DOA Estimation Based on Bayesian Compressive Sensing

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Abstract. In this paper, Bayesian Compressive Sensing algorithm is studied. To deal with signals with multiple snapshots, we extend traditional Bayesian algorithm under the condition of single snapshot to multi-snapshot Bayesian Compressed Sensing (MBCS) algorithm and apply MBCS algorithm to direction of arrival (DOA) estimation of narrowband signals and wideband signals. Simulation shows that the application of BCS to DOA has certain advantages in algorithm performance.

Keywords: Bayesian Compressed Sensing · DOA · Wideband signal

1 Introduction

Direction of Arrival (DOA) estimation is one of the important branches of array signal processing [1]. Since the Second World War, it has developed rapidly and has been widely used in military and civilian fields. The classic DOA estimation algorithm is divided into subspace division and subspace fitting two categories [2]. The typical algorithm of the former is the multiple signal classification (MUSIC) algorithm. Until now, a number of derivative algorithms have been proposed. The basic idea of the MUSIC algorithm is to decompose the received data into signal subspaces and noise subspaces that are orthogonal to each other and perform spectral peak searching based on the orthogonality to estimate DOA [3]. The representative algorithm of subspace fitting category is maximum likelihood (ML) algorithm. Compared with the subspace decomposition algorithm, the subspace fitting algorithm has better estimation accuracy under the condition of low signal to noise ratio. But because of the high complexity, subspace fitting algorithm is limited in engineering applications. In 1989, Roy and Kailayh proposed the rotation-invariant subspace (ESPRIT) algorithm which eliminates the spectral peak searching compared with MUSIC algorithm and increases the operation speed [4]. For the DOA estimation of wideband signals, the basic idea of most algorithms is to decompose wideband signal into multiple narrow band parts and deal with them separately. Incoherent signal subspace algorithm (ISSM) is the easiest way to estimate DOA of wideband signals. It divides the wideband signal into several narrowband signals, and then applies the MUSIC method to each narrowband signal directly. The coherent signal subspace algorithm (CSSM) proposed by Wang and Kaveh [5] presents the concept of focusing matrix and focusing frequency. By constructing the focusing matrix, the data of each frequency point is transferred to the

M. Jia et al. (Eds.): WiSATS 2019, LNICST 280, pp. 630–639, 2019.
https://doi.org/10.1007/978-3-030-19153-5_62

focusing frequency and each realize the estimation of the coherent signal DOA. Compressive Sensing (CS) was proposed by Donoho and Tao in 2006 and it has attracted extensive attention from the industry and research. Traditional DOA estimation algorithms have some common shortcomings and it is due to the characteristic of signal subspace theory. The rapid development of CS theory provides a new idea for DOA estimation. Combining CS with DOA estimation can effectively utilize the sparse spatial characteristics of the incoming wave direction and it can overcome the disadvantage of traditional methods to a certain extent. Tropp proposed orthogonal matching pursuit (OMP) algorithm which selects the most matching atomic in the measurement matrix and computes the residual. It can solve the disadvantage that the matching pursuit (MP) algorithm cannot guarantee orthogonality and cannot obtain the optimal solution. [6] introduces the spatial coupling in coding theory and proposes the compressed sensing algorithm in coding theory. In [7], A compressed sensing algorithm based on projection matrix optimization is proposed. [8] applies compressed sensing to DOA estimation under the assumption that the dimension of received data is less than the number of array elements; the literature [9] proposed a new measurement matrix model applied to DOA estimation, and it is proved that the new model performs better when the RIP property is satisfied. The literature [10] proposes DOA estimation algorithm for multi-band signal based on compressed sensing and the detection accuracy is improved. Bayesian Compressed Sensing (BCS) is one of the latest achievements in compressed sensing theory, which was proposed by Ji and Xue in 2008. The idea is to use Bayesian statistics to make a priori assumptions about the received signal and noise, and then calculate its maximum posterior probability. Babacan introduced the Laplace Prior based on the BCS framework and proposed the LP-BCS algorithm [11]; Wu proposed multi-task Bayesian Compressive Sensing Algorithm (CMT-BCS) for the complex signal. In this paper, we research the DOA estimation based on Bayesian Compressed Sensing.

In the following section, DOA estimation based on Bayesian Compressed Sensing is studied. Section 2 introduces the signal model. Section 3 describes different Bayesian Compressed Sensing algorithms. Section 4 is the simulation result of algorithm. Finally, Sect. 5 shows the conclusion.

2 Signal Model

In the problem for estimating DOA, the array data model is the basis of all algorithms. For different array models, the algorithm should be adjusted accordingly. In this paper, we assume the antenna array is a uniform line array, that is, all the antennas are arranged in a straight line and the space of each antenna is same. The number of array elements is M and the distance between the antenna elements is d. Treating the first array element as the reference point, we can obtain that the position of k^{th} array element can be expressed as follows.

$$r_k = (k-1) \cdot d \tag{1}$$

The array signal model is shown in Fig. 1.

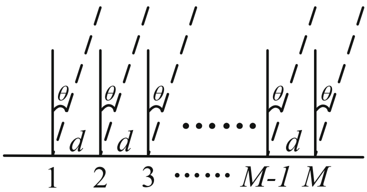

Fig. 1. Array signal model

2.1 Narrowband Signal Model

At present, there are several ways as follows to define narrowband signals.

(1) $B \ll f_0 \cdot B$ is the bandwidth of incoming signal and f_0 represents center frequency.
(2) $2v/c << 1/(TB) \cdot v$ is the speed of source relative to the array and c is the velocity of light.
(3) $(M-1)d/c << 1/B \cdot M$ represents the number of array elements, and d is the distance between elements.

In this paper, we adopt the first definition.

The incoming wave is far-field narrowband signal. For the i^{th} signal, the received data can be expressed as (2)

$$s_i(t-\tau) = u_i(t-\tau)e^{j(\omega_0(t-\tau)+\varphi_i(t-\tau))} \tag{2}$$

u represents the amplitude of received signal, and ω_0 is the frequency of signal. Signal phase is marked by φ and time delay is denoted by τ.

For different elements of array, there exists a delay between elements. With the first antenna as the reference point, the delay of the k^{th} antenna can be written as (3)

$$\tau_{ki} = (k-1) \cdot d \cdot \sin\theta_i / c \tag{3}$$

It is known that the amplitude and phase of narrowband signal are changing slowly and Eq. (4) can be obtained

$$\begin{aligned} u_i(t-\tau) &\approx u_i(t) \\ \varphi_i(t-\tau) &\approx \varphi_i(t) \end{aligned} \tag{4}$$

Then the received data of k^{th} antenna from i^{th} incoming wave can be written as (5)

$$s_i(t-\tau_{ki}) = u_i(t)e^{j(\omega_0(t-\tau)+\varphi_i(t))} = s_i(t)e^{-j\omega_0\tau_{ki}} = s_i(t)e^{-j\omega_0\frac{(k-1)\cdot d\cdot\sin\theta_i}{c}} \tag{5}$$

When the number of incoming wave is N, the received signal of the k^{th} antenna can be expressed as (6)

$$x_k(t) = \sum_{i=1}^{N} s_i(t - \tau_{ki}) + n_k(t) = \sum_{i=1}^{N} s_i(t) e^{-j\omega_0 \frac{(k-1)\cdot d \cdot \sin\theta_i}{c}} + n_k(t) \tag{6}$$

Then the received signal of array has the following form.

$$\begin{bmatrix} x_1(t) \\ \dots \\ x_M(t) \end{bmatrix} = \begin{bmatrix} e^{-j\omega_0\tau_{11}} & \dots & e^{-j\omega_0\tau_{1N}} \\ \dots & \dots & \dots \\ e^{-j\omega_0\tau_{M1}} & \dots & e^{-j\omega_0\tau_{MN}} \end{bmatrix} \begin{bmatrix} s_1(t) \\ \dots \\ s_N(t) \end{bmatrix} + \begin{bmatrix} n_1(t) \\ \dots \\ n_M(t) \end{bmatrix} \tag{7}$$

Equation (7) can be written in vector form as (8).

$$\mathbf{X} = \mathbf{AS} + \mathbf{N} \tag{8}$$

A represents array steering matrix whose size is $M \times N$. **S** is the signal to be estimated and **N** is Gaussian noise. The above is the narrowband signal model.

2.2 Wideband Signal Model

At present, there is no clear definition of the wideband signal. It is generally believed that signals which do not satisfy the definition of narrowband signals are wideband signals. For narrowband signals, the phase difference of the received signals of different array elements in the array is only related to the position of the array elements and the angle of the source. However, for the wideband signal, the phase difference of the received signals is not only related to the above two factors, but also related to the signal frequency and bandwidth. Therefore, the narrowband signal model cannot be applied to the wideband signal. It is usual to transform time domain form of signal into frequency domain form when we analyze wideband signals to use frequency and bandwidth information conveniently.

By performing DFT on the received signals, the received data of the k^{th} array can be expressed as (9)

$$X_k(f) = \sum_{i=1}^{N} S_i(f) e^{-jf\frac{(k-1)\times d \times \sin q_i}{c}} + N_k(f) \tag{9}$$

Assuming that the lower frequency of incoming wave is f_L and the upper frequency is f_H, we can know that the signal of each sub-band can be expressed as (10) when the wideband signal is divided into J sub-band.

$$\mathbf{X}(f_j) = \mathbf{A}(f_j, \theta)\mathbf{S}(f_j) + \mathbf{N}(f_j),\ j = 0, 1, \dots, J-1 \tag{10}$$

$\mathbf{A}(f_j, \theta)$ is the frequency domain form of array steering matrix at f_j and it can be expressed as (11).

$$\mathbf{A}\left(f_j,\theta\right) = \left[\mathbf{a}\left(f_j,\theta_1\right),\mathbf{a}\left(f_j,\theta_2\right),\ldots,\mathbf{a}\left(f_j,\theta_N\right)\right],\ j=0,1,\ldots,J-1 \tag{11}$$

$\mathbf{a}\left(f_j,\theta\right)$ is a column vector and has the following form.

$$\mathbf{a}\left(f_j,\theta_i\right) = \left[1, e^{-j\cdot f_j\frac{d\cdot\sin\theta_i}{c}},\ldots,e^{-j\cdot f_j\frac{(M-1)\cdot d\cdot\sin\theta_i}{c}}\right]^T,\ i=1,2,\ldots,N, j=0,1,\ldots,J-1 \tag{12}$$

The above is the array model for wideband signals.

3 Algorithm Description

3.1 MBCS Algorithm

DOA estimation must be performed under multiple snapshot conditions, this section presents a Bayesian Compression Sensing algorithm with multiple snapshots (MBCS) and apply it to narrowband and wideband DOA estimates.

Firstly, assuming that the noise is zero mean Gaussian white noise and the variance is σ^2, for each column of signal matrix $\mathbf{T}$ and sparse weight coefficient matrix $\mathbf{W}$, we have

$$p\left(\mathbf{t}_{\cdot j}|\boldsymbol{\omega}_{\cdot j}\right) = \left(2\pi\sigma^2\right)^{-N/2}\exp\left(-\frac{1}{2\sigma^2}\left\|\mathbf{t}_{\cdot j}-\boldsymbol{\Phi}\boldsymbol{\omega}_{\cdot j}\right\|^2\right) \tag{13}$$

Assume that each row element in the sparse weight coefficient matrix obeys Gaussian distribution whose mean is zero and variance is α_i. Then the distribution of $\mathbf{W}$ can be expressed as (14)

$$p(\mathbf{W};\boldsymbol{\alpha}) = \prod_{i=1}^{M} p(\boldsymbol{\omega}_i;\alpha_i) \tag{14}$$

The posterior distribution of the signal can be obtained as (15) based on likelihood estimation and prior estimation.

$$p\left(\boldsymbol{\omega}_{\cdot j}|\mathbf{t}_{\cdot j};\boldsymbol{\alpha}\right) = \frac{p\left(\boldsymbol{\omega}_{\cdot j},\mathbf{t}_{\cdot j};\boldsymbol{\alpha}\right)}{\int p\left(\boldsymbol{\omega}_{\cdot j},\mathbf{t}_{\cdot j};\boldsymbol{\alpha}\right)d\boldsymbol{\omega}_{\cdot j}} = N\left(\mu_{\cdot j},\boldsymbol{\Sigma}\right) \tag{15}$$

$\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are the mean and variance, and can be expressed as follows.

$$\begin{aligned}\boldsymbol{\mu} &= [\mu_1,\mu_2,\ldots,\mu_L] = E[\mathbf{W}|\mathbf{T};\boldsymbol{\alpha}] = \boldsymbol{\Lambda}\boldsymbol{\Phi}^T\mathbf{C}^{-1}\mathbf{T}\\ \boldsymbol{\Sigma} &= \mathrm{Cov}\left[\boldsymbol{\omega}_{\cdot j}|\mathbf{t}_{\cdot j};\boldsymbol{\alpha}\right] = \boldsymbol{\Lambda}-\boldsymbol{\Lambda}\boldsymbol{\Phi}^T\mathbf{C}^{-1}\boldsymbol{\Phi}\boldsymbol{\Lambda},\ j=1,2,\ldots,K\end{aligned} \tag{16}$$

$\boldsymbol{\Lambda} = \mathrm{diag}(\boldsymbol{\alpha})$, $\mathbf{C} = \sigma^2\mathbf{I}+\boldsymbol{\Phi}\boldsymbol{\Lambda}\boldsymbol{\Phi}^T$, and $\boldsymbol{\mu}$ is the parameter to be estimated.

By marginalizing likelihood function, the log-likelihood function of $\boldsymbol{\alpha}$ can be obtained as follows.

$$L(\boldsymbol{\alpha}) = -2\log_{10}\int p(\mathbf{T}|\mathbf{W})p(\mathbf{W};\boldsymbol{\alpha})d\mathbf{W}$$
$$= -2\log_{10}p(\mathbf{T};\boldsymbol{\alpha}) = K\log_{10}|\mathbf{C}| + \sum_{j=1}^{K}\boldsymbol{\omega}_{\cdot j}^{T}\mathbf{C}^{-1}\boldsymbol{\omega}_{\cdot j} \tag{17}$$

Solve the derivative of $L(\boldsymbol{\alpha})$ and make derivative equal to zero. The extreme point of $L(\boldsymbol{\alpha})$ is

$$\alpha_i^{new} = \frac{\frac{1}{K}\|\mu_i\|^2}{1-\alpha_i^{-1}\Sigma_{ii}} \tag{18}$$

Similarly, the estimated parameter σ^2 can be solved

$$\left(\sigma^2\right)^{new} = \frac{\frac{1}{K}||\mathbf{T}-\boldsymbol{\Phi}\boldsymbol{\mu}||_F^2}{N-M+\sum_{i=1}^{M}\frac{\Sigma_{ii}}{\gamma_i}} \tag{19}$$

3.2 DOA Estimation Based on MBCS Algorithm

Bayesian theory is only applicable to real-valued data. In order to apply it to DOA estimation, the signal model described in Sect. 2 cannot be directly used, and the signal model needs to be improved.

The complex signal data received by the array antenna is

$$\mathbf{y}_m = \mathbf{A}\mathbf{s}_m + \boldsymbol{\varepsilon}_m,\ m = 1, 2, \ldots, M \tag{20}$$

Convert the complex signal into the form shown in Eq. (21)

$$\mathbf{T}_m = \boldsymbol{\Phi}\mathbf{W} + \mathbf{N}_m,\ m = 1, 2, \ldots, M \tag{21}$$

$\mathbf{T}_m = [\mathrm{Re}\{\mathbf{y}_m\}, \mathrm{Im}\{\mathbf{y}_m\}]^T$ is the real matrix. $\mathbf{N}_m = [\mathrm{Re}\{\boldsymbol{\varepsilon}_m\}, \mathrm{Im}\{\boldsymbol{\varepsilon}_m\}]^T$ is the transformation of noise matrix, $\mathbf{W} = [\mathrm{Re}\{\mathbf{s}_m\}, \mathrm{Im}\{\mathbf{s}_m\}]^T$ is the transformation of signal matrix. The transformation of $\boldsymbol{\Phi}$ is the corresponding real steering matrix and can be written as

$$\boldsymbol{\Phi} = \begin{bmatrix} \mathrm{Re}\{\mathbf{A}\} & -\mathrm{Im}\{\mathbf{A}\} \\ \mathrm{Im}\{\mathbf{A}\} & \mathrm{Re}\{\mathbf{A}\} \end{bmatrix} \tag{22}$$

The algorithm steps applying LP-BCS to DOA estimation are as follows.

(1) Divide the angular space into equal spacing, and determine the real steering matrix $\boldsymbol{\Phi}$ according to (22)
(2) Transform the received data according to (21)

(3) Solve the sparse coefficient vector with the algorithm, the DOA information can be obtained

Figure 2 is the simulation result. The simulation condition is: incident angle are 10°, 50°; snapshot number is 10; signal to noise ratio is 0 dB; array element number 10; grid division 181 copies.

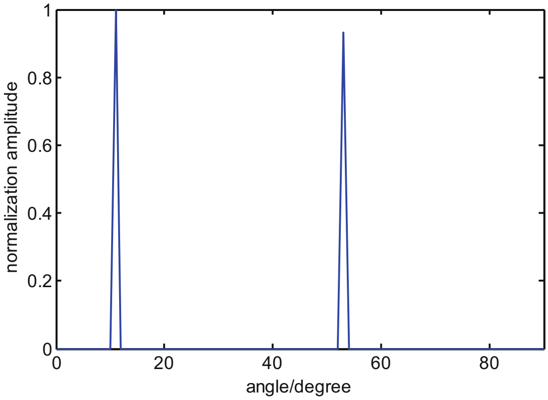

Fig. 2. Simulation result of MBCS

The steps for applying MBCS algorithm to wideband signal DOA estimation (MBCS-WDOA) are as follows:

(1) Sample the received signal with $L \times J$ points and divide signal into L groups.
(2) Transform the wideband signal into J narrowband signals by using discrete Fourier transform (DFT) to each group data.
(3) Construct measurement matrix of corresponding frequency point for each narrowband data.
(4) Apply MBCS algorithm to data for each frequency point and compute the result of each frequency point.
(5) Compute the average of the estimated values of each frequency point and solve the final result.

Figure 3 is the MBCS-WDOA simulation result. The simulation condition is: Incident angle is 40°; snapshot number is 6×128; signal to noise ratio is 0 dB; the number of array elements is 10; incident signal center frequency is 300 kHz; the bandwidth is 200 kHz.

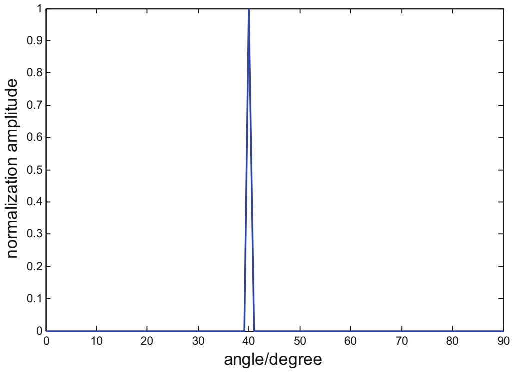

Fig. 3. Simulation result of MBCS-WDOA

4 Simulation Verification and Analysis

4.1 DOA Estimation for Narrowband Signal Based on MBCS

Figure 4 shows the estimation performance of MBCS, MUSIC and MMV-OMP. The simulation condition is: incidence angle is 10°; snapshot number is 10; array element number is 10.

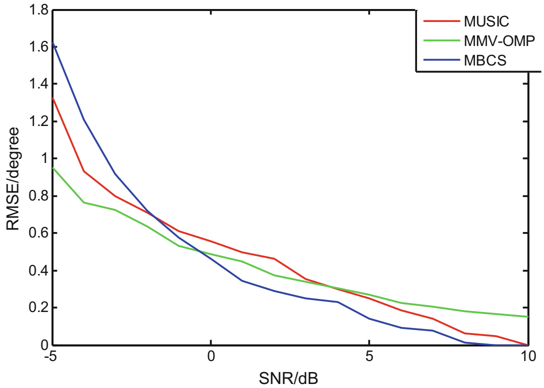

Fig. 4. Performance of MUSIC, MMV-OMP and MBCS

As can be seen from Fig. 4, at higher SNR, the performance of the MBCS is better than the MUSIC algorithm and MMV-OMP algorithm. This can be explained theoretically. The MBCS algorithm is essentially a process of learning the prior knowledge of signals and noise, that is, the estimation results of each time are related to the results of the previous calculation. When the signal-to-noise ratio is low, since the noise is random, and the MBCS algorithm has a simple a priori assumption on the signal, and the estimated hyper-parameter is less, the MBCS has a poor learning effect on the signal variance and the noise variance. The MUSIC algorithm utilizes subspace decomposition. Even if at low SNR, the eigenvalue of the signal subspace is still higher than that of noise subspace. However, with the increase of signal-to-noise ratio, the learning ability of MBCS is getting better. At this time, the advantage of MBCS multiple iteration learning to calculate the posterior probability is revealed, which can make full use of the better external environment.

4.2 DOA Estimation for Wideband Signal Based on MBCS

Figure 5 shows the estimation performance of MBCS, MUSIC and MMV-OMP. The simulation condition is: incidence angle is 10°; snapshot number is 6×128; array element number is 10; Incident signal center frequency is 300 kHz, and bandwidth is 200 kHz.

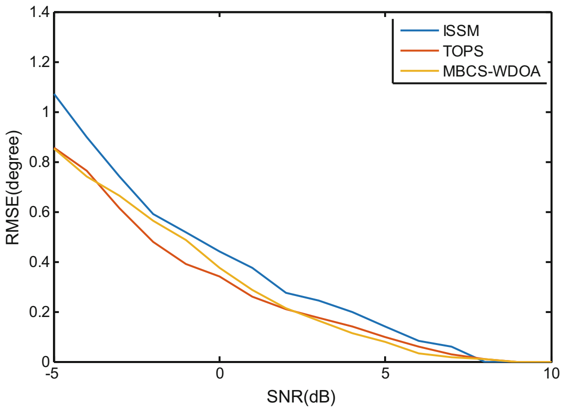

Fig. 5. Performance of MUSIC, MMV-OMP and MBCS

5 Conclusion

In this paper, we study Bayesian Compressed Sensing Theory and its application in DOA estimation. In the Bayesian Compressed Sensing Theory, we extend traditional Bayesian algorithm under the condition of single snapshot to multiple snapshots condition. Then MBCS algorithm is studied. Correspondingly, the likelihood function and parameter update formula of MBCS are given. In terms of DOA estimation, MBCS is applied in narrowband signal DOA estimation. At a higher signal-to-noise ratio, the performance is better than the other two compression sensing methods MMV-OMP and L1-SVD. The implementation scheme of MBCS-WDOA is given. Through simulation analysis, it is proved that its performance is better than ISSM, which is basically the same as the TOPS algorithm.

Acknowledgement. This work was supported by the Nation Science Foundation of China (Under Grant: 61671176).

References

1. Candès, E.J., Wakin, M.B.: An introduction to compressive sampling. IEEE Signal Process. Mag. **25**(2), 21–30 (2008)
2. Candès, E.J., Romberg, J., Tao, T.: Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information. IEEE Trans. Inf. Theory **52**(2), 489–509 (2006)
3. Vallet, P., Mestre, X., Loubaton, P.: Performance analysis of an improved MUSIC DoA estimator. IEEE Trans. Signal Process. **63**(23), 6407–6422 (2015)
4. Amiri, P.M., Ghofrani, S.: MUSIC algorithm for DOA estimation of coherent sources. IET Signal Process. **11**(4), 429–436 (2017)
5. Ghofrani, S., Amin, M.G., Zhang, Y.D.: High-resolution direction finding of non-stationary signals using matching pursuit. Signal Process. **93**(12), 3466–3478 (2013)
6. Parian, M.A., Ghofrani, S.: Using $\ell 1, 2$ mixed-norm MUSIC based on compressive sampling for direction of arrival estimation. In: IEEE International Symposium on Signal Processing and Information Technology, pp. 258–263 (2015)
7. Steinwandt, J., Roemer, F., Haardt, M., et al.: R-dimensional esprit-type algorithms for strictly second-order non-circular sources and their performance analysis. IEEE Trans. Signal Process. **62**(18), 4824–4838 (2014)
8. Donoho, D.L., Javanmard, A., Montanari, A.: Information-theoretically optimal compressed sensing via spatial coupling and approximate message passing. IEEE Trans. Inf. Theory **59** (11), 7434–7464 (2013)
9. Stanković, L., Orović, I., Stanković, S., et al.: Compressive sensing based separation of nonstationary and stationary signals overlapping in time-frequency. IEEE Trans. Signal Process. **61**(18), 4562–4572 (2013)
10. Li, G., Zhu, Z., Yang, D., et al.: On projection matrix optimization for compressive sensing systems. IEEE Trans. Signal Process. **61**(11), 2887–2898 (2013)
11. Ibrahim, M., Roemer, F., Del, G.G.: On the design of the measurement matrix for compressed sensing based DOA estimation. In: IEEE International Conference on Acoustics, Speech and Signal Processing, pp. 3631–3635 (2015)