

# Development of Trigonometry Teaching Materials with Knisley Mathematical Models

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**Abstract.** The aim of this study was to develop trigonometric teaching materials with a knisley mathematical model. The development of teaching materials for the knisley mathematical model includes three stages, namely preliminary design, design experiment, restrospective analysis. Furthermore, the teaching material was reviewed by three validators. Based on the results of the analysis of the validity of trigonometric teaching materials with a mathematical model, it can be concluded that teaching materials are categorized into clear categories and can be applied in learning activities.

**Keywords:** Trigonometric Teaching Materials, Knisley Mathematical Models

## 1 Introduction

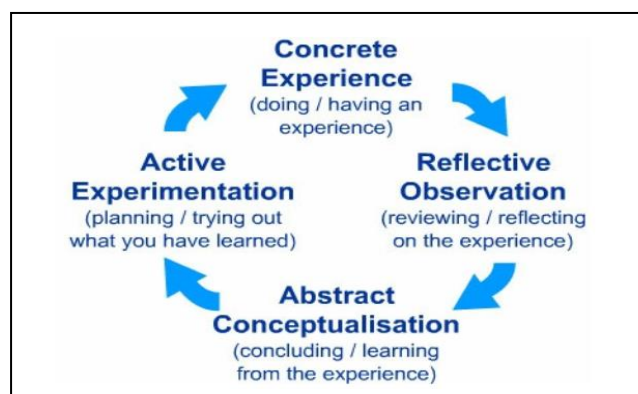
The higher education curriculum is a set of plans and arrangements regarding the objectives, contents, and teaching materials as well as the methods used as guidelines for the implementation of learning activities to achieve the goals of higher education [1]. Based on the understanding of the curriculum, one that needs to be considered is teaching material. Teaching materials are a means to help students learn well [2]. The teaching material means to achieve learning goals or learning outcomes. In addition [3] states that a teacher must find additional teaching materials needed if the availability of available teaching materials is considered insufficient. This means that the importance of certain teaching materials in teaching material to students. Learning outcomes of graduates of the study program are in accordance with national standards for higher education in 2014 [1] in the aspect of knowledge is the level of mastery, breadth, and depth of knowledge that characterizes the study program. One of the courses in the mathematics education study program is trigonometry. Trigonometry is one of the subjects studied in senior high school and in the National Examination [4].

The reality, most students who encounter the sine, cosine, tangent, and cotangent concepts for the first time in their high school or university education cannot relate these concepts to real-life situations and do not know where they come from [5]. With these things, the method used in teaching trigonometric concepts must prioritize the meaningfulness of learning, where students do not memorize concepts, do not understand concepts. In addition, [6] suggests that students be given information by the teacher indirectly, but plan learning activities that result in the formation of formal knowledge of mathematics built on students' understanding of mathematical concepts. Trigonometric concepts that are given and taught by the teacher to students by memorizing can add to the cognitive burden on students [7]. This, explains that the trigonometric concept cannot be understood only by memorizing which has an impact on the classical teaching method. In line with the view [8] which states that teaching in classical classes

on trigonometry concepts will show unfavorable effects at the college level. Therefore, most pre-service teachers have conceptual difficulties with situation trigonometric relationships that limit their ability to explain basic trigonometric identities or apply relationships in proving basic trigonometric identities [9]. This is supported by Mogari & Lupahla [10] say that "mathematical problems that are more complicated and difficult, do not have direct solutions, require productive thinking, are approached in ways that are more or less sophisticated, non-standard, involve unexpected and unknown solutions, requires an approach to insight and strategic thinking and involves the use of various mathematical concepts". While from the aspect of learning resources, a lot of mathematics books provided in bookstores merely give constructed steps without allocating any space for the students to be engaged in the process of finding the formulas[7]. Because of this, trigonometric teaching materials are needed that can help students and teachers to more easily understand the concept of trigonometry without having to memorize.

Therefore, mathematical material studied with success is influenced by many factors [11]. One of them held a renewal in how to deliver trigonometric lectures so that the output of trigonometric lectures was in accordance with the learning outcomes. One of them is by providing teaching material with mathematics. The renewal of the way in which the material is delivered is inseparable from the opinion of Piaget [10] seeing that the thought process is a gradual activity of intellectual function, which starts from thinking concretely into abstract thinking at the formal operational stage. [4] Kolb's learning cycle have four phase of the student activities, that is concrete-reflective related to allegorization, concrete-active related to integrators, abstracts-reflective related to analysis, and abstract-active relating to synthesizers.

[4] explained that the learning process of experience is Kolb's learning model, was adopted by Knisley, which is useful for learning mathematics. The following learning cycles are in the Knisley mathematical model [13]:



**Figure 1.** Four Stage Learning Cycle

From **Figure 1**, there are four basic learning cycles in the Knisley mathematical model, the learning cycle has meanings including: 1) Concrete Experience is a new experience of situation is encountered, or a reinterpretation of existing experience, 2) Reflective Observation of the new experience. Of particular importance are any inconsistencies between experience and understanding, 3) Abstract Conceptualisation is reflection gives rise to a new idea, or a modification of an existing abstract concept, and 4) Active Experimentation is the learner applies them to the world around them to see what results.

Kolb's learning cycle have been modified by the same means by [6] by fourth learning style mathematical Knisley:

1. Concrete-Reflective: a concept describe by teacher with figurative in familiar contexts based on perceptual student concepts.
2. Concrete-Active: The teacher gives assignments and encouragement so that students explore, experiment, measure, or compare so that they can distinguish this new concept from the concepts they have known
3. Abstract-Reflective: Students make or choose statements related to new concepts, give counter examples to deny false statements, and prove the correct statement together with the teacher
4. Abstract-Active: Students practice (practice) using new concepts to solve problems and develop strategies

Therefore, so that students do not consider trigonometric material to be a difficult mathematical material, for this reason we conduct research with the aim of developing trigonometric teaching materials with Knisley mathematics?

## 2 Method

### 2.1 Research Procedures

The procedure of this research includes several stages, [15] suggests that the stages are divided into three stages, namely preliminary design, design experiment, retrospective analysis. Following is the calculation of the validity of the development of teaching materials from the validator on all aspects assessed, presented in table form. Next, the average score is searched by using the formula (1), Muliyardi [16]:

$$R = \frac{\sum_{i=1}^n V_i}{n} \quad (1)$$

Information:

$R$  = average results of the evaluation of the validators

$V_i$  = score of the  $i$ -validator's assessment

$n$  = many validators

Then the average obtained is confirmed by the specified criteria. How to get criteria by following provisions such as, 1) range of scores ranging from 0 to 4; 2) criteria consist of five levels that are adjusted to the related aspects; and 3) the average range is divided into five interval classes.

For example, for aspects of the formulation of competency indicators criteria are used with the following terms: 1) If the average is  $> 3.20$  then the aspect considered is categorized very clearly; 2) If  $2.40 < \text{mean} \leq 3.20$  then it is categorized clearly; 3) If  $1.60 < \text{mean} \leq 2.40$  then it is categorized quite clearly; 4) If  $0.80 < \text{mean} \leq 1.60$  then it is not clear enough; and 5) If the mean is  $\leq 0.80$  then it is not clear. Then the average of all aspects for the module is calculated. To determine the validity level of the module, the following criteria are used, 1) If the average  $> 3.20$  then the workbook is categorized as very valid; 2) If  $2.40 < \text{mean} \leq 3.20$  then it is considered

valid; 3) If  $1.60 < \text{mean} \leq 2.40$  then it is considered quite valid; 4) If  $0.80 < \text{mean} \leq 1.60$  then it is categorized as less valid; and 5) If the mean is  $\leq 0.80$  then it is considered invalid.

## 2.2 Subjects and Research Locations

The subject of this research are 53 students from fourth semester 2018/2019 on mathematics education departement IKIP Siliwangi. This research done at IKIP Siliwangi. The subjects studied trigonometric materials using trigonometric teaching materials developed using the Knisley mathematical model then the researchers conducted interviews to strengthen the validity of the previous validators.

## 3 Result and Discussion

The development of trigonometry teaching materials through three stages in the preparation, before the teaching material is studied by students who are used as research subjects. The following are the results of the development of trigonometric teaching materials.

### 3.1 Result

The first stage, a preliminary design to develop a sequence of learning activities and design instruments to evaluate the learning process. The learning process starts from RPS analysis, analyzes trigonometric textbooks, reviews literature related to modules or teaching materials, and interviews with previous generation students to find out the obstacles in trigonometry lectures, conduct interviews with trigonometry lecturers. The second stage, an experimental design to explore and predict students' strategies and thoughts during the learning process. Explore by developing teaching materials by applying the Knisley mathematical model. Third stage, restrospective analysis to develop an evaluation of the success of learning activities that have been carried out. Evaluate teaching materials by validating to develop new results or novelty. The following aspects of validation were analyzed:

**Table 1.** Teaching Material Indicator Analysis

No	Aspect	Validator Assessment			Average	Criteria
		1	2	3		
1	Purpose of teaching materials	4	3	4	3.67	Very Clearly
2	Rationality of teaching materials	3	3	4	3.33	Very Clearly
3	Content of teaching material	3	3	3	3.00	Clearly
4	Characteristics of teaching materials	3	2	3	2.67	Clearly
5	Suitability of teaching materials	4	4	3	3.67	Very Clearly
6	Language	3	3	3	3.00	Clearly
7	Physical form of teaching material	3	3	3	3.00	Clearly
8	Flexibility of teaching materials	3	3	3	3.00	Clearly

**Table 1.** shows the averages on indicators of trigonometric teaching materials with knisley mathematical models. The teaching material was validated by three validators where the validator was a lecturer in trigonometry courses and expert experts. Then after the per-indicator data analysis, the validation values of each validator are analyzed as in the table below.

**Table 2.** Teaching Material Validity Recapitulation

No	Aspect	Validator Assessment		
		1	2	3
1	Purpose of teaching materials	4	3	4
2	Rationality of teaching materials	3	3	4
3	Content of teaching material	3	3	3
4	Characteristics of teaching materials	3	2	3
5	Suitability of teaching materials	4	4	3
6	Language	3	3	3
7	Physical form of teaching material	3	3	3
8	Flexibility of teaching materials	3	3	3
Validity		3.17		
Category		Valid		

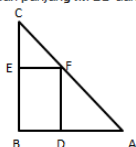
**Table 3.** Recapitulation of Suggestions from Validators

Validator	Suggestion
1	in the exercises on the subject section, give a few difficult or difficult questions it's really difficult and non-routine, in order to improve critical thinking skills
2	don't reduce the formula too much, because inhibits on many exercises. Because on basically trigonometry focuses on practice
3	it would be better if the formula decreases, do not be given lternatives/guidelines to improve critical thinking skills of students, so the tip in the process the questions will not focus on the decline the formula that is in the teaching material.

### 3.2 Discussion

Based on tables 1, 2, and 3, the results of the analysis of trigonometric teaching materials are used using a mathematical model. On indicators of trigonometric teaching materials it can be seen that the classification of indicators of teaching materials is explained very clearly and clearly. Furthermore, in the analysis of validity, teaching materials are categorized as valid. This means that the instructional materials prepared are sufficient to be used in learning, especially trigonometric materials. But based on the results of the validity analysis of several validators, some validator inputs were obtained, namely the practice of the questions on the teaching materials was not too challenging and would be better if added with non-routine questions. Whereas according to [17] non-routine problems are problems whose results are unpredictable. However, the aspect of teaching material characteristics with the Knisley mathematical model is still not visible because the validation of the three validators is not perfect. Likewise, the content, language, physical form and flexibility of teaching materials are still considered sufficient by the three validators. This is due to our limitations as researchers compile teaching materials, still lacking references especially in the discussion of trigonometric material.

1. Diketahui segitiga  $ABC$  siku-siku di  $C$ , sudut di  $A$  adalah  $\alpha$ , jika  $\cos \alpha = \frac{4}{5}$ . Tentukan perbandingan  $\sin \alpha$  dan  $\tan \alpha$ .
2. Pada gambar adalah segitiga siku-siku di titik  $B$  dengan panjang sisi  $AD = 4$ ,  $AF = 6$ ,  $FC = 3$ ,  $CE = 3$ . Berapakah panjang sisi  $BD$  dan  $BE$ ?



3. Dalam  $\triangle ABC$  diketahui bahwa  $\cos \angle A = \frac{3}{5}$  dan  $\cos \angle B = \frac{12}{13}$ . Berapakah nilai  $\cos \angle C$ ?
4. Dengan menggunakan gambar segitiga siku-siku dan  $\alpha$  salah satu sudutnya, hitunglah perbandingan trigonometri yang belum diketahui.
 

a. $\sin \alpha = p$	b. $\tan \alpha = s$	c. $\cos \alpha = q$
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**Fig. 1.** Examples of Problems Contained in Teaching Materials

From **Figure 1**, an example of the problems contained in trigonometry teaching materials that are commonly studied, then developed into critical thinking problems contained in the teaching material in **Figure 2**. The development of teaching materials is needed along with the development of science and technology today. This is in line with [18] who said that the development of science and technology has an impact on the ability to think critically, systematically, logically, think creatively and work together effectively that everyone needs to have.

1. Tentukan nilai  $\tan 120^\circ$  dengan menggunakan aturan sudut berelasi di kuadran II!
2. Tunjukkan bahwa syarat supaya grafik  $y = 2\cos(x + 30)^\circ$  dalam interval  $0^\circ \leq x \leq 360^\circ$  memiliki  $n$  maks dan  $n$  min nya adalah 2 dan -2!
3. Buktikan bahwa  $\csc^2 x \cdot \sec x = (1 + \cot^2 x) \sec x$
4. Suatu segitiga dengan panjang sisi berturut-turut adalah 5, 7, 9. Jika  $\beta$  adalah sudut yang berada pada depan sisi terpanjang, apakah masalah tersebut dapat dirubah ke dalam model matematika? tentukan  $\sin \beta$  dan  $\tan \beta$ !
5. Andaikan terdapat beberapa himpunan penyelesaian
 
$$\left\{x \mid 0 \leq x < \frac{\pi}{6}\right\}, \left\{x \mid \frac{\pi}{6} \leq x < \frac{\pi}{4}\right\}, \left\{x \mid \frac{\pi}{4} < x < \frac{2\pi}{3}\right\}, \left\{x \mid \frac{3\pi}{4} < x < \frac{7\pi}{6}\right\},$$

$$\left\{x \mid \frac{7\pi}{6} < x < \frac{5\pi}{4}\right\}$$
 data himpunan penyelesaian manakah yang tidak digunakan ketika menunjukkan bahwa  $\tan 2x < \sqrt{3}$ ,  $0 \leq x \leq 2\pi$

**Fig. 2.** Examples of Critical Thinking Problems Contained in Teaching Materials

It is hoped that the input can be used as a reference or case study in the development of subsequent teaching materials so that teaching materials become very clear and very valid and very suitable to be used as teaching materials in trigonometric lectures. The validity of all validators reaches an average of 3.17 with a clear category, meaning that the instructional materials are sufficiently fulfilled to be used in learning, especially trigonometric materials. Research is in line with [19] that lectures using teaching materials and students' tasks that have been developed following the stages of MPMK are quite effective. Besides this, learning with MPMK has a positive impact on learning achievement [20].

## 4 Conclusion

The development of trigonometric teaching materials arranged in clear terms means that trigonometric teaching materials with Knisley mathematical models are suitable for use in learning but there are still deficiencies in teaching materials that need to be refined and developed more deeply.

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