



# Parameter Estimation of Multiple Satellite Signals Based on Cyclic Spectrum

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**Abstract.** The cyclic spectral density function is a linear transformation that satisfies the principle of linear superposition. Based on this property, a new method for parameter estimation of multiple satellite signals is proposed. Firstly, the linear superposition characteristics of the cyclic spectrum are introduced. On the basis of this, the spectral characteristics of the mixed satellite signals are analyzed. According to the corresponding relationship between the cyclic spectrum line of multi-signals and the signal parameters. Finally, the parameter estimation of mixed signals is achieved indirectly through the method of detecting the line in specific cyclic frequency section of cyclic spectrum. The simulation results show that the new method has good performance under Gaussian noise.

**Keywords:** Satellite signals · Parameter estimation · Cyclic spectrum · Carrier frequency · Symbol rate

## 1 Introduction

Parameter estimation for signals is widely used in civil and military affairs, especially for space electronic reconnaissance, which is one of its core technologies. In the aspect of aerospace electronic reconnaissance application, the modern battlefield environment is complex and changeable, and the complex background environment, such as enemy interference, weather factors, and electronic interference of multi-type weapon equipment, which makes it possible that the intercepted satellite signals are mixed in two or more signals. Most of the existing signal parameter estimation methods are for single signal, which are no longer applicable for multi-signals, the existing methods of parameter estimation for multi-signals include cycle spectrum, high order cumulant, etc. [1–4], cyclic spectrum is not only insensitive to Gaussian noise [5–8], but also has linear superposition characteristics, which can represent the individual characteristics of this mixed signal very well.

The cyclic spectrum of the signal has discrete spectral lines at its cyclic frequency. By using the correspondence between the cyclic frequency and the signal parameters, the parameters of the signal can be estimated by extracting and analyzing the corresponding cyclic spectral line features. In addition, the cyclic spectrum of the mixed signal is equal to the superposition of the cyclic spectrum of each sub-signal component at the corresponding cyclic frequency. If the spectral line characteristics at the

cyclic frequency of the individual signal are analyzed, the other signals are equivalent to noise and will not affect the parameter estimation of the signal.

## 2 The Cyclic Spectrum Characteristics of Mixed Signals

Cyclic statistics are an effective tool for processing cyclostationary signals and have been widely used in signal processing. The cyclic spectrum is one of the important concepts, which can be used to describe the stationary characteristics of signal circulation while effectively suppressing noise. In addition, cyclic spectrum also has superposition and signal selectivity [9].

If multiple signals are independent of each other, then the cyclic spectrum of the mixed signals of these signals is equal to the sum of the individual signal cyclic spectrum. It is known from the definition of the cyclic spectrum that the cyclic spectral density function is a linear transformation, so it satisfies the principle of linear superposition. Taking the mixing of two signals as an example, the cyclic spectrum formula satisfies:

$$S_{ax_1 + bx_2}^\alpha(f) = aS_{x_1}^\alpha(f) + bS_{x_2}^\alpha(f) \quad (1)$$

In the above formula,  $S_x^\alpha(f)$  represents the cyclic spectrum of signal  $x$ ,  $\alpha$  represents the cyclic frequency,  $f$  represents the spectral frequency.

The cyclic spectrum of the signal is not zero only at its own cyclic frequency. Therefore, the cyclic spectrum has signal selectivity, or mixed signals with different cyclic frequencies have separability on the cyclic spectrum. For example, the cyclic frequency of signal  $x_1$  and  $x_2$  are respectively  $\alpha_1$  and  $\alpha_2$ , then:

$$\begin{cases} S_{x_1}^{\alpha_2}(f) = 0 \\ S_{x_2}^{\alpha_1}(f) = 0 \end{cases} \quad (2)$$

In addition, considering the cyclic spectrum of the mixed signals after the two signals are superimposed, the following results are obtained:

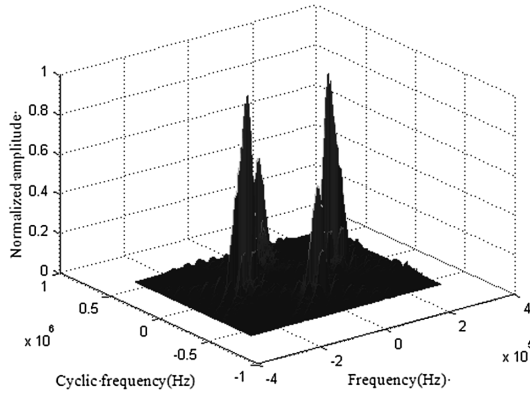
$$\begin{cases} S_{x_1 + x_2}^{\alpha_1}(f) = S_{x_1}^{\alpha_1}(f) + S_{x_2}^{\alpha_1}(f) = S_{x_1}^{\alpha_1}(f) \\ S_{x_1 + x_2}^{\alpha_2}(f) = S_{x_1}^{\alpha_2}(f) + S_{x_2}^{\alpha_2}(f) = S_{x_2}^{\alpha_2}(f) \end{cases} \quad (3)$$

The above analysis also has similar conclusions for the superposition form of multiple signals. The above properties of cyclic spectrum make it suitable for multi-signals processing. These properties are also the theoretical basis for the study of satellite multi-signals parameter estimation algorithms based on cyclic spectrum.

Taking the commonly used BPSK and QPSK signals in satellite communication as an example. Assuming that the received signal is a mixed form of these two signals, combined with formula (1–3), the cycle spectrum of the mixed signal can be obtained as follows:

$$S_{\text{BQ}}^{\alpha}(f) = \begin{cases} \frac{E_b}{4T_b} [Q_b(f - f_{cb} + \frac{\alpha}{2})Q_b^*(f - f_{cb} - \frac{\alpha}{2}) + \\ Q_b(f + f_{cb} + \frac{\alpha}{2})Q_b^*(f + f_{cb} - \frac{\alpha}{2})], \alpha = \frac{n}{T_b} \\ \frac{E_q}{4T_q} [Q_q(f - f_{cq} + \frac{\alpha}{2})Q_q^*(f - f_{cq} - \frac{\alpha}{2}) + \\ Q_q(f + f_{cq} + \frac{\alpha}{2})Q_q^*(f + f_{cq} - \frac{\alpha}{2})], \alpha = \frac{n}{T_q} \\ \frac{E_b}{4T_b} [e^{j2\varphi_{0b}}Q_b(f - f_{cb} + \frac{\alpha}{2})Q_b^*(f + f_{cb} - \frac{\alpha}{2}) + \\ e^{-j2\varphi_{0b}}Q_b(f + f_{cb} + \frac{\alpha}{2})Q_b^*(f - f_{cb} - \frac{\alpha}{2})] \\ \alpha = \pm 2f_{cb} + \frac{n}{T_b} \end{cases} \quad (4)$$

In the above formula,  $f_{cb}$  and  $f_{cq}$  respectively represent the carrier frequency of BPSK and QPSK;  $T_b$  and  $T_q$  denotes the symbol period of BPSK and QPSK signals respectively;  $Q_b$  and  $Q_q$  respectively represent sinc functions corresponding to BPSK and QPSK signals;  $E_b$  and  $E_q$  respectively represent the average power of BPSK and QPSK signals. Figure 1 gives a more vivid description of the cyclic spectrum characteristics of mixed signals.



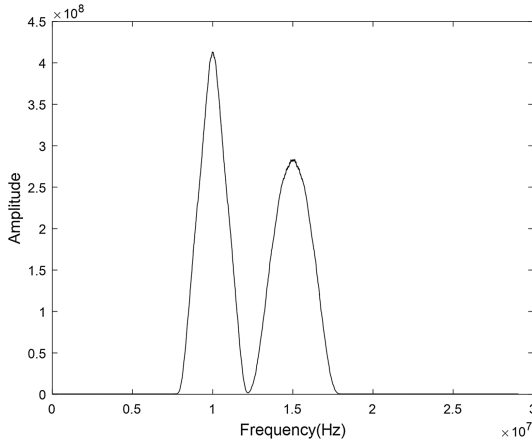
**Fig. 1.** Cyclic spectrum of mixed signals

In addition, it can be seen from formula (4) and Fig. 1 that the cyclic spectrum of mixed signals has symmetry. In the following paper, for the convenience of calculation and discussion, it only targets at  $(\alpha \geq 0, f \geq 0)$  region.

In the zero cyclic frequency ( $\alpha = 0$ ) section of the cyclic spectrum, taking into account the common spectral characteristics of these two signals, that is

$$S_{\text{BPSK} + \text{QPSK}}^{\alpha}(f) = \begin{cases} \frac{E_b}{4T_b} |Q_b(f - f_{cb})|^2, \alpha = 0 \\ \frac{E_q}{4T_q} |Q_q(f - f_{cq})|^2, \alpha = 0 \end{cases} \quad (5)$$

As can be seen from the above equation, when  $f = f_{cb}$  or  $f = f_{cq}$ , the above formula obtains the maximum value. Therefore, the peak values are searched at the zero cyclic frequency ( $\alpha = 0$ ) section of the cyclic spectrum, and the frequencies corresponding to the peaks are the carrier frequency estimation values of the two signals, as shown in the Fig. 2.

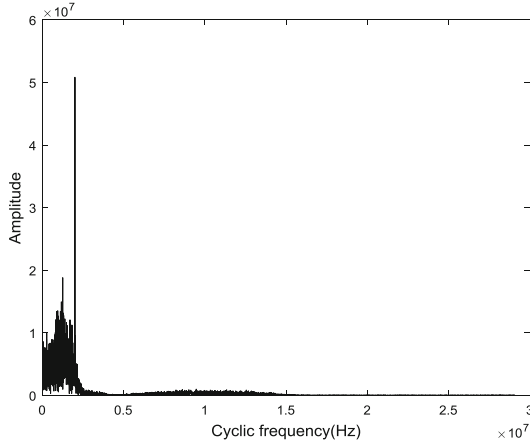


**Fig. 2.** The zero cyclic frequency section

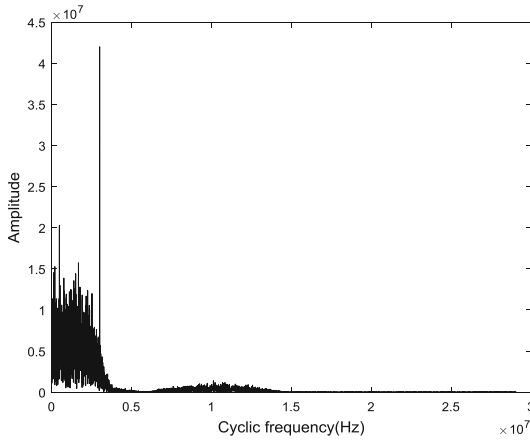
In the  $f = f_{cb}$  or  $f = f_{cq}$  section of cyclic spectrum, considering the common spectral characteristics of both signals, formula (4) can be simplified as

$$S_{\text{BPSK} + \text{QPSK}}^\alpha(f) = \begin{cases} \frac{E_b}{4T_b} \left| Q_b\left(\frac{\alpha}{2}\right) \right|^2, \alpha = \frac{n}{T_b} \\ \frac{E_q}{4T_q} \left| Q_q\left(\frac{\alpha}{2}\right) \right|^2, \alpha = \frac{n}{T_q} \end{cases} \quad (6)$$

It can be seen from the above formula that, except  $\alpha = 0$ , the spectral section obtains the maximum value at  $\alpha = \frac{1}{T_b}$  and  $\alpha = \frac{1}{T_q}$ . Therefore, the maximum value is searched in the  $f = f_{cb}$  or  $f = f_{cq}$  section of cyclic spectrum, and the frequency corresponding to the maximum value is the symbol rate estimation value, as shown in Figs. 3 and 4.



**Fig. 3.** The  $f = f_{cb}$  section of cyclic spectrum



**Fig. 4.** The  $f = f_{cq}$  section of cyclic spectrum

### 3 Description of Algorithm

According to the above analysis, the parameter estimation steps of the mixed signal are as follows:

- (1) Calculate cyclic spectrum of the received signal;
- (2) Search for peaks in the zero cyclic frequency section of cyclic spectrum, and frequencies corresponding to peak are the carrier frequency  $\hat{f}_{cb}$  and  $\hat{f}_{cb}$ ;
- (3) Search for the maximum value in the  $f = \hat{f}_{cb}$  and  $f = \hat{f}_{cb}$  section of cyclic spectrum respectively, and the frequencies corresponding to the maximum values are symbol rate  $\hat{R}_{bb}$  and  $\hat{R}_{bq}$ .

### 4 Simulation Analysis

In order to verify the effectiveness of the proposed algorithm, this paper evaluates the performance of the proposed algorithm by using MATLAB. Signal parameters are: sampling frequency is 60 MHz, carrier frequency of BPSK signal is 10 MHz, symbol rate is 2 Mbit/s, carrier frequency of QPSK signal is 15 MHz, symbol rate is 3 Mbit/s. The normalized root mean square error is used as the evaluation standard. The parameter estimation results of the mixed signal are as follows (Figs. 5 and 6).

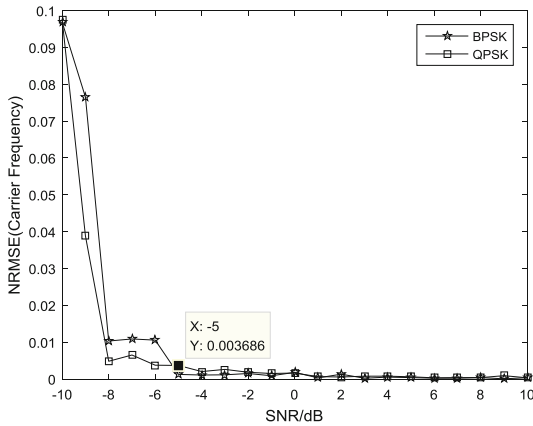


Fig. 5. Estimation results of carrier frequency

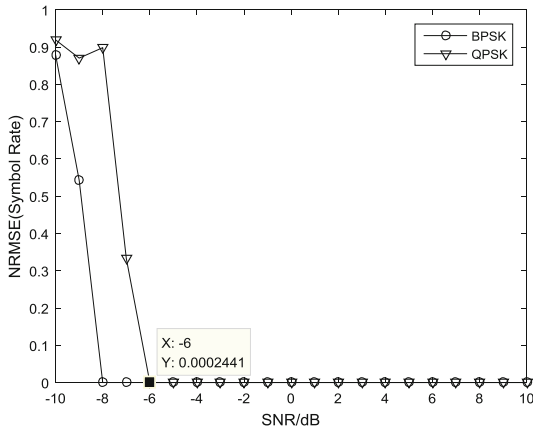


Fig. 6. Estimation results of symbol rate

From the above simulation results, it can be seen that in the whole range of SNR, the carrier frequency estimation is better than 1% when the SNR is larger than  $-5$  dB, and the symbol rate estimation is better than 0.1% when the SNR is larger than  $-6$  dB, indicating that the algorithm has better estimation performance under low SNR. In addition, the symbol rate estimation performance is better than the carrier frequency estimation. The reason is mainly that the symbol rate is estimated in the cyclic frequency domain, and this area is not sensitive to noise.

## 5 Conclusion

In this paper, a parameter estimation algorithm for mixed satellite signals based on cyclic spectrum is proposed, and the effective parameter estimation of mixed signals is realized by using the linear superposition of cyclic spectrum and the potential correspondence between the spectral lines in different cyclic spectrum sections and signal parameters. The simulation results prove the validity of the method. At the same time, the method in this paper can be extended to multiple signal mixtures, with a wide range of applications.

## References

1. Yan, X., Feng, G., Wu, H.C., Xiang, W., Wang, Q.: Innovative robust modulation classification using graph-based cyclic-spectrum analysis. *IEEE Commun. Lett.* **21**(1), 16–19 (2017)
2. Yang, W.C., Yang, X.Q., Kuang, Y.: Research on parameter estimation of MPSK signals based on the generalized second-order cyclic spectrum. In: 2014 URSI General Assembly and Scientific Symposium (URSI GASS), Beijing, China, pp. 1–4 (2014)
3. Dong, J., Wei, X., Zhang, Q., Zhao, L.: Speech enhancement algorithm based on higher-order cumulants parameter estimation. *Int. J. Innov. Comput. Inf. Control* **5**(9), 2725–2733 (2009)
4. Kouame, D., Girault, J.M.: Improvement of cumulant-based parameter estimation. In: 2001 IEEE International Conference on Acoustics, Speech, and Signal Processing, Salt Lake City, UT, USA, pp. 3989–3992 (2001)
5. Zhang, W., Li, K., Jiang, W.: Parameter estimation of periodic frequency modulated signal based on cyclic spectrum density and its application on micro-Doppler signatures extraction. In: 2014 International Conference on Information and Communications Technologies, Nanjing, China, pp. 1–7 (2014)
6. Vučić, D., Vukotić, S., Erić, M.: Cyclic spectral analysis of OFDM/OQAM signals. *Int. J. Electron. Commun.* **73**(3), 139–143 (2017)
7. Peng, J., Han, Z., Sun, J.: A fast cyclic spectrum detection algorithm for MWC based on Lorentzian Norm. In: Sun, G., Liu, S. (eds.) ADHIP 2017. LNICST, vol. 219, pp. 177–188. Springer, Cham (2018). [https://doi.org/10.1007/978-3-319-73317-3\\_22](https://doi.org/10.1007/978-3-319-73317-3_22)
8. Narieda, S.: Low complexity cyclic auto-correlation function computation for spectrum sensing. *IEICE Commun. Express* **6**(6), 387–392 (2017)
9. Cohen, D., Pollak, L., Eldar, Y.C.: Carrier frequency and bandwidth estimation of cyclostationary multiband signals. In: 2016 IEEE International Conference on Acoustics, Speech and Signal Processing, Shanghai, China, pp. 3716–3720 (2016)