A Low-Complexity Channel Estimation Method Based on Subspace for Large-Scale MIMO Systems

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Abstract. In large-scale multiple-input multiple-output (LS-MIMO) systems, singular value decomposition (SVD) or eigenvalue decomposition (EVD) are common channel estimation schemes. However, the computational complexity of two estimators limits the application in LS-MIMO systems. Motivated by this, in order to reduce the complexity, a novel method that combines fast single compensation approximated power iteration (FSCAPI) algorithm with iterative least square with projection (ILSP), FSCAPI-ILSP, is proposed in this paper, In the proposed method, the received signals subspace is estimated by the FSCAPI algorithm firstly, then the initial channel estimation is obtained by the pilot signals. Finally, we combine it with the ILSP algorithm to improve the accuracy of the channel estimation. Compared with the conventional methods, the proposed scheme degrades the computational complexity significantly. Simulated results indicate the provided method is better than its counterparts and improves the channel estimation accuracy effectively.

Keywords: Channel estimation · Semi-blind · Large scale MIMO · Subspace tracking

1 Introduction

Due to high demand for speed and spectrum utilization in the next-generation communication systems, the concept of large scale multiple-input multiple-out (LS-MIMO) system which is also called massive MIMO has been deemed as an important approach

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for future communication system networks [1–3]. In such systems, at the base station (BS), there is large number of antennas equipped, which is able to serve tens of users in the same frequency band simultaneously [4–7]. Therefore, it achieves higher spectral efficiency [8] and significantly improves the capacity and reliability of wireless systems. LS-MIMO technology can significantly degrade the transmit power, and enhance the power efficiency. For the LS-MIMO system, the channel vectors are superiority of LS-MIMO is reflected with the antennas increase, channel vectors are approximately orthogonal under rich scattering environments. In addition, as the number of antennas at the BS increases, the influences caused by noise and the fast fading tend to be negligible [9].

To obtain higher performance and reduce pilot overhead [10], the orthogonal pilot symbols are restricted due to the finite coherence time. Thus, adjacent cells typically use non-orthogonal or reuse the same orthogonal pilot sequences for channel estimation, which causes pilot contamination [11] However, the imperfect channel state information (CSI) will degrade the accuracy of channel estimation. Numerous papers have been given that accurate CSI determines the quality of LS-MIMO systems. Therefore, it is vitally important for such systems to obtain high-accuracy channel estimation [12–15]. However, in LS-MIMO systems, the channels need to be estimated may be excessive. Consequently, effective channel estimation methods for reducing the complexity are urgently required.

Many methods have been proposed for resolving the problem in channel estimation. Specifically, in [16, 17] compressive sensing method was introduced to estimate the channel to degrade the complexity. In [18], the authors proposed a method based on EVD, which assumed that channel vectors are perfectly orthogonal, thus each channel vector can be uniquely characterized by an eigenvector having at most a multiplicative scalar ambiguity, which will be resolved with a few pilot sequences. But the channel vectors are only approximately orthogonal in practice. In [19], the authors proposed the SVD-based scheme, and the analysis showed that the inter-cell interference (ICI) can be completely eliminated with an infinite antennas and data symbols, but there is still a residual ICI. Therefore, semi-blind channel estimation (SBCE) is able to substantially degrading ICI impact in practical LS-MIMO systems require further investigation. Unfortunately, the disadvantage of the EVD and SVD is that the computational complexity is too high, which is $O(M^3)$, M represents the quantity of the antennas. So as M increase, it is not practical to use the two methods for LS-MIMO systems.

In this paper, we concentrate on mitigating the computational complexity and further improving the estimation accuracy, a low-complexity subspace adaptive SBCE method is introduced for LS-MIMO, which is comprised two parts designed to degrade the complexity and improve the performance of channel estimation, respectively. Primary contributions of each part are summarized as below:

(1) In [20], to carry out pilot decontamination, the received signal is projected into the subspace. It was indicated that pilot contamination is able to degrade effectively by means of subspace projection when the data sequences length and the BS antennas are enough large. Moreover, the number of dimensions of the received signals is degraded by the subspace projection processing. However, to obtain the signal subspace, the SVD procedure is required on a high-dimensional matrix,

which is consisted of all the received symbols. Therefore, we should find a better computationally effective method to determine the dominant subspace. our proposed scheme estimates the received signals subspace by using the FSCAPI algorithm [21], which has quickly convergence and better signal subspace tracking performance, compared to that of [18, 19], it can achieve lower computational complexity and accelerate the estimation of signal subspace. In [21], to obtain the signal subspace faster, the FSCAPI algorithm is proposed, which is referred as paper [19]. Compared to latter, the former has better performance in computational complexity and tracking.

(2) Having obtained the signal subspace, an initial estimation can be got with short uplink pilot symbols, however, there will be a deviation which caused by finite sample data rather than real data. Therefore, to degrade the estimation error, we combine it with the ILSP signal detection algorithm [22] to improve the performance. The procedures of the ILSP algorithm are summarized as below. Firstly, it takes use of the obtained initial channel estimate which has discussed above to carry out data detection, and then, the detected data are used to re-estimate the channel. Considering single-cell systems performance is the upper bound of the multi-cell systems [17] in LS-MIMO systems with multi-user. So, in this paper, we mainly research the single-cell system.

The remainder of the paper is consisted as follows: Sect. 2 presents the system model. Section 3 describes the proposed subspace-based low-complexity SBCE method and extends complexity analysis. In Sect. 4, we establish and discuss the simulation results. Finally, the conclusion is derived in Sect. 5.

Notation: In this paper, vectors (matrices) are represented as lower-case (upper-case) boldface letters. Superscripts $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$, and $(\cdot)^+$ are the conjugate, transpose, Hermitian, and the Moore-Penrose inversion operators, respectively. $\mathbf{I}_M$ is the size $M \times M$ identity matrix, $[\cdot]_i$ denotes matrix i-th column, the (i, j)-th element of a matrix is representing by $[\cdot]_{i,j}$, the Euclidean norm of a vector is $\|\cdot\|$, the operator $\mathbf{E}\{\cdot\}$ is the expectation.

2 System Model

Given a typical single-cell LS-MIMO system with multi-user shares the same frequency band, considering a frequency-flat fading uplink transmission in a communication system works at mechanism called time-division duplex (TDD). Thus downlink (DL) channels are able to obtain by transposing the uplink (UL) channels because of the reciprocity. At the BS, there are M antennas, and K users each equipped single antenna are served simultaneously, usually $K \ll M$ is assumed. Then the uplink received signal vector $\mathbf{r} \in \mathrm{C}^{M \times 1}$ is

$$\mathbf{r} = \sqrt{p_u}\mathbf{G}\mathbf{s} + \mathbf{w}, \tag{1}$$

the uplink channel matrix represents by $\mathbf{G} \in \mathrm{C}^{M \times K}$, between user k and antenna m, the channel coefficient is represented by $g_{m,k} = [\mathbf{G}]_{m,k}$ The elements of $\mathbf{G}$ are considered to be independent and identically distributed (i.i.d.), and circularly-symmetric Gaussian random variables with zero mean and unit variance ($CN(0, 1)$), $\mathbf{s} \in \mathrm{C}^{K \times 1}$ is transmitted symbols (p_{u} is the signal-to-noise ratio(SNR)), the additive white Gaussian noise (AWGN) is $\mathbf{w} \in \mathrm{C}^{M \times 1}$ and satisfies $CN(0, \mathbf{I})$ distribution at the BS.

3 Subspace-Based Low-Complexity Channel Estimation

In this part, based on FSCAPI-ILSP, we present a subspace-based low-complexity SBCE scheme for multiuser LS-MIMO systems. Specifically, the proposed algorithm is mainly composed of two steps. In the first step, the received vectors subspace can be obtained by using FSCAPI algorithm to degrade the computational complexity, and then the initial channel estimation matrix is able to be obtained. In the second stage, to further obtain better performance, ILSP algorithm is combined. The details of the proposed scheme are presented in the following.

3.1 Problem Formulation and Resolve

The SBCE fully exploits both the pilots and the data symbols, thus it outperforms the totally blind channel estimator. According to large numbers theorem, as antennas at the BS are very big, for example as $M \to \infty$, the channel vectors will become asymptotically orthogonal between the BS and users, i.e. $(1/M)\mathbf{G}^{\mathrm{H}}\mathbf{G} \to \mathbf{I}_k$. This is a vitally feature in LS-MIMO systems. It is helpful to analyze subspace-based low-complexity SBCE. In particular, the subspace-based SBCE exploits the received signals characteristic, which is able to achieve better tradeoff between performance and complexity. The received signal $\mathbf{r}$ covariance matrix is computed as

$$\mathbf{cov}_r = \mathrm{E}\{\mathbf{r}\mathbf{r}^{\mathrm{H}}\} \tag{2}$$

However, in practical, the sample covariance matrix is usually substituted by Eq. (3). Therefore, the estimation of the covariance matrix $\hat{\mathbf{C}}_r$ in (3) will be approximated as

$$\hat{\mathbf{C}}_r = \frac{1}{N}\sum_{t=1}^{N}[\mathbf{r}(t)][\mathbf{r}(t)]^{\mathrm{H}} \tag{3}$$

As N tends to infinity, the matrix $\hat{\mathbf{C}}_r$ will gradually converge to the real. Since $\hat{\mathbf{C}}_r$ is a Hermitian matrix, and its SVD is given by $\hat{\mathbf{C}}_r = \mathbf{R}\Sigma\mathbf{R}^{\mathrm{H}}$, where the column vectors of $\mathbf{R} \in \mathrm{C}^{M \times M}$ are the singular vectors of $\hat{\mathbf{C}}_r$, $\mathbf{\Sigma} \in \mathrm{C}^{M \times M}$ represents a diagonal matrix, which elements are consisted of the M singular values. More specifically, $\mathbf{R}$ can be divided into $\mathbf{R} = [\mathbf{R}_s, \mathbf{R}_n]$, where $\mathbf{R}_s \in \mathrm{C}^{M \times K}$ represents the signal subspace, the noise subspace is represented by $\mathbf{R}_n \in \mathrm{C}^{M \times (M-K)}$. In [19], we can know that $\mathbf{R}_s$ is the largest K singular value that correspond to the singular vectors of $\hat{\mathbf{C}}_r$, which can be used to estimate $\mathbf{G}$, the corresponding channel estimation [24] is

$$\tilde{\mathbf{G}} = \mathbf{R}_s \mathbf{E}_j, \tag{4}$$

where $\mathbf{E}_j$ is considered as the ambiguity matrix. Thus as long as $\mathbf{E}_j$ is known, the channel matrix $\mathbf{G}$ can be estimated from $\mathbf{R}_s$. Therefore, the short uplink pilots are invoked for resolving the ambiguity matrix $\mathbf{E}_j$. At the BS, the received pilot symbols $\mathbf{Y}^{\mathrm{p}} \in \mathrm{C}^{M \times N\mathrm{p}}$ can be expressed as

$$\mathbf{Y}^{\mathrm{p}} = \sqrt{p_{\mathrm{t}}}\mathbf{G}\,\mathbf{\Phi} + \mathbf{N}, \tag{5}$$

where pilot matrix $\mathbf{\Phi} \in \mathrm{C}^{K \times N\mathrm{p}}$ satisfies $\mathbf{\Phi}\mathbf{\Phi}^{\mathrm{H}} = \mathbf{I}_k$, and N_{p} is the pilots sequences length, which are transmitted by users. p_{t} is the transmit power, noise matrix $\mathbf{N} \in \mathrm{C}^{M \times N\mathrm{p}}$ follows i.i.d. $CN(0, 1)$. According to (5), $\mathbf{G}$ can be estimated by invoking $\mathbf{Y}^{\mathrm{p}}$ as

$$\hat{\mathbf{G}}^{\mathrm{p}} = \frac{1}{\sqrt{p_{\mathrm{t}}}}\mathbf{Y}^{\mathrm{p}}\mathbf{\Phi}^{\mathrm{H}}. \tag{6}$$

For resolving the ambiguity matrix $\mathbf{E}_j$ in the signal subspace $\mathbf{R}_s$, based on the estimation channel with the aid of pilot, we can address the ambiguity matrix as

$$\mathbf{E}_j = (\mathbf{R}_s)^{\mathrm{H}}\hat{\mathbf{G}}^{\mathrm{p}}. \tag{7}$$

Thus, from Eqs. (4) and (7), the corresponding channel estimation is

$$\tilde{\mathbf{G}} = \mathbf{R}_s\mathbf{E}_j = \mathbf{R}_s(\mathbf{R}_s)^{\mathrm{H}}\hat{\mathbf{G}}^{\mathrm{p}}. \tag{8}$$

From what has discussed above, it can be seen that a key factor is to get the signal subspace estimation from the received signal in the SBCE methods. The commonly used channel estimation methods based on EVD or SVD are not easy to carry out for real-time application in practical systems due to high computational complexity.

3.2 The Proposed Method and Performance Analysis

For the purpose of obtaining the estimation of the signal subspace faster and reduce the deviation caused by using finite sample data rather than real data, we propose a method that combines the ILSP signal detection algorithm with the FSCAPI algorithm to reduce computational complexity and improve channel estimation accuracy. In this part, the complexity of the FSCAPI-ILSP is analyzed. Additionally, the proposed method is compared to other SBCEs. In [21, 23], the authors obtain the signal subspace estimation of the received signal by using FSCAPI algorithm, which will adopt in this paper.

As discussed above, because the covariance matrix is made of the finite sample data rather than real data, there must be a deviation. Thus, joint FSCAPI-based method and ILSP method is introduced. Defining $K \times N$ dimensional matrix $\mathbf{S}_l$ is the data signals sent by K users to the BS, and channel $\mathbf{G}_l$ is $M \times K$-dimensional matrix, $\mathbf{W}_l$ is the noise matrix, so the received signal $\mathbf{Y}_l$ is expressed as

$$\mathbf{Y}_l = \sqrt{p_u}\mathbf{G}_l\mathbf{S}_l + \mathbf{W}_l \tag{9}$$

where $\mathbf{Y}_l$ = [$\mathbf{r}(1)$, $\mathbf{r}(2)$, …, $\mathbf{r}(N)$], $\mathbf{S}_l$= [$\mathbf{s}(1)$, $\mathbf{s}(2)$, …, $\mathbf{s}(N)$], $\mathbf{W}_l$= [$\mathbf{w}(1)$, $\mathbf{w}(2)$, …, $\mathbf{w}(N)$]. The operation procedures of ILSP method is introduced as below. First of all, assuming the channel $\mathbf{G}_l$ is estimated from the initial procedure by using FSCAPI, and next the data are detected by using least-squares method, and the result can be written as

$$\tilde{\mathbf{S}}_l = \arg\min_{S_l \in \chi} \left\| \frac{1}{\sqrt{p_u}} \tilde{\mathbf{G}}_l^+ \mathbf{Y}_l - \mathbf{S}_l \right\|_F^2 \tag{10}$$

The values of the transmitted signal is represented by the set χ, next, the detected signals $\tilde{\mathrm{S}}_l$ are considered as the real transmitted signals and used to re-estimated the channel via least-squares, so the estimation of the channel matrix is

$$\tilde{\mathbf{G}}_l = \frac{1}{\sqrt{p_u}} \mathbf{Y}_l \tilde{\mathbf{S}}_l^+ \tag{11}$$

From Eqs. (10) and (11), our problem can be solved by using the ILSP algorithm [22]. And making use of the initial estimation channel by FSCAPI scheme, the joint FSCAPI and ILSP algorithm is proposed. The primary procedures of the proposed algorithm are summarized in **Algorithm 1**.

3.3 Analysis of Computational Complexity

Both EVD and SVD algorithms are requiring NM^2 complex multiplication when calculating the auto-correlation matrix of received signal samples. If the dimension of the matrix is $M \times M$, calculating the accurate EVD and SVD, the complex multiplications is needed $(4/3)M^3$ and $11M^3$ separately, and the SBCE algorithm in [18] needs $8MK^2N_\mathrm{p} + (4N_\mathrm{p} + 1)MK + O(K^3)$ complex multiplications to calculate the ambiguity matrix. MK complex multiplications are also required when calculate final estimation of $\mathbf{G}$. In [19], to get pilot-based channel estimation, the SBCE algorithm based on SVD demands $MKN_\mathrm{p} + MK$ complex multiplications, and it also requires $2MK^2$ complex multiplications to get the estimation of $\mathbf{G}$. The computational complexity in [11] is primarily depended on SVD, LS algorithm and the iteration times. In Table 1, it is shown the different complexity about various estimation schemes.

Algorithm 1 The FSCAPI-ILSP Based Channel estimation algorithm

Step 1) Calculate the estimation channel based on pilot using (6).
Step 2) Calculate received signals subspace.
For $t = 1,2,\ldots,N$
Given: $\mathbf{r}(t)$.
Using the FSCAPI Algorithm in [24] to obtain the $\mathbf{\Lambda}$(t)
End for
Step 3) Let $\mathbf{R}_s = \mathbf{\Lambda}(t)$, resolve the ambiguity matrix $\mathbf{E}_j$ using (7)
Step 4) Obtain the initial channel estimation matrix using (8)
Step 5) Initialize, choose number of iterations K_{step}, assume $k = 0$,
For: k+=1

$$\tilde{\boldsymbol{S}}_{l,k} = \arg\min_{\boldsymbol{S}_l \in \chi} \left\| \frac{1}{\sqrt{\mathrm{p_u}}} \tilde{\mathbf{G}}_{l,k}^{+} \mathbf{Y}_l - \boldsymbol{S}_l \right\|_F^2$$

$$\tilde{\mathbf{G}}_{l,k} = \frac{1}{\sqrt{\mathrm{p_u}}} \mathbf{Y}_l \tilde{\boldsymbol{S}}_{l,k}^{+}$$

Repeat until k = K_{step}

Table 1. Computational complexity compared in different algorithms

Algorithm	Complexity
SVD-ILSP [11]	$K_{\text{step}}(MK(N_{\text{p}} + 1)) + 11M^3 + NM^2 + 2MK^2 + MKN_{\text{p}} + MK$
EVD-BASED [18]	$(4/3)M^3 + NM^2 + 8MK^2N_{\text{p}} + (4N_{\text{p}} + 2)MK + O(K^3)$
SVD-BASED [19]	$11M^3 + NM^2 + 2MK^2 + MKN_{\text{p}} + MK$
FSCAPI-BASED [23]	$2M^2K + MK(N_{\text{p}} + 1) + 3(MK + K^2) + 2M + 6K$
PROPOSED	$K_{\text{step}}(MK(N_{\text{p}} + 1)) + 2M^2K + MK(N_{\text{p}} + 1) + 3(MK + K^2) + 2M + 6K$

In Table 1, the complexity is compared in different Algorithms with the proposed scheme by taking the length of pilot N_{p} into account. Due to the BS antennas M are much more than the users K in LS-MIMO systems usually, thus the proposed scheme enjoys lower complexity than others methods

4 Numerical Results

The Monte Carlo simulation is conducted to analyze the proposed scheme performance in large-scale multi-user MIMO system, only single-cell system is considered. The normalized mean square error (NMSE) is regarded as the performance metric is formulated as

$$NMSE = \frac{tr\{(\tilde{\mathbf{G}} - \mathbf{G})^H (\tilde{\mathbf{G}} - \mathbf{G})\}}{MK}. \tag{12}$$

A comparative analysis of channel estimation performance is performed under various signal-to-noise ratios (SNR) and different antenna numbers, respectively. Assuming the terminals are $K = 10$ in each cell, and $\mu = 0.995$ is the value of forgetting factor, pilot symbols are set to $N_p = K$ of each user. BPSK modulation is used for the transmission of data symbols.

Figure 1 shows the NMSE performance versus the received SNR under various channel estimation schemes, where the BS antennas are assumed to be $M = 100$, and the samples are assumed to be $N = 100$. As we can see, the performance of the EVD estimator almost unchanged with the SNR is increasing, that is because the ambiguity matrix is approximates assumed to be a diagonal matrix, but in real conditional, this assumption is invalid. As the simulation results shown, we can conclude that the proposed scheme performance is nearly same with the SVD-ILSP estimator, and they also show a tendency to decrease linearly, and also outperform than others estimators.

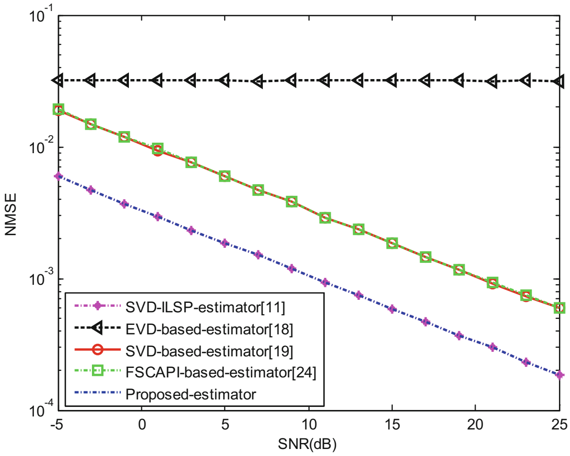

Fig. 1. The performance of NMSE for different schemes versus SNRs with $M = 100$, $N = 100$ are compared.

Figure 2 depicts different channel estimators performance with SNR = 20 dB versus different BS antennas. It shows that the performance of the NMSE in all methods are degrading with the antennas increasing, and it further proves the proposed scheme is better than SVD, EVD and FSCAPI schemes versus different antennas. But on the other hand, we can observe that with BS antennas are increasing, the NMSE reduction is not so obviously, the main reason of this phenomenon is caused by the large-scale fading coefficient of the users.

Figure 3 compares the computational complexity of different algorithms versus number of receive antennas with the samples set $N = 100$, the pilot symbols are setting

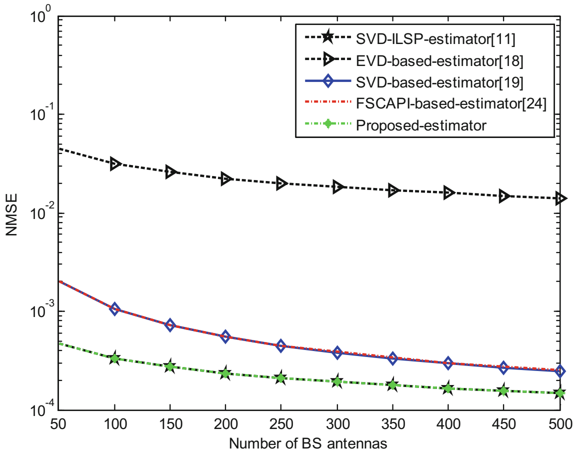

Fig. 2. NMSE comparison with different methods versus antennas with the fixed SNR = 20 dB.

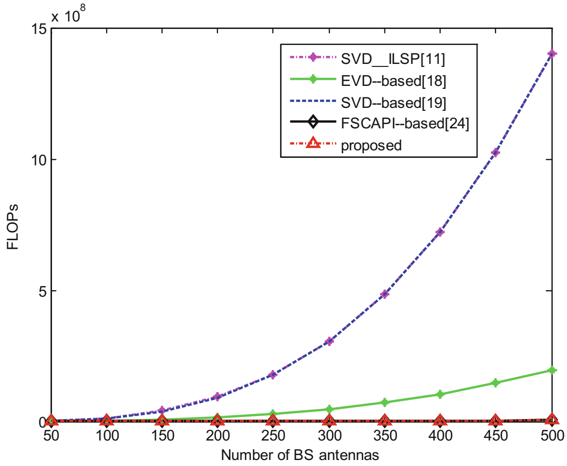

Fig. 3. Comparison of FLOPs in different algorithms versus receive antennas M, with $N_P = K$=10, N = 100, K_{step} = 5

to $N_p = K$=10 of each user, the number of iterations K_{step} = 5. It is observed that the proposed scheme is much lower complexity than SVD-based and SVD-ILSP, especially when M is increasing, the computational complexities of the latter two schemes are too high while the proposed scheme is slightly increased. Therefore, our proposed scheme is less computational complexity as compared with other schemes.

5 Conclusion

In this paper, a subspace-based adaptive SBCE scheme is introduced to reduce the computational complexity and improve the accuracy of LS-MIMO multi-user systems. The proposed scheme estimates the column space of the channel matrix firstly, which is relying on the FSCAPI algorithm to obtain the received vectors subspace and it requires lower computational complexity than EVD or SVD decomposition. And then, the channel matrix is estimated with the aid of a short uplink pilot. Finally, the ILSP algorithm is used to increase the accuracy of the estimation. Simulation results have shown that compared with other schemes, the proposed approach exhibits a better performance significantly, and reduces the computational complexity effectively in LS-MIMO system.

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