Optimized Power Allocation for Weighted Complementary Coded-CDMA Systems

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Abstract. Complementary codes (CCs) have opened up a whole new frontier in Code Division Multiple Access (CDMA) techniques due to the ideal correlation properties. However, the equal gain combination must be satisfied in such CDMA system, which constrains the system performance over frequency selective fading channels. This paper aims to propose a set of weighted complementary codes (WCCs) to enable variable combination parameters while maintaining ideal correlation properties. Such new WCCs can provide power allocation with more freedom, and more importantly, an optimized power allocation can improve the bit error probability performance of CDMA systems as proved at the end of this paper.

Keywords: Weighted complementary codes · WCC-CDMA system · Multiple access interference · Multi-path interference · Power allocation

1 Introduction

CDMA systems have been considered as interference-limited systems in the past decade, due to the fact that ideal correlation properties cannot be achieved with one-dimensional spreading codes. As a consequence, these codes could lead to multi-path interference (MPI) and multiple access interference (MAI), as they only considered periodic auto-correlation functions (ACFs) and periodic cross-correlation functions (CCFs) while aperiodic ACFs and/or CCFs were neglected [1]. A possible solution was to design a spreading code based on CCs [2–4], which have taken account of periodic and aperiodic ACFs and/or CCFs. The systems

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employing such spreading codes are named Complementary Coded-CDMA (CC-CDMA) systems, which are interference-free [5]. However, orthogonality cannot be recovered in frequency selective channels, which again introduces the interference problems. The previous constructions of CCs were so inflexible that the gains of subcodes of each channel had to be equal [6,7], which means that the channels can only result in an equal gain regardless of how they perform. Therefore, CC-CDMA systems needed to satisfy an equal gain of correlation coefficients. The previous academic literature only attempted to resolve this problem through the design optimization of traditional CC-CDMA systems, where the designs were highly complicated and the use of diversity gain was insufficient. In this sense, it is vital to develop much smarter codes while keeping the properties of periodic and aperiodic ACFs and/or CCFs at the same time.

This paper aims to solve the problem by proposing a new construction of WCCs to enable the allocation of more power to better channels and thereby reduce the BER. WCC-CDMA systems no longer require equal transmitter powers and equal combination gains of channels, because of the ideal correlation properties of subcodes. According to the water filling algorithm, the allocation of more power to better channels can result in the improvement of capacity, so that there will be a declined bit error rate (BER). Systems employing a set of WCCs are weighted complementary coded-CDMA (WCC-CDMA) systems.

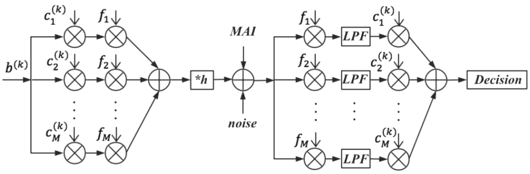

Fig. 1. The architecture of an FDM CC-CDMA system with k users taking transceiver

2 System Model

This paper considers the same CC-CDMA architecture as that in [8]. Figure 1 shows a classic frequency division multiplex (FDM) CC-CDMA system architecture. The signature codes are a set of CCs $\mathbb{C}(M, N, K)$, where M is the flock size, which is the number of element sequences; N is the code length, which is the same length of each element sequence; and K is the set size, which is the maximum number of users. MN is the "congregated length" of a CC, and it determines the processing gain of the corresponding CC-CDMA system [9]. The CC assigned to user k is represented by $C^{(k)} = \left\{c_m^{(k)}\right\}_{m=1}^{M}$, $k \in \{1, 2, \cdots, K\}$, the mth sequence is $c_m^{(k)} = [c_{m,1}^{(k)}, c_{m,2}^{(k)}, \cdots, c_{m,N}^{(k)}]$, where $c_{m,n}^{(k)} \in \{-1, 1\}$, $m \in \{1, \cdots, M\}$, and $n \in \{1, \cdots, N\}$.

The correlation properties of CCs are expressed as

$$\rho(C^{(k_1)}, C^{(k_2)}; \tau) = \sum_{m=1}^{M} \phi(c_m^{(k_1)}, c_m^{(k_2)}; \tau) \quad (1)$$

where $C^{(k_1)}, C^{(k_2)} \in C(M, N, K)$, $k_1, k_2 \in \{1, 2, \cdots, K\}$, τ is the delay of element sequences, and $\phi(c_m^{(k_1)}, c_m^{(k_2)}; \tau)$ is the aperiodic correlation function of $c_m^{(k_1)}$ and $c_m^{(k_2)}$. When $k_1 = k_2$, (1) is the aperiodic auto-correlation function (ACF); otherwise, it is the aperiodic cross-correlation function (CCF).

The ACF of a CC is ideal if it is a delta function. The CCF of a set of CC is ideal if and only if it is a zero function. The ideal correlation properties can be expressed as

$$\rho(C^{(k_1)}, C^{(k_2)}; \tau) = \begin{cases} MN, & k_1 = k_2 \ and \ \tau = 0, \\ 0, & otherwise. \end{cases} \quad (2)$$

where $-N < \tau < N$, and $\forall k_1, k_2 \in \{1, \ldots, K\}$.

Let $b^{(k)}(i)$ be the source data for user k, where $b^{(k)}(i) \in \{-1, 1\}$, $i \in \{0, 1, \cdots, B-1\}$, and B is data block length. The source data is spread by $C^{(k)}$, as

$$s_m^{(k)}(t) = \sqrt{p} \sum_{i=0}^{B-1} b^{(k)}(i) C_m^{(k)}(t - iT_b) \quad (3)$$

where p is the transmitter power, T_b is bit interval. $C_m^{(k)}(t)$ is chip waveform of the mth element sequence, as

$$C_m^{(k)}(t) = \sum_{n=1}^{N} c_{m,n}^{(k)} q(t - nT_c + T_c) \quad (4)$$

where $q(t)$ is the impulse of chip waveform-shaping filter, as

$$q(t) = \begin{cases} \frac{1}{\sqrt{T_C}}, 0 \le t \le T_C, \\ 0, \quad otherwise. \end{cases} \quad (5)$$

At the receiver of user k, the signal received can be written as

$$r_m^{(k)}(t) = h_m \sum_{k=1}^{K} s_m^{(k)}(t) + n_m(t) \quad (6)$$

where $h_m = |h_m| e^{j\varphi_m}$ is the channel coefficient from base stations to the receiver of user k, $n_m(t)$ is white Gaussian noise with zero mean and variance $\sigma_n^2 = N_0/T_c$, where N_0 is the power spectrum density of the complex additive white Gaussian noise (AWGN), assuming that M noises samples are uncorrelated.

3 Scalable Complementary Sets of Sequence

This section starts with the introduction of special CCs namely, WCCs, and prove that the correlation properties of WCCs is perfect with a delay τ, where $\tau = 0, 2, 4, \cdots$.

It has been known that a Hadamard Matrix H_N is an N-order matrix, satisfying

$$H_N H_N^T = H_N^T H_N = N I_N \tag{7}$$

where $H_N \in \{-1, 1\}$, H_N^T is the transpose of H_N and I_N is the identity matrix of order N. (7) implies the orthogonality of any two sequences given by rows of H_N. The Sylvester-type Hadamard matrices with $N = 2^n (n \geq 0)$ can be generated as

$$H_{2N} = \begin{bmatrix} H_N & H_N \\ H_N & -H_N \end{bmatrix} \tag{8}$$

Inspired by (8), the construction of Hadamard matrices of order $N = 2^n (n \geq 0)$ can be formulated by a new recursive procedure as

$$H_{2N} = \begin{bmatrix} H_N & \tilde{H}_N \\ H_N & -\tilde{H}_N \end{bmatrix} \tag{9}$$

where $\tilde{H}_N = P_N H_N Q_N$, P_N and Q_N are orthogonal matrices as

$$P_N P_N^T = P_N^T P_N = Q_N Q_N^T = Q_N^T Q_N = I_N \tag{10}$$

assuming that

$$P_N = \begin{bmatrix} 0 & I_{\frac{N}{2}} \\ I_{\frac{N}{2}} & 0 \end{bmatrix} \tag{11}$$

and

$$Q_N = I_N \tag{12}$$

(11) and (12) represent that $\tilde{H}_N$ results from the exchange of the upper and lower half part of H_N. Therefore, if H_N is a CC of N-order, then $\tilde{H}_N$ is a CC of N-order. Let $H_2^{(0)} = \begin{bmatrix} +1 & +1 \\ +1 & -1 \end{bmatrix}$ and $H_2^{(1)} = \tilde{H}_2^{(0)} = \begin{bmatrix} +1 & -1 \\ +1 & +1 \end{bmatrix}$, a CC set of 2-order can be formed. Through (9), we obtain a CC set of N-order from the set of N-1-order, recursively. The CC set of N-order can be expressed

$$c_{m,n}^{(k)} = (-1)^{\sum_{r=0}^{L-2} (n_{r+1} \oplus m_r \oplus k_r) n_r + (m_{L-1} \oplus k_{L-1}) n_{L-1}} \tag{13}$$

where $m \in \{1, 2, \cdots, M\}$, $n \in \{1, 2, \cdots, N\}$, $k \in \{1, 2, \cdots, K\}$, $K = M = N = 2^L (L \geq 1)$, and m in the radix-2 form is expressed as [10]. Therefore

$$m \equiv (m_{L-1} m_{L-2} \cdots m_0) = \sum_{r=0}^{L-1} 2^r m_r \tag{14}$$

Now, we add a coefficient to (1) and (2), and get

$$\rho(C^{(k_1)}, C^{(k_2)}; \tau; W) = \sum_{m=1}^{M} w_m \phi(c_m^{(k_1)}, c_m^{(k_2)}; \tau) \tag{15}$$

$$\rho(C^{(k_1)}, C^{(k_2)}; \tau; W) = \begin{cases} MN, & k_1 = k_2 \ and \ \tau = 0, \\ 0, & otherwise. \end{cases} \tag{16}$$

Let $L = 2$, we get a set of C_4. Substituting C_4 into (16), we can obtain $W = \{w_1, \cdots, w_4\}$ satisfying

$$w_1 + w_2 = w_3 + w_4 \tag{17}$$

when τ is an even number ($\tau = 0, 2, 4, \cdots$).

In order to prove $\rho(H_N^{(k_1)}, H_N^{(k_2)}; \tau; w) = 0$, we need to prove that the open statement $s(n)$ is true, where $s(n)$ denotes a mathematical statement of $\rho(H_{2^n}^{(k_1)}, H_{2^n}^{(k_2)}; \tau; w) = 0$, $N = 2^n$, $n = 1, 2, \cdots$, $\tau = 2 - N, \cdots, 0, 2, \cdots, N - 2$, $k_1 \neq k_2$ and $W = (w_1, \cdots, w_N)$ satisfying

$$w_1 + w_2 = w_3 + w_4 = \cdots = w_{N-1} + w_N \tag{18}$$

For $n = 1$

$$s(1) : \rho({H_2}^{(k_1)}, {H_2}^{(k_2)}; \tau; w) = 0 \tag{19}$$

So, $s(1)$ is true. Assuming s(n) is true for $n = N$,

$$s(N) : \rho({H_{2^N}}^{(k_1)}, {H_{2^N}}^{(k_2)}; \tau; w) = 0 \tag{20}$$

To establish the truth of $s(N + 1)$, we need to show that

if $1 \leq k < 2^N$

$$H_{2^{N+1}}^{(k)} = \begin{bmatrix} H_{2^N}^{(k')} & H_{2^N}^{(g)} \\ H_{2^N}^{(k')} & -H_{2^N}^{(g)} \end{bmatrix} \tag{21}$$

if $2^N \leq k < 2^{N+1}$

$$H_{2^{N+1}}^{(k)} = \begin{bmatrix} H_{2^N}^{(k')} & -H_{2^N}^{(g)} \\ H_{2^N}^{(k')} & H_{2^N}^{(g)} \end{bmatrix} \tag{22}$$

where $k' \equiv k(\text{mod} 2^N)$ and $g = \left|2^N - k'\right|$.

$$s(N + 1) : \rho({H_{2^{N+1}}}^{(k_1)}, {H_{2^{N+1}}}^{(k_2)}; \tau; w) = 0 \tag{23}$$

Consequently, $s(N)$ is true for all $n \in Z^+$ based on the Principle of Finite Induction.

4 QPSK in CC-CDMA System

In the section above, we have proved the ideal correlation properties of WCCs, while chip delay τ is an even number. In this section, we will prove that the chip delay τ is an even number by QPSK modulation. Figure 2 shows the chip delay by BPSK and QPSK. It is obvious that the chip delay τ is even by QPSK. In addition, the chip delay is even in some other higher order modulation schemes, for example 16QAM.

The signal after QPSK is

$$\begin{aligned} s_m^{(g)}(t) &= I_m^{(g)}(t) + jQ_m^{(g)}(t) \\ &= \sum_{i=1}^{B} b_k(i) C_{I,m}^{(g)}(t - iT_s + T_s) \\ &\quad + j \sum_{i=1}^{B} b_k(i) C_{Q,m}^{(g)}(t - iT_s + T_s) \end{aligned} \tag{24}$$

where T_s is the symbol interval and the source data is spread by $C_{I,m}^{(g)}(t)$ and $C_{Q,m}^{(g)}(t)$.

$$C_{I,m}^{(g)}(t) = \sum_{i=1}^{N/2} c_{m,2i-1}^{(g)} q(t - iT_c + T_c) \tag{25}$$

$$C_{Q,m}^{(g)}(t) = \sum_{i=1}^{N/2} c_{m,2i}^{(g)} q(t - iT_c + T_c) \tag{26}$$

At the receiver of user g, the signal received can be written as

$$\widehat{I}_m^{(g)}(t) = \sum_{l=1}^{L} \sum_{k=1}^{K} h_l^{(k)} I_m^{(k)}(t - \tau_{k,l}) + n_{I,m}(t) \tag{27}$$

$$\widehat{Q}_m^{(g)}(t) = \sum_{l=1}^{L} \sum_{k=1}^{K} h_l^{(k)} Q_m^{(k)}(t - \tau_{k,l}) + n_{Q,m}(t) \tag{28}$$

where $n_{I,m}(t)$ and $n_{Q,m}(t)$ are noise after demodulation through channels I and Q, respectively. $h_l^{(k)}$ is the fading coefficient of the lth path of the channel for signal sent by user k. $\tau_{k,l}$ is the delay on the lth path of user k. In this paper, only the effect of asynchronous fading channels on baseband signals is considered. Therefore, for user p and the mth sequence, the first paths of channels I and Q are despread as

$$\Im_{m,1}^{(p)} = \frac{1}{MN} \sum_{k=1}^{K} \sum_{l=1}^{L} h_l^k b_k \sum_{i=1}^{N/2\ -\delta} g_{m,2i-1}^{(k)} g_{m,2i+2\delta-1}^{(p)} + \omega_{I,m} \tag{29}$$

$$\mathfrak{Q}_{m,1}^{(p)} = \frac{1}{MN}\sum_{k=1}^{K}\sum_{l=1}^{L} h_l^k b_k \sum_{i=1}^{N/2-\delta} g_{m,2i}^{(k)} g_{m,2i+2\delta}^{(p)} + \omega_{Q,m} \tag{30}$$

where $\delta = (\tau_{p,1} - \tau_{k,l})/T_c$, $\delta \in \{0, 1, \cdots, \frac{N}{2} - 1\}$

$$\begin{aligned}\widehat{b}_{p,1} &= \sum_{m=1}^{M} (\Im_{m,1}^{(p)} + \mathfrak{Q}_{m,1}^{(p)}) \\ &= \frac{1}{MN}\sum_{k=1}^{K}\sum_{l=1}^{L} h_l^k b_k \sum_{m=1}^{M}\sum_{n=1}^{N/2-\delta} \underbrace{g_{m,n}^{(k)} g_{m,n+2\delta}^{(p)}}_{\rho(G^{(k)},G^{(p)};2\delta)} + \omega\end{aligned} \tag{31}$$

It is obvious that two neighboring chips are modulated to the same symbol by QPSK. Moreover the delay caused by asynchronous communication or multi-path transmission is an even number at the receiver.

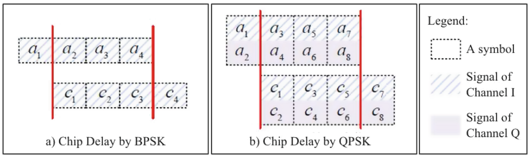

Fig. 2. Chip delay by BPSK and QPSK modulation

5 Power Allocation in WCC-CDMA System

Based on the discussions in the above two sections, WCCs modulated by QPSK can achieve ideal correlation properties of subcodes, which equal transmitter powers and equal combination gains of channels are no longer required.

This section starts with two combining algorithms at the receiver. Maximum ratio combining (MRC) in a CC-CDMA system is employed to get a maximum frequency diversity gain. M filtered signal can be combined with MRC as

$$\begin{aligned}\widehat{b}_{mrc}^{(g)}(j) &= \sum_{m=1}^{M} [h_m^{(g)}] \cdot y_m^{(g)}(j) \\ &= N\sqrt{\overline{p_r}^{(g)}} \sum_{m=1}^{M} |h_m^{(g)}| b^{(g)}(j) + I_{mrc}^{(g)} + \omega_{mrc}\end{aligned} \tag{32}$$

where '∗' denotes complex conjugate operation, $I_{mrc}^{(g)}$ is the multi-user interference (MUI) under MRC, ω_{mrc} is the noise, and $\overline{p_r}^{(g)} = \frac{E_b}{MNT_c}$.

Equal gain combining (EGC) in a CC-CDMA system is employed to equalize frequency diversity gain. M filtered signal can be combined with EGC as

$$\begin{aligned}\widehat{b}_{egc}^{(g)}(j) &= \sum_{m=1}^{M} [h_m^{(g)}] * y_m^{(g)}(j)/|h_m^{(g)}| \\ &= N\sqrt{\overline{p_r}^{(g)}} \sum_{m=1}^{M} b^{(g)}(j)|h_m^{(g)}| + I_{egc}^{(g)} + \omega_{egc}\end{aligned} \tag{33}$$

From the above analysis, it can be suggested that the MUI problem still exists despite the fact that CCs correlation properties are perfect. The problem is that the ideal correlation of CCs cannot be regained by combining algorithm at a receiver, because channel gains vary with different sub-carriers of users. In order to solve this problem, this paper will propose a technology of power allocation.

As defined in (18), we can obtain one constraint equation as

$$\sqrt{P_1} + \sqrt{P_2} = \cdots = \sqrt{P_{M-1}} + \sqrt{P_M} \tag{34}$$

where P_m is the transmitter power for the mth channel. Assuming that

$$\sum_{m=1}^{M} P_m = M \tag{35}$$

As proposed by R. G. Gallager

$$C = \frac{B}{N} \sum_{m=1}^{M} \log_2(1 + H_m P_m) \tag{36}$$

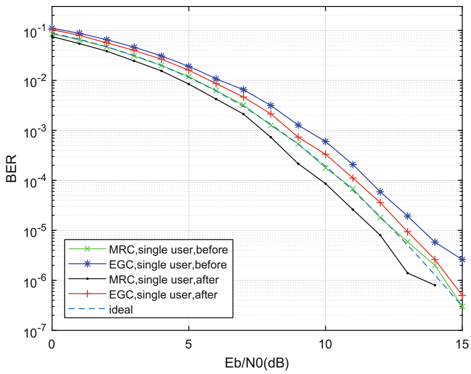

Fig. 3. BER performance of power allocation of weighted-combined uplink CC-CDMA systems for a single user in an asynchronous scenario

where C is the total channel capacity of CC-CDMA system and B is the bandwidth of the system. Under the constraint condition that the total power is constant, the maximum of C can be described as

$$C^* = \max \frac{B}{N} \sum_{m=1}^{M} \log_2(1 + H_m P_m) \tag{37}$$

Construct a function L as

$$L = \sum_{m=1}^{M} \log_2(1 + H_m P_m) - \lambda(\sum_{m=1}^{M} P_m - P_{total}) \tag{38}$$

where $P_{total} = M$. Let $\frac{\partial L}{\partial P_m} = 0$, we get

$$P_m + \frac{1}{H_m} = P_n + \frac{1}{H_n} \tag{39}$$

Considering (34), we make a trade-off between the maximum total channel capacity and the amount of calculation, as follows

$$\begin{cases} \grave{P}_{2k-1} = [\frac{s}{2} + \frac{1}{s}(\frac{1}{H_{2k}} - \frac{1}{H_{2k-1}})]^2 \\ \grave{P}_{2k} = [\frac{s}{2} + \frac{1}{s}(\frac{1}{H_{2k-1}} - \frac{1}{H_{2k}})]^2 \end{cases} \tag{40}$$

where we assume $s = \sqrt{\grave{P}_{2k-1}} + \sqrt{\grave{P}_{2k}}$ is a constant and $k = (1, \cdots, \frac{M}{2})$. Substituting (40) into (35)

$$P_m = \frac{M\grave{P}_m}{\sum\limits_{m=1}^{M} \grave{P}_m} \tag{41}$$

where P_m is the power allocation algorithm for WCC-CDMA systems.

6 Numerical Results and Discussions

In this section, we will verify the aforementioned correctness of the analytical results through simulation. In addition to a set of WCCs $\mathbb{C}(M, N, K)$ (13), the uplink communication with an uncoded QPSK modulation also need to be considered in the simulation. Under the condition that perfect channel state information (CSI) is considered available at the receiver, Figs. 3 and 4 illustrate the simulation results.

Figure 3 compares BER performance of a single-user CC-CDMA system under different combining algorithms with the BER performance of WCC-CDMA through optimized power allocation. It can be observed that both EGC and MRC for a single user can be improved after power allocation. In particular, it should be noted that MRC is even better than the ideal curve of BER (i.e. the best performance of BER without power allocation).

Figure 4 compares BER performance of a CC-CDMA system for multiple users under different detecting algorithms with the BER performance of WCC-CDMA through power allocation. The simulation result shows that, after power allocation, BER of MRC for multiple users is nearly the same when EbN0 is low, and slightly worse when EbN0 is high; however, BER of EGC for multiple users is improved regardless of EbN0.

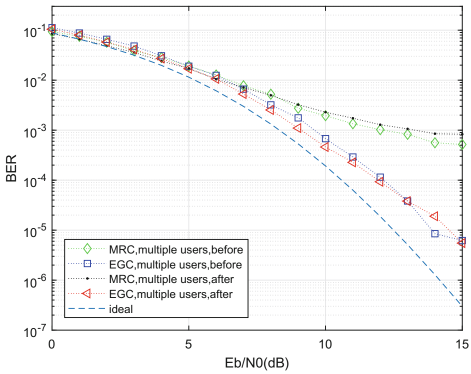

Fig. 4. BER performance of power allocation of weighted-combined uplink CC-CDMA systems for 8 users in an asynchronous scenario

7 Conclusion

This paper proposed a type of weighted complementary codes (WCCs), which enables the variation of combination parameters while remaining under the constraint of ideal correlation properties. It has been proven that the constructed WCCs can bring more freedom on the variation of power allocation, which can therefore improve the BER performance of a CC-CDMA system. Given consideration to the unicity of WCCs constructed in this paper, the construction of generalized WCCs is recommended for a possible direction of future work.

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