R/I-Capon for Low-Complexity Direction of Arrival Estimation with Real-Valued Computation

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Abstract. The problem of low-complexity direction-of-arrival (DOA) estimation without knowing sources number is addressed. Real/imaginary-part of array covariance matrix (ACM) can be remodeled as a whole ACM of signal received by virtual array aimed at fast DOA estimate. Based on such a virtual signal model, a novel real/imaginary-part Capon (R/I-Capon) involving the inverse of only the real/imaginary-part of the estimated ACM (EACM) is derived. In-depth insights are provided to prove that the rank of real/imaginary-part of EACM is always equal or greater than that of whole EACM, which indicates that R/I-Capon exceeds conventional Capon under the circumstance with small numbers of snapshots. Further discussion indicates that R/I-Capon is also capable of decreasing about 75% complexity with any array structures, which shows an enforcement advantage over state-of-the-art prototype methods. Simulations are finally conducted to verify theoretical analysis and to show practicability of proposed algorithm.

Keywords: Orthogonal projection · Cumulative sum · Cumulative multiplication · Singular value decomposition · Intersection

1 Introduction

Subspace-based direction of arrival (DOA) algorithms including MUSIC [2,3], root-MUSIC [4,5] and ESPRIT [6] generally desire number of sources to be detected in advance or to be known [7], when sources number is misjudged, performance of those estimators can get deteriorated signally [8]. Beamforming techniques such as Capon [9] can be applied for DOA estimation without knowing sources number, but they have to compute inverse of estimated array covariance matrix (EACM) as well as complex product of steering vector, which involve higher complexity than MUSIC. Unitary techniques with real-valued

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computation are able to decrease complexity by a quarter as compared to their complex-valued versions [10,11]. Unfortunately, those algorithms are based upon centro-symmetrical arrays (CSAs), [13].

This paper aims for solving problem of low-complexity DOA estimation without knowing sources number, in which real-valued computation is attempted to be implemented with arbitrary array structures. To this end, the real/imaginary-part of the EACM is used instead of the whole EACM to simplify the optimization problem by a new real/imaginary-part Capon (R/I-Capon). Theoretical analysis and numerical simulations show that R/I-Capon is able to decrease about 75% complexity with improved precision in the condition of small numbers of snapshots.

2 Signal Models and Conventional Capon

Consider a linear array with M antennas placed at $\mathbb{X} = \left[x_1, x_2, \ldots, x_M\right]$, where $\mathbb{X}$ is arbitrary array[1] including uniform linear array and sparse array, but is assumed to own rank-$(M-1)$ ambiguity restriction [14]. Assume $K, K < M$ uncorrelated narrow-band signals with DOAs $\boldsymbol{\theta} = \left[\theta_1, \theta_2, \ldots, \theta_K\right]$ impinging on array, and the output of array at $t, t \in [1, T]$ is expressed as [1–13]

$$\mathbf{x}(t) = \mathbf{A}(\boldsymbol{\theta})\,\mathbf{s}(t) + \mathbf{n}(t), \tag{1}$$

where $\mathbf{s}(t)$ is $K \times 1$ signal, $\mathbf{n}(t) \sim \mathcal{CN}\left(\mathbf{0}, \sigma_n^2 \mathbf{I}_M\right)$ is $M \times 1$ additive noise, σ_n^2 is noise power, $\mathbf{I}_M$ is $M \times M$ identity matrix, $\mathbf{A}(\boldsymbol{\theta}) = \left[\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_K)\right]$ is $M \times K$ array manifold with

$$\mathbf{a}(\theta_k) = \left[e^{j\frac{2\pi}{\lambda}x_1 \sin\theta_k}, e^{j\frac{2\pi}{\lambda}x_2 \sin\theta_k}, \cdots, e^{j\frac{2\pi}{\lambda}x_M \sin\theta_k}\right]^T \tag{2}$$

denoting $M \times 1$ steering vector, where $(\cdot)^T$ is transpose. The $M \times M$ array covariance matrix (ACM) is given by

$$\mathbf{R}_{xx} = \mathrm{E}\left[\mathbf{x}(t)\,\mathbf{x}^H(t)\right] = \mathbf{A}(\boldsymbol{\theta})\,\mathbf{R}_{ss}\mathbf{A}^H(\boldsymbol{\theta}) + \sigma_n^2 \mathbf{I}_M, \tag{3}$$

where $(\cdot)^H$ is conjugate transpose and $\mathbf{R}_{ss} = \mathrm{E}\left[\mathbf{s}(t)\,\mathbf{s}^H(t)\right]$ is the $K \times K$ source covariance matrix. Note that $\mathbf{R}_{ss}$ is a real diagonal matrix as $\mathbf{s}(t)$ is uncorrelated. In practice, ACM is estimated by EACM from T snapshots of observed data as

$$\widehat{\mathbf{R}}_{xx} = \frac{1}{T}\sum_{t=1}^{T} \mathbf{x}(t)\,\mathbf{x}^H(t). \tag{4}$$

When K is known by signal number detection methods [15] in advance, subspace-based methods utilize eigenvalue decomposition (EVD) and obtain

[1] The new method can be easily extended to arbitrary plane array for two-dimensional DOA estimate by introducing two electrical DOAs [13].

signal-noise orthogonal matrix to find DOAs. However, performance of subspace-based methods will get deteriorated without the correct K [8].

Instead of detecting signal number K, Capon [9] suggests a weighted output of array $\mathbf{y}(t) = \mathbf{w}^H \mathbf{x}(t)$ and searches DOAs by minimizing power of $\mathbf{y}(t)$ as

$$\min_{\mathbf{w}} \ \mathrm{E}\big[\mathbf{y}(t)\,\mathbf{y}^H(t)\big] = \mathbf{w}^H \mathbf{R}_{xx} \mathbf{w} \quad \text{s.t. } \mathbf{w}^H \mathbf{a}(\theta) = 1. \tag{5}$$

By solving (5), DOAs can be estimated over $[-\pi/2, \pi/2]$ by seeking peaks of

$$f_{\text{Capon}}(\theta) = \frac{1}{\mathbf{a}^H(\theta)\,\widehat{\mathbf{R}}_{xx}^{-1}\mathbf{a}(\theta)} \tag{6}$$

The sources number and DOAs can be obtained by peak number of $f_{\text{Capon}}(\theta)$ and their locations.

3 Proposed R/I-Capon Algorithm

3.1 Virtual Signal Models

It can be shown that real- and imaginary-parts of ACM contain virtual signal models can be used for DOA estimate. Using $\mathrm{Re}(\mathbf{R}_{xx}) = \frac{1}{2}(\mathbf{R} + \mathbf{R}^*)$ as well as (3), we have

$$\begin{aligned}
\mathrm{Re}(\mathbf{R}_{xx}) &= \frac{1}{2}\big[\mathbf{A}(\boldsymbol{\theta})\,\mathbf{R}_{ss}\mathbf{A}^H(\boldsymbol{\theta}) + \mathbf{A}^*(\boldsymbol{\theta})\,\mathbf{R}_{ss}\mathbf{A}^T(\boldsymbol{\theta})\big] + \sigma_n^2 \mathbf{I}_M \\
&= \underbrace{\big[\mathbf{A}(\boldsymbol{\theta}) \;\; \mathbf{A}^*(\boldsymbol{\theta})\big]}_{M\times 2K} \times \underbrace{\frac{1}{2}\begin{bmatrix} \mathbf{R}_{ss} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{ss} \end{bmatrix}}_{2K\times 2K} \times \underbrace{\begin{bmatrix} \mathbf{A}^H(\boldsymbol{\theta}) \\ \mathbf{A}^T(\boldsymbol{\theta}) \end{bmatrix}}_{2K\times M} + \sigma_n^2 \mathbf{I}_M \\
&= \mathbf{A}_1(\boldsymbol{\theta})\mathcal{R}_{ss,1}\mathbf{A}_1^H(\boldsymbol{\theta}) + \sigma_n^2 \mathbf{I}_M,
\end{aligned} \tag{7}$$

where $\mathbf{0}$ is a $K \times K$ matrix with all zero elements, $\mathcal{R}_{ss,1}$ and matrix $\mathbf{A}_1(\boldsymbol{\theta})$ are defined as

$$\mathcal{R}_{ss,1} = \frac{1}{2}\begin{bmatrix} \mathbf{R}_{ss} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{ss} \end{bmatrix} \in \mathbb{C}^{2K\times 2K} \tag{8-1}$$

$$\mathbf{A}_1(\boldsymbol{\theta}) = \big[\mathbf{A}(\boldsymbol{\theta}) \;\; \mathbf{A}^*(\boldsymbol{\theta})\big] \in \mathbb{C}^{M\times 2K}. \tag{8-2}$$

Using $\mathrm{E}\big[\mathbf{s}(t)\,\mathbf{s}^H(t)\big] = \mathbf{R}_{ss}$ and $\mathrm{E}\big[\mathbf{s}(t)\,\mathbf{s}^T(t)\big] = 0$, $\mathcal{R}_{ss,1}$ can be rewritten as

$$\begin{aligned}
\mathcal{R}_{ss,1} &= \mathrm{E}\Bigg\{ \underbrace{\begin{bmatrix} \mathbf{s}(t) \\ \mathbf{s}^*(t) \end{bmatrix}}_{2K\times 1} \times \underbrace{\big[\mathbf{s}^H(t) \;\; \mathbf{s}^T(t)\big]}_{1\times 2K} \Bigg\} \\
&= \mathrm{E}[\mathbf{s}_1(t)\,\mathbf{s}_1^H(t)],
\end{aligned} \tag{9}$$

where $\mathbf{s}_1(t) = \begin{bmatrix} \mathbf{s}(t) \\ \mathbf{s}^*(t) \end{bmatrix}$ is a $2K \times 1$ vector. Inserting (9) into (7) and using $\mathrm{E}\left[\mathbf{n}(t)\mathbf{n}^H(t)\right] = \sigma_n^2 \mathbf{I}_M$ leads to

$$\begin{aligned} \mathrm{Re}(\mathbf{R}_{xx}) &= \mathbf{A}_1(\boldsymbol{\theta})\mathcal{R}_{ss,1}\mathbf{A}_1^H(\boldsymbol{\theta}) + \sigma_n^2\mathbf{I}_M \\ &= \mathbf{A}_1(\boldsymbol{\theta})\mathrm{E}[\mathbf{s}_1(t)\mathbf{s}_1^H(t)]\mathbf{A}_1(\boldsymbol{\theta})^H + \mathrm{E}\left[\mathbf{n}(t)\mathbf{n}^H(t)\right] \\ &= \mathrm{E}\left\{\left[\mathbf{A}_1(\boldsymbol{\theta})\mathbf{s}_1(t) + \mathbf{n}(t)\right] \times \left[\mathbf{A}_1(\boldsymbol{\theta})\mathbf{s}_1(t) + \mathbf{n}(t)\right]^H\right\} \\ &= \mathrm{E}\left[\mathbf{x}_1(t)\mathbf{x}_1^H(t)\right], \end{aligned} \tag{10}$$

where $\mathbf{x}_1(t)$ is a $2K \times 1$ vector, given by

$$\mathbf{x}_1(t) = \mathbf{A}_1(\boldsymbol{\theta})\mathbf{s}_1(t) + \mathbf{n}(t). \tag{11}$$

Comparing (11) with (1) and (10) with (3), we can conclude that real-part of ACM can be completely treated as the whole ACM with a virtual manifold $\mathbf{A}_1(\boldsymbol{\theta})$ [16]. $\mathbf{x}_1(t)$ and $\mathbf{s}_1(t)$ are observed data and incident signal via this virtual array, respectively. Signal covariance matrix are noted as $\mathcal{R}_{ss,1}$.

Likewise defining

$$\mathcal{R}_{ss,2} = \frac{j}{2}\begin{bmatrix} \mathbf{R}_{ss} & \mathbf{0} \\ \mathbf{0} & -\mathbf{R}_{ss} \end{bmatrix} \in \mathbb{C}^{2K\times 2K} \tag{12-1}$$

$$\mathbf{A}_2(\boldsymbol{\theta}) = \begin{bmatrix} \mathbf{A}^*(\boldsymbol{\theta}) & \mathbf{A}(\boldsymbol{\theta}) \end{bmatrix} \in \mathbb{C}^{M\times 2K} \tag{12-2}$$

$$\mathbf{s}_2(t) = \frac{1}{2}\begin{bmatrix} (1+j)\mathbf{s}(t) \\ (1-j)\mathbf{s}^*(t) \end{bmatrix} \in \mathbb{C}^{2K\times 1} \tag{12-3}$$

$$\mathbf{x}_2(t) = \mathbf{A}_2(\boldsymbol{\theta})\mathbf{s}_2(t) + \mathbf{n}(t) \in \mathbb{C}^{2K\times 1}, \tag{12-4}$$

and using $\mathrm{Im}(\mathbf{R}_{xx}) = \frac{j}{2}(\mathbf{R}^* - \mathbf{R})$, one can easily obtain

$$\mathrm{Im}(\mathbf{R}_{xx}) + \sigma_n^2\mathbf{I}_M = \mathrm{E}\left[\mathbf{x}_2(t)\mathbf{x}_2^H(t)\right]. \tag{13}$$

Hence, $\mathrm{Im}(\mathbf{R}_{xx}) + \sigma_n^2\mathbf{I}_M$ can be also regarded as the whole ACM and those manifold is $\mathbf{A}_2(\boldsymbol{\theta})$. Similarly, $\mathbf{x}_2(t)$ and $\mathbf{s}_2(t)$ are severally observed data and incident signal as well as its covariance matrix noted as $\mathcal{R}_{ss,2}$.

3.2 Proposed *R/I*-Capon Algorithm

Defining a new weighted array output $\mathbf{y}_1(t) = \mathbf{w}^H\mathbf{x}_1(t)$ using (11) on virtual array, optimization problem is shown as

$$\min_{\mathbf{w}} \ \mathrm{E}\left[\mathbf{y}_1(t)\mathbf{y}_1^H(t)\right] = \mathbf{w}^H\mathrm{Re}(\mathbf{R}_{xx})\mathbf{w} \tag{14-1}$$

$$\text{s.t.} \quad \mathbf{w}^H\mathbf{a}_1(\theta) = 1, \tag{14-2}$$

where $\mathbf{a}_1(\theta)$ is virtual steering vector composing $\mathbf{A}_1(\boldsymbol{\theta})$. Noting $\forall\boldsymbol{\theta}$, $\mathbf{A}^*(\boldsymbol{\theta}) = \mathbf{A}(-\boldsymbol{\theta})$ holds, thus $\mathbf{A}_1(\boldsymbol{\theta}) = \begin{bmatrix} \mathbf{A}(\boldsymbol{\theta}) & \mathbf{A}(-\boldsymbol{\theta}) \end{bmatrix}$ and $\mathbf{A}_1(\boldsymbol{\theta})$ can be treated as new manifold combined with $\mathbf{A}(\boldsymbol{\theta})$ and $\mathbf{A}(-\boldsymbol{\theta})$. Equally, $\mathbf{a}_1(\boldsymbol{\theta})$ can be treated as new steering vector combined with $\mathbf{a}(\boldsymbol{\theta})$ and $\mathbf{a}(-\boldsymbol{\theta})$. Then, (19-1) and (19-2) can be simplified as[2]

$$\min_{\mathbf{w}} \ \mathbf{w}^H \mathrm{Re}(\mathbf{R}_{xx})\mathbf{w} \quad \text{s.t. } \mathbf{w}^H \mathbf{a}(\theta) = 1. \tag{15}$$

Now, adopting Lagrange multiplier technique, we have

$$h(\mathbf{w}) = \mathbf{w}^H \mathrm{Re}(\mathbf{R}_{xx})\mathbf{w} - \mu \left[\mathbf{w}^H \mathbf{a}(\theta) - 1\right], \tag{16}$$

where μ is Lagrange multiplier. By making $\partial h_1(\mathbf{w})/\partial \mathbf{w} = 0$, (16) can be simplified as

$$\mathbf{w}_{\mathrm{opt}} = \frac{\mathrm{Re}^{-1}(\mathbf{R}_{xx})\mathbf{a}(\theta)}{\mathbf{a}^H(\theta)\,\mathrm{Re}^{-1}(\mathbf{R}_{xx})\mathbf{a}(\theta)}. \tag{17}$$

Combining (17) and $\mathbf{w}^H \mathrm{Re}(\mathbf{R}_{xx})\mathbf{w}$, the problem can be solved as

$$f_{R/I\text{-Capon}}(\theta) = \frac{1}{\mathbf{a}^H(\theta)\,\mathrm{Re}^{-1}(\widehat{\mathbf{R}}_{xx})\mathbf{a}(\theta)}. \tag{18}$$

Taking (13) into account, we can get another weighted output $\mathbf{y}_2(t) = \mathbf{w}^H \mathbf{x}_2(t)$ and the optimization problem can be rewrote as

$$\min_{\mathbf{w}} \ \mathrm{E}\left[\mathbf{y}_2(t)\,\mathbf{y}_2^H(t)\right] = \mathbf{w}^H \left[\mathrm{Im}(\mathbf{R}_{xx}) + \sigma_n^2 \mathbf{I}_M\right]\mathbf{w}$$

$$= \mathbf{w}^H \mathrm{Im}(\mathbf{R}_{xx})\mathbf{w} + \sigma_n^2 \|\mathbf{w}\|^2 \tag{19-1}$$

$$\text{s.t.} \quad \mathbf{w}^H \mathbf{a}_2(\theta) = 1, \tag{19-2}$$

Since $\mathbf{a}_2(\theta)$ covers both $\mathbf{a}(\boldsymbol{\theta})$ and $\mathbf{a}(-\boldsymbol{\theta})$, (19-1) and (19-2) can be simplified as

$$\min_{\mathbf{w}} \ \mathbf{w}^H \mathrm{Im}(\mathbf{R}_{xx})\mathbf{w} + \sigma_n^2 \|\mathbf{w}\|^2 \quad \text{s.t. } \mathbf{w}^H \mathbf{a}(\theta) = 1. \tag{20}$$

By applying Lagrange multiplier technique to solve (20), proposed R/I-Capon can be equivalently given by[3]

$$f_{R/I\text{-Capon}}(\theta) = \frac{1}{\mathbf{a}^H(\theta)\left[\mathrm{Im}(\widehat{\mathbf{R}}_{xx}) + \widehat{\sigma}_n^2 \mathbf{I}_M\right]^{-1}\mathbf{a}(\theta)}. \tag{21}$$

Considering $\widehat{\theta}$ varies over $[0, \pi/2]$ and $f_{R/I\text{-Capon}}(-\widehat{\theta}) = f_{R/I\text{-Capon}}(-\widehat{\theta})$, $\widehat{\theta} \in [-\pi/2, 0]$ could also be true DOAs. Moreover, $\widehat{\theta}$ and $-\widehat{\theta}$ could be both true DOAs when two sources come from $\widehat{\theta}$ and $-\widehat{\theta}$ incidentally. To deal with ambiguity problem, conventional beamformer (CBF) [1] is appropriate for selecting true DOA by maximizing $\|\mathbf{a}^H(\theta)\,\widehat{\mathbf{R}}_{xx}\mathbf{a}(\theta)\|$.

Detailed steps of proposed algorithm are summarized in Table 1.

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[2] The simplification leads to an estimation ambiguity problem, which is analyzed and solved at the end of this subsection.

[3] It is suggested to use (18) rather than (21) in practice since (21) contains a noise part $\widehat{\sigma}_n^2 \mathbf{I}_M$ which should be estimated in advance.

Table 1. Detailed steps of R/I-Capon algorithm

• **Step 1**	Compute $\widehat{\mathbf{R}}_{xx} = \frac{1}{T}\sum_{t=1}^{T}\mathbf{x}(t)\mathbf{x}^H(t)$ and obtain $\mathrm{Re}(\widehat{\mathbf{R}}_{xx})$
• **Step 2**	Search peaks of $f_{R/I\text{-Capon}}(\theta) = \frac{1}{\mathbf{a}^H(\theta)\mathrm{Re}^{-1}(\widehat{\mathbf{R}}_{xx})\mathbf{a}(\theta)}$ over $[0, \pi/2]$ to obtain the candidate angles $\widehat{\theta}_i, i \in [1, Q], Q \leqslant \lfloor \frac{K}{2} \rfloor$
• **Step 3**	For each $\widehat{\theta}_i, i \in [1, Q]$, select true DOA between $\widehat{\theta}_i$ and $-\widehat{\theta}_i$ from maximization of product $\Vert \mathbf{a}^H(\theta)\widehat{\mathbf{R}}_{xx}\mathbf{a}(\theta)\Vert$

3.3 Rank Enhancement and Complexity Reduction

Comparing (18) and (21) with (6), it can be predicted that performance of R/I-Capon may behave worse than conventional Capon because of the half-exploitation of the EACM in R/I-Capon. However, simulation shows that new method has a similar performance compared to Capon. Besides, following theorem illustrates the outperformances of R/I-Capon over the conventional Capon in the situation of small numbers of snapshots.

Theorem 1. *Let the EACM be computed using T snapshots data by (4) in practice, the rank of the real/imaginary-part of the EACM is always no less than that of the entire EACM such that*[4]

$$\begin{cases} \mathrm{rank}\left[\mathrm{Re}\left(\widehat{\mathbf{R}}_{\mathbf{xx}}\right)\right] \geqslant \mathrm{rank}\left(\widehat{\mathbf{R}}_{xx}\right) \\ \mathrm{rank}\left[\mathrm{Im}\left(\widehat{\mathbf{R}}_{\mathbf{xx}}\right) + \widehat{\sigma}_n^2\mathbf{I}_M\right] \geqslant \mathrm{rank}\left(\widehat{\mathbf{R}}_{xx}\right) \end{cases}. \tag{22}$$

4 Simulation Results

Throughout numerical simulations, 500 independent Monte Carlo trials were used to compare performances between conventional Capon and proposed algorithm[5], where the root mean square error (RMSE) can be defined by

$$\mathrm{RMSE} \triangleq \sqrt{\frac{1}{500}\sum_{k=1}^{K}\sum_{i=1}^{500}\left(\widehat{\theta}_{k,i} - \theta_k\right)^2}, \tag{23}$$

where $\widehat{\theta}_{k,i}$ is the ith estimated value for kth DOA θ_k.

Figure 1 plots the RMSEs against the signal-to-noise ratios (SNRs), and Fig. 2 plots those against T's. Figure 1 shows a similar performance of R/I-Capon compared with conventional Capon over SNR = 0 dB–40 dB. With a moderate SNR = 10 dB, it is observed from Fig. 2 that our method achieves better results than conventional Capon all along different T's, especially in scenario within $T \leqslant 100$.

[4] In fact, we also have $\mathrm{rank}\left[\mathrm{Im}\left(\widehat{\mathbf{R}}_{\mathbf{xx}}\right)\right] \geqslant \mathrm{rank}\left(\widehat{\mathbf{R}}_{xx}\right)$.

[5] A half-wavelength uniform linear array (ULA) with $M = 12$ sensors are used to find $K = 2$ sources at $\theta_1 = 20°$ and $\theta_1 = 30°$, where the standard MUSIC [2] is applied for a common comparison reference.

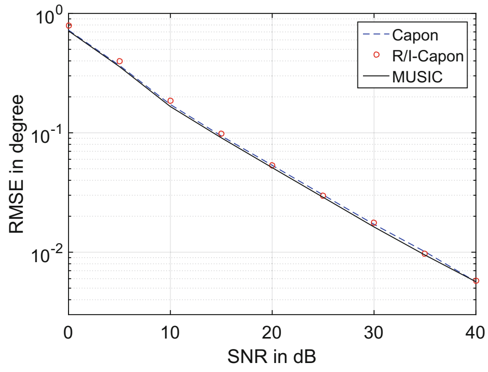

Fig. 1. RMSE against the SNR, $T = 100$ snapshots.

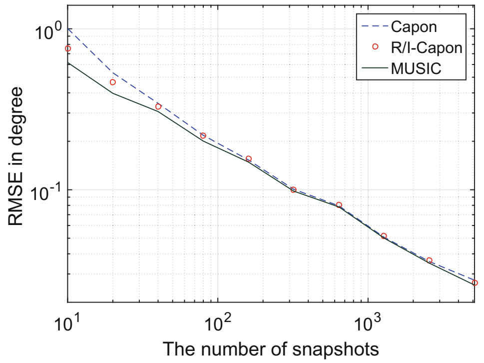

Fig. 2. RMSE against the number of snapshots, SNR = 10 dB.

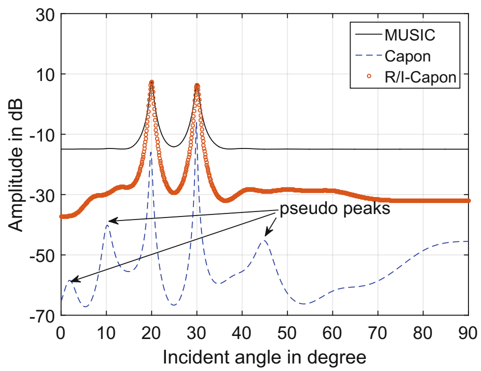

Fig. 3. Direction finding with $T = 20$ snapshots and SNR = 10 dB.

To see more clearly performance of new algorithm with small snapshots, Fig. 3 compares spectrums of different algorithms with $T = 20$. From Fig. 3, it is apparent that conventional Capon appears pseudo peaks while R/I-Capon does

Table 2. Matrix rank against small number of snapshots, SNR = 10 dB.

Items	$T = 30$	$T = 25$	$T = 20$	$T = 15$	$T = 10$
$\text{rank}(\widehat{\mathbf{R}}_{\mathbf{xx}})$	12	10	10	10	7
$\text{rank}[\text{Re}(\widehat{\mathbf{R}}_{\mathbf{xx}})]$	12	12	12	12	9

Table 3. Comparison of CPU time in second, SNR = 10 dB, $T = 100$.

Items	$M = 8$	$M = 12$	$M = 16$	$M = 20$	$M = 24$
Capon	1.8017	3.1400	3.3097	3.2956	3.5283
R/I-Capon	0.5003	0.8121	0.8201	0.8510	0.8806
MUSIC	1.3358	2.5361	2.6361	2.8725	3.0456

not at $T = 20$ snapshots. Moreover, we find the pseudo peak shows repetitively in conventional Capon spectral.

To further verify results observed from Figs. 2 and 3, Table 2 compares $\text{rank}(\widehat{\mathbf{R}}_{\mathbf{xx}})$ and $\text{rank}[\text{Re}(\widehat{\mathbf{R}}_{\mathbf{xx}})]$ with different small T's. It is shown from the table that $\text{Re}(\widehat{\mathbf{R}}_{\mathbf{xx}})$ does has an enhanced rank as compared to $\widehat{\mathbf{R}}_{\mathbf{xx}}$, which verifies the conclusions of Theorem 1.

Finally, Table 3 compares the CPU times of different algorithms against the number of sensors. Table 3 shows that R/I-Capon costs about 4 times lower simulation times than Capon in expectations.

5 Conclusions

A novel R/I-Capon algorithm for DOA estimation without investigation for number of sources has been proposed. The new algorithm develops only real part of ACM to decrease complexity by a factor about four. It has been demonstrated by Theoretical analysis as well as numerical simulations that R/I-Capon also shows a better property than Capon at small snapshots owing to the rank enhancement of R/I-Capon in comparison to conventional Capon.

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