–50 s time period, the root mean square error of the estimated position of the algorithm is obtained. After Monte Carlo experiments, Fig. 2 is an RMSE plot of the



**Fig. 1.** Estimate target plots of improved algorithm

proposed algorithm for estimating multiple target locations at different times. As the motion time of target increased in the airspace, the root mean square error of target parameter estimation is increased, and the positioning effect is gradually deteriorated.

Setting the signal-to-noise ratio range to 0–24 dB when the time  $t = 1$  s,  $t = 2$  s,  $t = 3$  s and  $t = 4$  s in motion, the root mean square error of target 1, target 2, target 3 and target 4 at different time is obtained by simulation. In Fig. 3, with the signal-to-noise ratio increasing, the positioning performance is improved continuously. When the signal-to-noise ratio range to 0–10 dB, it has a great influence on the positioning performance of the algorithm. The algorithm is less affected relatively by it when the signal-to-noise ratio is 0 dB. When the signal-to-noise ratio is constant, the positioning performance decreased with the extension of the target motion time.

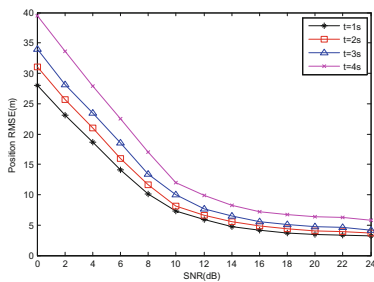


Fig. 2. The root mean square error

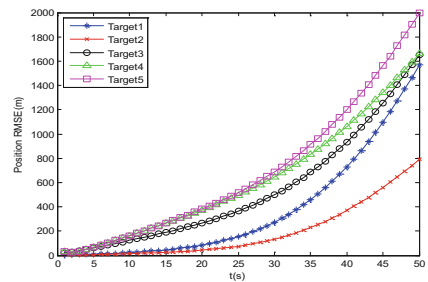


Fig. 3. Position effect under different SNR

## 5 Conclusion

In this paper, we have proposed an improved multi-target locating method in MIMO radar. Firstly, the maximum likelihood estimation algorithm is used to process the echo to obtain the target position parameter estimation. Then the iterative method is used to solve the ML equation to obtain the position parameter estimation. The localization algorithm based on fuzzy C cluster and Kalman filtering obtains the target parameter estimation. The algorithm is simulated by MATLAB software. The effects of SNR, receiving/transmitting antenna and target motion time on estimation accuracy are analyzed. The simulation results show that the proposed algorithm can locate multi-target and improved effectively the resolution ratio of positioning adjacent targets in the airspace.

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