



# A Novel AoA Estimation Algorithm Based on Phase Compensation of Linear Array

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**Abstract.** This paper presents a novel algorithm for angle of arrive (AoA) estimation by employing the phase of received signal. First, the phase of the received signal is compensated in all directions. Then, the AoA is estimated by evaluating the fluctuation of the compensated phases from sensors of the array. Meanwhile, since matrix decomposition is not required, the complexity is greatly reduced compared to the conventional methods. Our implementation and evaluation on commodity WiFi devices demonstrate that the proposed algorithm achieves better or comparable performance to SpotFi. In terms of estimation accuracy, the proposed algorithm can estimate the incident angle of the multipath and the coherent signals effectively. In terms of complexity, since matrix decomposition is not required, the complexity is greatly reduced compared to the conventional methods.

**Keywords:** Wi-Fi · Indoor localization · Channel state information · Angle of arrival · Coherent signal

## 1 Introduction

Recent years, the ever-fast development of indoor localization [1–6] has attracted people’s attention. Mainstream indoor positioning techniques are based on ultra wide band (UWB), fingerprinting, radio frequency identification (RFID), MEMS and Wi-Fi. Those positioning techniques based applications are required to be accurate, universal and efficient for meeting user needs and Summary Cognitive radio has attracted considerable attention because of its ability to make full use of the available spectrum resources for wireless terrestrial communication networks [7–9].

The accuracy of ArrayTrack [10], SpotFi [11] and UWB [12] based system is sufficient in most cases. But ArrayTrack needs 16 antennas to overcome the multipath effect which is not universal for modifying the hardware. UWB based system is more real-time and accurate than ArrayTrack and SpotFi while the related

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expensive equipment makes it unattractive. SpotFi uses ToF based approaches and AoA based approaches for joint estimation, which needs matrix decomposition and smoothing to remove the influence of coherent signals and results in SpotFi inefficient and time complex. In sample environment, SpotFi can achieve better estimation accuracy than other methods. However, for complex indoor environment, due to the effect of coherent signal and noise, the estimation accuracy decreased.

In order to reduce the influence of coherent signal on angle of arrival estimation and solving the problem of high algorithm complexity caused by a large number of operations in SpotFi. We present a novel AoA estimation algorithm, which uses phase compensation to solve the coherent signal and algorithm complexity problems of SpotFi effectively.

The three key contributions of this paper are listed as follow:

- (1) The proposed algorithm solves the problem of AoA estimation with respect to the coherent signal and multipath signal effectively.
- (2) The AoA estimation efficiency is improved without any loss of accuracy.
- (3) The time complexity of the algorithm is greatly reduced.

The rest of this paper is organized as follows. The related work about localization technologies is introduced in Sect. 1. The proposed one-dimensional and 2D AoA estimation is described in Sect. 2. Section 3 conducts the experiments and analyzes the results. And Sect. 4 draws the conclusion and provides some future works.

## 2 System Description

In the indoor environment, the signal could arrive at the receiver after multiple reflections, and the arriving directions of the signals can be estimated by considering the phase shift. Our system makes full use of the phase difference to estimate incident angle.

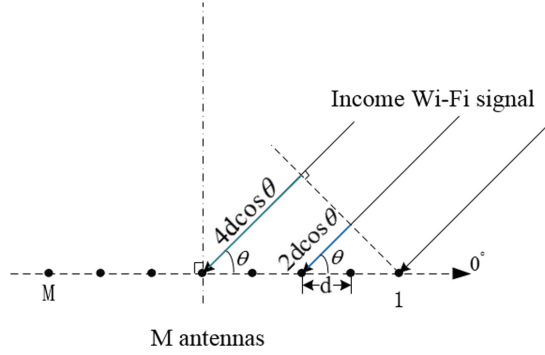
### 2.1 AoA Estimation Based on Linear Array

Let  $H(f)$  be the channel frequency response (CFR) as the signal is influenced in transmission channel with both amplitude and phase, and the amplitude attenuation is attributed to the energy loss during signal propagation while the phase change is caused by ToF of signal. Thus, can be denoted as:

$$H(f) = |H(f)| \exp(-j \times 2\pi \times \sin(\angle H(f))) \quad (1)$$

where  $|H(f)|$  and  $\angle H(f)$  is amplitude and phase of the channel frequency response  $H(f)$  respectively.

More generally, we suppose there are  $M$  antennas placed in a uniform equidistant linear array with equal spacing  $d$  as shown in Fig. 1. For each propagation path, the ToF contains the time proportional to the distance between the transmitter and receiver, and the time is related to incident angle of signal. Let  $\Theta$  be



**Fig. 1.** Uniform linear array diagram

the incident angle of signal corresponding to the normal direction of the linear array and  $\gamma$  be the signal attenuation during path traveling.

Combining with Eq. (1), the signal arriving at first antenna of the array can be expressed as:

$$S_{r_1}(f) = \gamma S(f)_s e^{-j2\pi \times \tau_d \times f} \tag{2}$$

where  $\tau_d$  is the time from the transmitter to the first antenna;  $f$  is the frequency of transmitted signal. Due to the AoA and array spacing, the signal arriving at the second antenna needs to fly another distance about  $d \times \cos \theta$  which results in a phase shift about  $\Phi(\theta) = e^{-j2\pi \times (d \times \cos \theta / c) \times f}$ . Thus the signal received by the second antenna is shown below:

$$S_{r_2}(f) = S_{r_1}(f) e^{-j2\pi \times (d \times \cos \theta / c) \times f} \tag{3}$$

In a uniform line array, the signal phase received from each antenna can be recorded as:

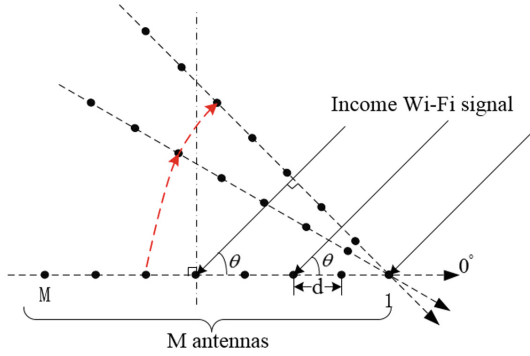
$$P(\theta) = [1 \ \Phi(\theta) \ \dots \ \Phi(\theta)^{M-1}]^T \tag{4}$$

where  $\Phi(\theta) = e^{-j2\pi \times d \times \sin(\theta) \times f / c}$ , and  $c$  is the speed of light and  $f$  is the frequency of the transmitted signal. Considering the first antenna as a reference, we make up the phase difference of the received signal at  $m$ -th antenna by multiplying  $e^{-j2\pi \times [(m-1)d \times \cos(\theta) / c] \times f}$  to the received signal, and the compensation operation can be denoted as:

$$Q(\omega) = [1 \ \Phi(\theta) \ \dots \ \Phi(\theta)^{M-1}]^T \cdot C(\omega) \tag{5}$$

$$C(\omega) = \left[ 1 \ e^{-j2\pi \times d \times \cos(\omega) / c \times f} \ \dots \ e^{-j2\pi \times (M-1)d \times \cos(\omega) / c \times f} \right] \tag{6}$$

where  $\omega$  ranges from  $0^\circ$  to  $180^\circ$ , and  $k$  is the number of each antenna in the linear array. The phase compensation for each antenna is equivalent to compensate the ToF of signal by rotating the linear array in clockwise direction, as shown in Fig. 2.

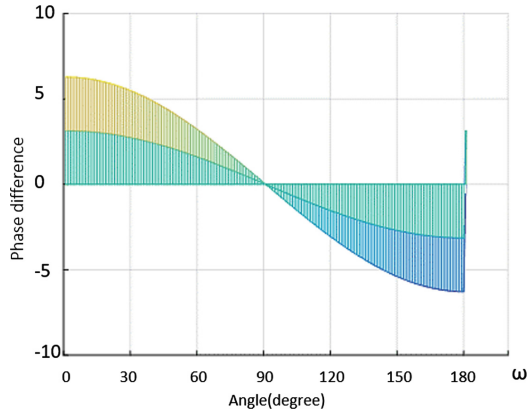


**Fig. 2.** The antenna array rotates clockwise until it is perpendicular to the incident signal, where the phase difference between the incident signal and the different antennas is smallest.

With the angle rotation, the compensated phase on each antenna and the phase difference of any two antennas change. As  $\omega$  approaches to the true incidence angle, the linear array will be gradually perpendicular to the incident signal and the phase differences among different antennas will become smaller. When  $\omega$  equals to the expected AoA, the linear array becomes perpendicular to the signal direction and the ToF at all antennas is the same. Thus the phase shift is formulated into:

$$f(\omega) = 1/\sqrt{\frac{1}{M} \sum_{n=0}^{M-1} (\Phi(\theta)^n \times e^{-j2\pi \times n \times d \times \cos(\omega)/c \times f} - \bar{Q})^2} \quad (7)$$

where  $\bar{Q}$  refers to the mean of elements from  $Q(\omega)$ . For each rotation angle, we use Eq. (7) to measure the dispersion degree of the phase values from all antennas. For instance, we uses 3 antennas to estimate a signal with an angle of arrival of  $90^\circ$ . When the antenna array begins to rotate clockwise, rotation angle  $\omega$  changes from  $0^\circ$  to  $180^\circ$ , the phases dispersion degree of three antennas changes from  $-6$  to  $6$  as shown in Fig. 3. Construct a compensation signal with the same structure as the original compensated signal by using the parameter  $\omega$ , calculate the variance between the compensate signal and the mean of the original compensated signal. We will find that when  $\omega = 90^\circ$ , the signal phase difference between the compensated signal and the original compensated signal reach a minimum value of 0, and the variance of the phase difference between the compensated signal and the original compensated signal is also a minimum value, the reciprocal of the variance  $f(\omega)$  reaches the highest value. Therefore, We can get the angle of arrival according to the angle corresponding to the maximum value and the angle of compensated.



**Fig. 3.** When the phase compensation angle is equal to the angle of incidence, the phase different between all antennas is 0 and the signal phase from all antennas will be the same.

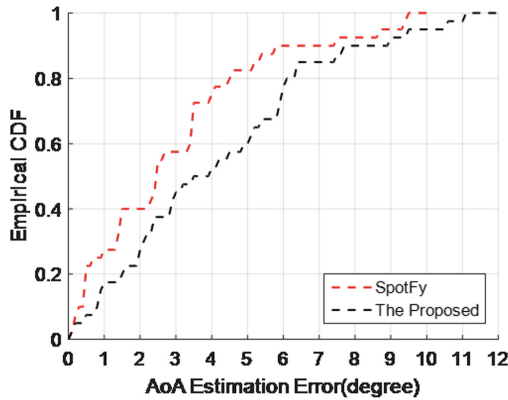
### 3 Experimental Results

In this section, first, we explore the estimation accuracy of proposed algorithm with linear array and compare it with SpotFi. Then, case studies are performed in a typical indoor environment and performance comparisons are made with SpotFi under different parameter settings.

#### 3.1 Experiment Based on Linear Array

We set 4 Access Points (AP) with three antennas at a height of 2 m operating as the signal receivers. The central frequency is 5.2G and we use only 30 of the subcarriers and single carrier modulation with signal noise ratio (SNR)  $\xi = 10$  dB. We compare the performance of proposed AoA estimation algorithm with SpotFi.

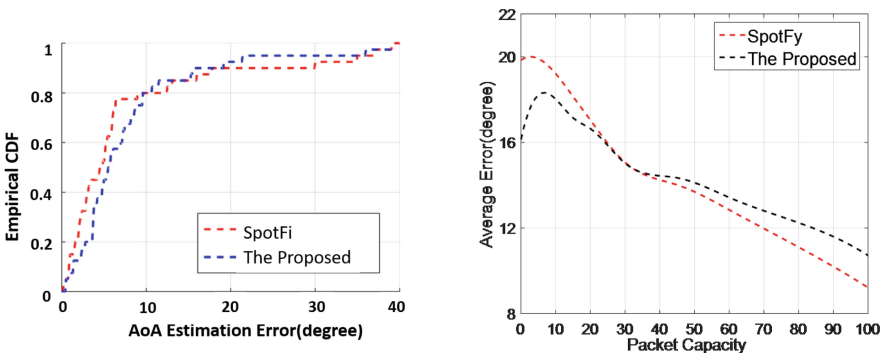
We chose SpotFi with three antennas as the comparison point because among recently proposed designs it is the best performing system which can be deployed without any hardware or firmware modifications at the APs. First, we compare the performance of the proposed algorithm and SpotFi with incoherent signal. As can be seen from Fig. 4, SpotFi is more accurate with error of 60% less than  $4^\circ$  compared to  $5^\circ$  in the same case with the proposed algorithm. Under this circumstance, by phase compensation, the proposed system works effectively without smoothing between carriers although the accuracy of proposed algorithm is slightly lower than SpotFi. For coherent signals, however, its performance is superior to SpotFi. We will explain this in the following experiments. In addition, the proposed algorithm can calculate the incident angles of coherent signals without smoothing, and it only operates on signal phase without matrix decomposition which is the main reason for complexity reduction.



**Fig. 4.** SpotFi is more accurate with error of 60% less than  $4^\circ$  and the proposed algorithm is more accurate with error of 60% about  $5^\circ$  in the same time.

For coherent signal, we conduct case study in a typical indoor environment to verify the capability of the proposed algorithm. There are 3 columns, each of which has 9 test points. 27 test points are uniformly calibrated with every two adjacent points spacing 0.6 m in each column and the distance interval between the neighboring columns of test points is about 3 m.

The results of the experiment are as follows Fig. 5. From Fig. 5(a), we can see that although both SpotFi and the proposed algorithm contain some large estimation errors due to the complex indoor environment, the proposed algorithm can guarantee a satisfactory estimate accuracy. The proposed algorithm is more



(a) The performance of the AoA estimation error (b) The performance of packet capacity error

**Fig. 5.** (a) The performance of the AoA estimation error of SpotFi and the proposed algorithm is compared. (b) The effect of the number of packets on the accuracy of angle estimation is compared, and the proposed algorithm is more accurate with less than 25 packet.

accurate when the AoA estimation error is more than 10. However, as shown in the Fig. 5(a), SpotFi is more accurate than the proposed algorithm when the AoA estimation error is less than 10. Because of the influence of environment and other factors, there will be some errors in the experimental results of the measured data. At the same time, we study the influence of the packet number on the angle estimation accuracy in Fig. 5(b). As can be seen from Fig. 5(b), the proposed algorithm is more accurate with less than 25 packets. As the number of packets increases, the accuracy gradually rises. This is because SpotFi needs to use incoherent data in a large number of packets to perform operations. When the number of packets is small, the useful information in the packets is less so that the estimation accuracy is low. The proposed algorithm can use both coherent and incoherent signals to calculate at the same time, and the estimation accuracy is higher when the number of packets is small. Meanwhile, the proposed algorithm does not need matrix decomposition, thus reducing the complexity of the algorithm.

## 4 Conclusion

In this paper, we present a novel algorithm to estimate the incident signal angle based on phase compensation. By phase compensation for each antenna in the antenna array, the antenna array rotates perpendicular to the incident signal, the phase different between all antennas reaches the lowest and the AoA is estimated. Therefore, the proposed algorithm can deal with multipath and coherent signals effectively, the AoA estimation is more accuracy than SpotFi and the complexity is greatly reduced since there is no complex matrix decomposition and coherent signal processing involved.

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