

Jamming game with incomplete information about the jammer*

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ABSTRACT

In the present work we deal with the concept of jamming with incomplete information about the jammer. We consider two scenarios. In the first one a jammer could be either present in the environment bringing extra background noise or absent. The user has only statistical knowledge about either presence or absence of the jammer. Namely, the user knows that in the environment only a natural background noise could be with probability γ meanwhile with probability $1 - \gamma$ a jammer could come into the action distributing an extra noise of the total power \bar{J} among the channels. In the second scenario the user does not know exactly the total jamming power. Namely, the user knows that with probability γ it could be \bar{J}^1 and with probability $1 - \gamma$ it could be \bar{J}^2 . All the problems are modelled as non-zero sum games. The equilibrium strategies are found in closed form.

Categories and Subject Descriptors

C.2.1 [Computer-communication networks]: Wireless communication

General Terms

Theory

Keywords

Wireless Networks, Power Control, Nash Equilibrium, Incomplete Information, Jamming, SINR

1. INTRODUCTION

*The work was supported by EGIDE ECO-NET grant no.18933SL "Game Theory for Wireless Networks" and RFBR Grant no.09-01-00334-a.

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Gamecomm 2009, October 23, 2009 - Pisa, Italy.

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The problem of jamming plays an important role in ensuring the quality and security of wireless communications, especially at this moment when wireless networks are quickly becoming ubiquitous. The recent literature covers a variety of jamming problems [3], [5], [1], [9], [2], [13], [14], [15].

Since jamming can be considered as a game in which a jammer is playing against a user (transmitter) who would like to transmit signal with good quality and at the same time with a reasonable amount of energy, game theory is an appropriate tool for dealing with jamming. Here we investigate the effect of partially available information about the jammer.

As an objective function for the transmitter we consider SINR. To the best of our knowledge, the SINR as an objective function in the power control game was only considered in [7, 12]. In [12] all users have a single common channel and choose between several base stations. And in [7] the authors has considered the power control game between users, not between a user and a jammer. We note that in the regime of low SINR the present objective can serve as an approximation to the Shannon capacity. A central motivation to consider SINR as an objective function and not Shannon capacity, is that current technology for voice over wireless does not try to achieve Shannon capacity but rather uses given codecs that can adapt the transmission rate to the SINR; these turn out to adapt the rate in a way that is linear in the SINR over a wide range of throughput. The SINR has therefore been used very often to represent directly the throughput see [17, 11]. The validity of this can be seen e.g. in [10, p. 151, 222, 239]. As we see from [10, Fig. 10.4, p. 222], the ratio between the throughput and the SINR is close to a constant throughout long range of bit rates. For example, between 16Kbps and 256Kbps, the maximum variation around the median value is less than 20%.

In the present work we deal with the concept of jamming with incomplete information about the jammer. We consider two scenarios. In the first one a jammer could be either present in the environment bringing extra background noise to the natural one or absent. The user has only statistical knowledge about either presence or absence of the jammer. Namely, the user knows that in the environment only a natural background noise could be with probability γ meanwhile with probability $1 - \gamma$ a jammer could come into the action distributing an extra noise of the total power \bar{J} among the channels. In the second scenario the user does not know exactly the total jamming power. Namely, he knows that

with probability γ it could be \bar{J}^1 and with probability $1 - \gamma$ it could be \bar{J}^2 . All the problems are modelled as non-zero sum games.

It is worth to mention that the considered jamming game relates to resource allocation games which have a lot of application with military flavour (say, Colonel Blotto game [16] or Star War game [8]) and in search theory [8]. Also, the jamming game principally differs with the game of several cooperative jammers [6]. In the several jammers game their optimal strategies are time-shared meanwhile in the considered game the optimal jammers strategies tell to jam the same set of the channels besides an extra set of the channels which are jammed by bigger expected jamming power.

We finally note our game serves as an extension of the standard water-filling scheme for the case of incomplete information with SINR as objective which allows us to find solution in closed form for the jamming scenario. It would be worth to mention that jamming and selfish transmission scenarios with the Shannon capacity as an objective and complete information in the framework of water-filling scheme were investigated in [3, 4].

2. THE USER DOES NOT KNOW WHETHER THE JAMMER IS PRESENT OR NOT

In this scenario one user (transmitter) should assign different power levels for different channels to maximize the objective function v . In the environment a jammer could be either present bringing extra background noise to the natural one or absent. The user has only statistical knowledge about either presence or absence of the jammer. Namely, the user knows that in the environment only a natural background noise could be with probability γ meanwhile with probability $1 - \gamma$ a jammer could come into the action distributing an extra noise of the total power \bar{J} among the channels. So, the pure strategy of the user is $T = (T_1, \dots, T_n)$ with $T_i \geq 0, i \in [1, n]$ such that

$$\sum_{i=1}^n T_i = \bar{T}, \quad (1)$$

where $\bar{T} > 0$ is the total available power for the user to transmit, n is the number of the channels and T_i is the power level assigned for channel i . The strategy of jammer is $J = (J_1, \dots, J_n)$ with $J_i \geq 0, i \in [1, n]$ such that

$$\sum_{i=1}^n J_i = \bar{J}, \quad (2)$$

where $\bar{J} > 0$ is the total jamming power.

The payoff to the user is his expected SINR, so it is given as follows:

$$v_T(T, J) = \gamma \sum_{i=1}^n \frac{\alpha_i T_i}{N^0} + (1 - \gamma) \sum_{i=1}^n \frac{\alpha_i T_i}{N^0 + \beta_i J_i}. \quad (3)$$

We do not deal here with zero-sum game since jammer does not take care about how to decrease the expected payoff of user (so, to minimize $-v_T(T, J)$) but he wants to bring real harm for user transmission, thus he wants to minimize the user SINR. Thus, we deal with non-zero-sum game framework and the cost to the jammer is the user SINR, so his

payoff is the user SINR taken with negative sign and it is given as follows:

$$v_J(T, J) = - \sum_{i=1}^n \frac{\alpha_i T_i}{N^0 + \beta_i J_i}, \quad (4)$$

where N^0 is the background noise level and $\alpha_i > 0$ and $\beta_i > 0$ are fading channel gains of user and jammer for channel i . We will assume that all the fading channel gains α_i and β_i , the noise level N^0 , the total powers \bar{T} and \bar{J} and the probability γ the jammer comes into action are known to both players.

We shall look for a Nash equilibrium, that is, we want to find $(T^*, J^*) \in A \times B$ such that for any $(T, J) \in A \times B$,

$$\begin{aligned} v_T(T, J^*) &\leq v_T(T^*, J^*), \\ v_J(T^*, J) &\leq v_J(T^*, J^*) \end{aligned}$$

where A and B are the sets of all the strategies of the user and the jammer, respectively.

Note that payoff (3) is the linear on T . Also,

$$\frac{\partial v_J}{\partial J} = - \frac{2\alpha_i \beta_i^2 T_i}{(N^0 + \beta_i J_i)^3} < 0.$$

Thus, we can apply a mix of linear and non-linear optimization approaches to get the following result.

THEOREM 1. *(T, J) is an equilibrium if and only if there are ω (the minimal expected induced noise) and ν (the Lagrangian multiplier) such that*

$$T_i \begin{cases} \geq 0, & \gamma \frac{\alpha_i}{N^0} + (1 - \gamma) \frac{\alpha_i}{N^0 + \beta_i J_i} = \omega, \\ = 0, & \gamma \frac{\alpha_i}{N^0} + (1 - \gamma) \frac{\alpha_i}{N^0 + \beta_i J_i} < \omega \end{cases} \quad (5)$$

and

$$\frac{\alpha_i \beta_i T_i}{(N^0 + \beta_i J_i)^2} \begin{cases} = \nu, & J_i > 0, \\ < \nu, & J_i = 0. \end{cases} \quad (6)$$

It is clear by (6) that the jammer is going to harm only the channels employed by the user for transmission. Then, by (5), the jamming equilibrium strategy has to have the form as follows:

$$J_i = J_i(\omega) := \frac{\alpha_i}{\beta_i} \left(\frac{1 - \gamma}{\omega - \gamma \alpha_i / N^0} - \frac{N^0}{\alpha_i} \right) \text{ while } J_i > 0, \quad (7)$$

where ω has to be given as the root of the equation

$$\sum_{i=1}^n J_i(\omega) = \bar{J}.$$

After finding the jamming equilibrium strategy, by (6), we obtain the following relation for the user equilibrium strategy:

$$\frac{1}{(1 - \gamma)^2} T_i \frac{\beta_i}{\alpha_i} \left(\omega - \gamma \frac{\alpha_i}{N^0} \right)^2 = \nu \text{ while } T_i > 0.$$

Thus,

$$T_i = \frac{\alpha_i}{\beta_i} \frac{\nu}{\left(\omega - \gamma \frac{\alpha_i}{N^0} \right)^2} \quad (8)$$

Summing up the last relation yields:

$$\nu = \frac{\bar{T}}{\sum_{T_i > 0} \frac{\alpha_i}{\beta_i} \frac{1}{\left(\omega - \gamma \frac{\alpha_i}{N^0}\right)^2}}.$$

This jointly with (7) and (8) imply the following result describing the equilibrium in closed form.

THEOREM 2. *The game has the unique equilibrium (T, J) where the equilibrium jamming strategy is given as follows*

$$J_i = \frac{\alpha_i}{\beta_i} \left[\frac{1 - \gamma}{\omega - \gamma \alpha_i / N^0} - \frac{N^0}{\alpha_i} \right]_+ \text{ for } i \in [1, n]$$

with $\omega = \omega_*$ is the unique root of the following equation in $[\gamma \max \alpha_i / N^0, \infty)$

$$\sum_{i=1}^n \frac{\alpha_i}{\beta_i} \left[\frac{1 - \gamma}{\omega - \gamma \alpha_i / N^0} - \frac{N^0}{\alpha_i} \right]_+ = \bar{J}$$

and the equilibrium user strategy is given as follows:

$$T_i = \begin{cases} \frac{1}{\sum_{m \in I^J(\omega)} \frac{\beta_m}{(\omega - \gamma \alpha_m / N^0)^2} \alpha_m} \frac{\beta_i}{(\omega - \gamma \alpha_i / N^0)^2} \alpha_i \bar{T}, & i \in I^J(\omega) \\ 0, & i \notin I^J(\omega). \end{cases}$$

where

$$I^J(\omega) = \{i \in [1, n] : \gamma \alpha_i / N^0 < \omega\}.$$

3. NUMERICAL EXAMPLES

In this section we present some numerical examples of the equilibrium strategies. We consider the case when the jammer is near with five channels ($n = 5$). In this scenario $\alpha = (1, 2, 3, 4, 5)$, $\beta = (5, 4, 3, 2, 1)$, $N^0 = 1$ and $\bar{T} = 10$. In Table 1, 2 and 3 we give strategies of the user and jammer for different value of γ for three cases of the total jamming power, namely, (a) with jamming power $\bar{J} = 2$ essential smaller the user's power (Table 1), (b) with jamming power $\bar{J} = 7$ comparable to the user's power (Table 2) and (c) with jamming power $\bar{J} = 20$ essential larger the user's power (Table 3).

Of course with increasing jamming power more channels become involved in jamming. But with increasing of γ the effect of increasing jamming power on the equilibrium strategies vanishes.

4. THE USER DOES NOT KNOW EXACTLY THE TOTAL JAMMING POWER

In this section we consider the scenario where the user does not know exactly the total jamming power. Namely, he knows that with probability γ it could be \bar{J}^1 and with probability $1 - \gamma$ it could be \bar{J}^2 . This situation can be described as if the user plays with an agent whose nature is unknown

Table 1: Dependence of the strategies on γ for $\bar{J} = 2$

γ	J/T	1	2	3	4	5
0.1	J	0.00	0.00	0.11	0.43	1.46
	T	0.00	0.00	5.56	3.07	1.37
0.2	J	0.00	0.00	0.07	0.40	1.53
	T	0.00	0.00	5.17	3.20	1.62
0.3	J	0.00	0.00	0.03	0.36	1.62
	T	0.00	0.00	4.72	3.30	1.98
0.4	J	0.00	0.00	0.00	0.30	1.70
	T	0.00	0.00	0.00	5.75	4.25
0.5	J	0.00	0.00	0.00	0.22	1.78
	T	0.00	0.00	0.00	5.08	4.92
0.6	J	0.00	0.00	0.00	0.12	1.88
	T	0.00	0.00	0.00	4.18	5.82
0.7	J	0.00	0.00	0.00	0.00	2.00
	T	0.00	0.00	0.00	3.03	6.97
0.8	J	0.00	0.00	0.00	0.00	2.00
	T	0.00	0.00	0.00	0.00	10.00
0.9	J	0.00	0.00	0.00	0.00	2.00
	T	0.00	0.00	0.00	0.00	10.00

Table 2: Dependence of the strategies on γ for $\bar{J} = 7$

γ	J/T	1	2	3	4	5
0.1	J	0.00	0.16	0.58	1.53	4.72
	T	0.00	4.60	2.79	1.73	0.88
0.2	J	0.00	0.07	0.43	1.38	5.13
	T	0.00	3.82	2.71	2.06	1.41
0.3	J	0.00	0.00	0.28	1.18	5.54
	T	0.00	0.00	3.54	3.27	3.19
0.4	J	0.00	0.00	0.15	0.94	5.91
	T	0.00	0.00	2.70	2.95	4.35
0.5	J	0.00	0.00	0.04	0.69	6.28
	T	0.00	0.00	1.88	2.38	5.74
0.6	J	0.00	0.00	0.00	0.43	6.57
	T	0.00	0.00	0.00	1.89	8.11
0.7	J	0.00	0.00	0.00	0.17	6.83
	T	0.00	0.00	0.00	1.04	8.96
0.8	J	0.00	0.00	0.00	0.00	7.00
	T	0.00	0.00	0.00	0.00	10.00
0.9	J	0.00	0.00	0.00	0.00	7.00
	T	0.00	0.00	0.00	0.00	10.00

or unknown type of jammer. Then we can think that in the environment a jammer of two types $k = 1, 2$ is possible with strategies $J^k = (J_1^k, \dots, J_n^k)$ such that $J_i^k \geq 0, i \in [1, n]$ and

$$\sum_{i=1}^n J_i^k = \bar{J}^k. \quad (9)$$

The payoff to the user is his expected SINR, so it is given as follows

$$v_T(T, J) = \gamma \sum_{i=1}^n \frac{\alpha_i T_i}{N^0 + \beta_i J_i^1} + (1 - \gamma) \sum_{i=1}^n \frac{\alpha_i T_i}{N^0 + \beta_i J_i^2}. \quad (10)$$

The payoff of the jammer of type k with the total jamming power $\bar{J}^k, k = 1, 2$ is the user's SINR taken with negative sign. So, it is given as follows:

$$v_J^k(T, J) = - \sum_{i=1}^n \frac{\alpha_i T_i}{N^0 + \beta_i J_i^k}. \quad (11)$$

Table 3: Dependence of the strategies on γ for $\bar{J} = 20$

γ	J/P	1	2	3	4	5
0.1	J	0.06	0.50	1.47	4.00	13.98
	P	4.07	2.21	1.59	1.24	0.88
0.2	J	0.00	0.23	0.92	3.14	15.71
	P	0.00	2.50	2.15	2.28	3.07
0.3	J	0.00	0.07	0.55	2.33	17.05
	P	0.00	1.57	1.49	1.93	5.00
0.4	J	0.00	0.00	0.29	1.65	18.05
	P	0.00	0.00	1.01	1.49	7.49
0.5	J	0.00	0.00	0.11	1.10	18.79
	P	0.00	0.00	0.54	0.87	8.59
0.6	J	0.00	0.00	0.00	0.65	19.35
	P	0.00	0.00	0.00	0.47	9.53
0.7	J	0.00	0.00	0.00	0.28	19.72
	P	0.00	0.00	0.00	0.21	9.79
0.8	J	0.00	0.00	0.00	0.00	20.00
	P	0.00	0.00	0.00	0.00	10.00
0.9	J	0.00	0.00	0.00	0.00	20.00
	P	0.00	0.00	0.00	0.00	10.00

Thus, we deal here with a one shot game with lack of in-

formation on user side about the type of jammer. We shall look for a Bayesian equilibrium, that is, we want to find $(T^*, J^{1*}, J^{2*}) \in A \times B$ such that for any $(T, J^1, J^2) \in A \times B^1 \times B^2$,

$$\begin{aligned} v_T(T, J^{1*}, J^{2*}) &\leq v_T(T^*, J^{1*}, J^{2*}), \\ v_J^1(T^*, J^1, J^{2*}) &\leq v_J^1(T^*, J^{1*}, J^{2*}), \\ v_J^2(T^*, J^{1*}, J^2) &\leq v_J^2(T^*, J^{1*}, J^{2*}), \end{aligned}$$

where A and B^1 and B^2 are the sets of all the strategies of the user and the jammer, respectively.

Note that the payoff (10) is the linear on T . Also, note that

$$\frac{\partial v_J^k}{\partial J^k} = -\frac{2\alpha_i\beta_i^2 T_i}{(N^0 + \beta_i J_i^k)^3}.$$

Thus, we can apply a mix of linear and non-linear optimization approaches to get the following result.

THEOREM 3. (T, J^1, J^2) is an equilibrium if and only if there are ω (the minimal expected induced noise), and ν^1 and ν^2 (the Lagrangian multiplier) such that

$$T_i \begin{cases} \geq 0, & \gamma \frac{\alpha_i}{N^0 + \beta_i J_i^1} + (1-\gamma) \frac{\alpha_i}{N^0 + \beta_i J_i^2} = \omega, \\ = 0, & \gamma \frac{\alpha_i}{N^0 + \beta_i J_i^1} + (1-\gamma) \frac{\alpha_i}{N^0 + \beta_i J_i^2} < \omega \end{cases} \quad (12)$$

and

$$\frac{\alpha_i\beta_i T_i}{(N^0 + \beta_i J_i^1)^2} \begin{cases} = \nu^1, & J_i^1 > 0, \\ < \nu^1, & J_i^1 = 0, \end{cases} \quad (13)$$

$$\frac{\alpha_i\beta_i T_i}{(N^0 + \beta_i J_i^2)^2} \begin{cases} = \nu^2, & J_i^2 > 0, \\ < \nu^2, & J_i^2 = 0. \end{cases} \quad (14)$$

Then we have the following result describing the equilibrium

THEOREM 4. Let (T, J^1, J^2) be an equilibrium.

(a) If $T_i = 0$ then

$$J_i^1 = 0 = J_i^2$$

and $i \in I_{00}$ with

$$I_{00} = \left\{ i \in [1, n] : \frac{\alpha_i}{N^0} \leq \omega \right\},$$

(b) If $T_i > 0, J_i^1 > 0$ and $J_i^2 = 0$ then

$$J_i^1 = \frac{\alpha_i}{\beta_i} \left(\frac{\gamma}{\omega - (1-\gamma)\alpha_i/N^0} - \frac{N^0}{\alpha_i} \right) \quad (15)$$

$$T_i = \left(\frac{\gamma}{\omega - (1-\gamma)\alpha_i/N^0} \right)^2 \frac{\alpha_i \nu^1}{\beta_i} \quad (16)$$

and $i \in I_{10}$ with

$$I_{10} = \left\{ i \in [1, n] : \left(1 - \gamma + \gamma \sqrt{\frac{\nu^1}{\nu^2}} \right) \frac{\alpha_i}{N^0} \leq \omega < \frac{\alpha_i}{N^0} \right\}.$$

(c) If $T_i > 0, J_i^2 > 0$ and $J_i^1 = 0$ then

$$J_i^2 = \frac{\alpha_i}{\beta_i} \left(\frac{1-\gamma}{\omega - \gamma\alpha_i/N^0} - \frac{N^0}{\alpha_i} \right)$$

$$T_i = \left(\frac{1-\gamma}{\omega - \gamma\alpha_i/N^0} \right)^2 \frac{\alpha_i \nu^2}{\beta_i}$$

and $i \in I_{01}$ with

$$I_{01} = \left\{ i \in [1, n] : \left(\gamma + (1-\gamma) \sqrt{\frac{\nu^2}{\nu^1}} \right) \frac{\alpha_i}{N^0} \leq \omega < \frac{\alpha_i}{N^0} \right\}.$$

(d) If $T_i > 0, J_i^1 > 0$ and $J_i^2 > 0$ then

$$J_i^1 = \frac{\alpha_i}{\beta_i} \left(\frac{\gamma + (1-\gamma) \sqrt{\nu^2/\nu^1}}{\omega} - \frac{N^0}{\alpha_i} \right), \quad (17)$$

$$J_i^2 = \frac{\alpha_i}{\beta_i} \left(\frac{\gamma \sqrt{\nu^1/\nu^2} + 1 - \gamma}{\omega} - \frac{N^0}{\alpha_i} \right), \quad (18)$$

$$T_i = \frac{\alpha_i}{\beta_i} \left(\frac{\gamma \sqrt{\nu^1} + (1-\gamma) \sqrt{\nu^2}}{\omega} \right)^2 \quad (19)$$

and $i \in I_{11}$ with

$$I_{11} = \left\{ i \in [1, n] : \omega < \left(1 - \gamma + \gamma \sqrt{\frac{\nu^1}{\nu^2}} \right) \frac{\alpha_i}{N^0}, \right. \\ \left. \omega < \left(\gamma + (1-\gamma) \sqrt{\frac{\nu^2}{\nu^1}} \right) \frac{\alpha_i}{N^0} \right\}.$$

Proof: (a) If $T_i = 0$ then by (13) and (14) $J_i^1 = 0$ and $J_i^2 = 0$. Also, by (12), $\alpha_i/N^0 \leq \omega$, so $i \in I_{00}$.

(b) If $T_i > 0, J_i^1 > 0$ and $J_i^2 = 0$ then (12), (13) and (14) implies that

$$\gamma \frac{\alpha_i}{N^0 + \beta_i J_i^1} + (1-\gamma) \frac{\alpha_i}{N^0} = \omega, \quad (20)$$

$$\frac{\alpha_i\beta_i T_i}{(N^0 + \beta_i J_i^1)^2} = \nu^1 \quad (21)$$

and

$$\frac{\alpha_i \beta_i T_i}{(N^0)^2} \leq \nu^2. \quad (22)$$

Then, by (20), (15) holds. Substituting (15) into (21) implies (16).

Since $J_i > 0$ then, by (20),

$$\alpha_i / N^0 > \omega \quad (23)$$

and, by (15),

$$\frac{\gamma}{\omega - (1 - \gamma)\alpha_i / N^0} > \frac{N^0}{\alpha_i} \quad (24)$$

and

$$\omega > (1 - \gamma)\alpha_i / N^0. \quad (25)$$

Dividing (22) by (21) and substituting (15) yields

$$\left(\frac{\gamma \alpha_i}{\omega N^0 - (1 - \gamma)\alpha_i} \right)^2 \frac{\nu^1}{\beta_i} \leq \nu^2. \quad (26)$$

So, i has to satisfy the conditions (23)-(26). Let simplify these conditions. Note that by (23) and (25), the inequalities (24) and (23) are equivalent each other. By (24), (26) can be rewritten in the following equivalent form

$$(1 - \gamma + \sqrt{\nu^1 / \nu^2} \gamma) \alpha_i / N^0 \leq \omega. \quad (27)$$

Then, $i \in I_{10}$ follows from (23), (25) and (27).

(c) follows from (b) by symmetry.

(d) If $T_i > 0$, $J_i^1 > 0$ and $J_i^2 > 0$ then (12), (13) and (14) imply that

$$\gamma \frac{\alpha_i}{N^0 + \beta_i J_i^1} + (1 - \gamma) \frac{\alpha_i}{N^0 + \beta_i J_i^2} = \omega, \quad (28)$$

$$\frac{\alpha_i \beta_i T_i}{(N^0 + \beta_i J_i^1)^2} = \nu^1 \quad (29)$$

and

$$\frac{\alpha_i \beta_i T_i}{(N^0 + \beta_i J_i^2)^2} = \nu^2. \quad (30)$$

Dividing (29) by (30) implies that

$$N^0 + \beta_i J_i^2 = (N^0 + \beta_i J_i^1) \sqrt{\frac{\nu^2}{\nu^1}}. \quad (31)$$

Substituting (31) into (28) implies (17) and (18). Then, (19) follows from (17) and (29). Since $J_i^1 > 0$ and $J_i^2 > 0$, by (17) and (18), we obtain that

$$\frac{N^0}{\alpha_i} \leq \frac{\gamma \sqrt{\nu^1} + (1 - \gamma) \sqrt{\nu^2}}{\sqrt{\nu^1} \omega},$$

$$\frac{N^0}{\alpha_i} \leq \frac{\gamma \sqrt{\nu^1} + (1 - \gamma) \sqrt{\nu^2}}{\sqrt{\nu^2} \omega}.$$

Thus, $i \in I_{11}$. This completes the proof (d) and Theorem 4.

Theorem 4 implies the following nice properties of the sets I where jammers are going to apply efforts telling that the jammers equilibrium strategy are non-symmetrical.

THEOREM 5. (a) Let $\nu^1 > \nu^2$ then

$$I_{10} = \emptyset,$$

$$I_{11} = \left\{ i \in [1, n] : \omega < \left(\gamma + (1 - \gamma) \sqrt{\frac{\nu^2}{\nu^1}} \right) \frac{\alpha_i}{N^0} \right\}.$$

(b) Let $\nu^1 < \nu^2$ then

$$I_{01} = \emptyset,$$

$$I_{11} = \left\{ i \in [1, n] : \omega < \left(1 - \gamma + \gamma \sqrt{\frac{\nu^1}{\nu^2}} \right) \frac{\alpha_i}{N^0} \right\}.$$

To find the optimal ω , ν^1 and ν^2 we have to solve the following system of the equations:

$$H_J^1(\omega, \nu^2 / \nu^1) = \bar{J}^1, \quad (32)$$

$$H_J^2(\omega, \nu^1 / \nu^2) = \bar{J}^2, \quad (33)$$

$$H_T(\omega, \nu^1, \nu^2) = \bar{T}, \quad (34)$$

where

$$H_J^1(\omega, \nu^2 / \nu^1) = \sum_{i \in I_{10}} \frac{\alpha_i}{\beta_i} \left(\frac{\gamma}{\omega - (1 - \gamma)\alpha_i / N^0} - \frac{N^0}{\alpha_i} \right) + \sum_{i \in I_{11}} \frac{\alpha_i}{\beta_i} \left(\frac{\gamma + (1 - \gamma) \sqrt{\nu^2 / \nu^1}}{\omega} - \frac{N^0}{\alpha_i} \right),$$

$$H_J^2(\omega, \nu^1 / \nu^2) = \sum_{i \in I_{01}} \frac{\alpha_i}{\beta_i} \left(\frac{1 - \gamma}{\omega - \gamma \alpha_i / N^0} - \frac{N^0}{\alpha_i} \right) + \sum_{i \in I_{11}} \frac{\alpha_i}{\beta_i} \left(\frac{\gamma \sqrt{\nu^1 / \nu^2} + 1 - \gamma}{\omega} - \frac{N^0}{\alpha_i} \right),$$

$$H_T(\omega, \nu^1, \nu^2) = \nu^1 \sum_{i \in I_{10}} \left(\left(\frac{\gamma}{\omega - (1 - \gamma)\alpha_i / N^0} \right)^2 \frac{\alpha_i}{\beta_i} \right) + \nu^2 \sum_{i \in I_{01}} \left(\left(\frac{1 - \gamma}{\omega - \gamma \alpha_i / N^0} \right)^2 \frac{\alpha_i}{\beta_i} \right) + \left(\frac{\gamma \sqrt{\nu^1} + (1 - \gamma) \sqrt{\nu^2}}{\omega} \right)^2 \sum_{i \in I_{11}} \frac{\alpha_i}{\beta_i}.$$

Function $H_J^1(\omega, \tau)$ has the following properties. It is continuous on ω and τ , it is decreasing on ω and increasing on τ . For a fixed $\tau > 0$ $H_J^1(\omega, \tau) = 0$ for enough big ω and $H_J^1(0+, \tau) = \infty$. Thus, for a fixed $\tau > 0$ there is $\omega^1(\tau)$ such that

$$H_J^1(\omega^1(\tau), \tau) = \bar{J}^1.$$

It is clear that $\omega^1(\tau)$ is continuous increasing function such that

$$\omega^1(\infty) = \infty, \omega^1(0) = \bar{\omega}^1,$$

where $\bar{\omega}^1$ is the unique root of the following water-filling equation

$$R^1(\omega) := \sum_{i=1}^n \frac{\alpha_i}{\beta_i} \left[\frac{\gamma}{\omega} - \frac{N^0}{\alpha_i} \right]_+ = \bar{J}^1. \quad (35)$$

Similarly, for a fixed $\tau > 0$ there is $\omega^2(\tau)$ such that

$$H_J^2(\omega^2(\tau), \tau) = \bar{J}^2.$$

Also, $\omega^2(\tau)$ is continuous increasing function such that

$$\omega^2(\infty) = \infty, \omega^2(0) = \bar{\omega}^2,$$

where $\omega = \bar{\omega}^2$ is the unique root of the following water-filling equation

$$R^2(\omega) := \sum_{i=1}^n \frac{\alpha_i}{\beta_i} \left[\frac{1-\gamma}{\omega} - \frac{N^0}{\alpha_i} \right]_+ = \bar{J}^2. \quad (36)$$

Thus, $\omega^k(1/\tau)$ for $k = 1, 2$ is decreasing such that $\omega^k(1/0+) = \infty$ and $\omega^k(1/\infty) = \bar{\omega}^k$. Then, we can define

$$\nu^2/\nu^1 = \tau_* \quad (37)$$

where $\tau = \tau_*$ is the unique root of the equation

$$\omega^1(\tau) = \omega^2(1/\tau) \text{ for } \bar{\omega}^1 \leq \bar{\omega}^2 \quad (38)$$

and

$$\omega^2(\tau) = \omega^1(1/\tau) \text{ for } \bar{\omega}^1 > \bar{\omega}^2. \quad (39)$$

Then we obtain the optimal $\omega = \omega_*$ as follows

$$\omega_* = \omega^1(\tau_*). \quad (40)$$

To get the optimal ν^1 we note that

$$H_T(\omega, \nu^1, \nu^2) = \nu^1 \bar{H}_T(\omega, \nu^1/\nu^2), \quad (41)$$

where

$$\begin{aligned} \bar{H}_T(\omega, \nu^2/\nu^1) &= \sum_{i \in I_{10}} \left(\left(\frac{\gamma}{\omega - (1-\gamma)\alpha_i/N^0} \right)^2 \frac{\alpha_i}{\beta_i} \right) \\ &+ \frac{\nu^2}{\nu^1} \sum_{i \in I_{01}} \left(\left(\frac{1-\gamma}{\omega - \gamma\alpha_i/N^0} \right)^2 \frac{\alpha_i}{\beta_i} \right) \\ &+ \left(\frac{\gamma + (1-\gamma)\sqrt{\nu^2/\nu^1}}{\omega} \right)^2 \sum_{i \in I_{11}} \frac{\alpha_i}{\beta_i}. \end{aligned}$$

So, we can define the optimal $\nu^1 = \nu_*^1$ as follows

$$\nu_*^1 = \frac{\bar{T}}{\bar{H}_T(\omega_*, \tau_*)}$$

Then, that is clear that

$$H_T(\omega_*, \nu_*^1, \nu_*^2) = \bar{T}. \quad (42)$$

with

$$\nu_*^2 = \nu_*^1 \tau_* \quad (43)$$

Thus, we have proved the following theorem describing the equilibrium.

THEOREM 6. *The game with uncertainty about the total jamming power has the unique equilibrium where the optimal ω_* , ν_*^1 and ν_*^2 are given by (40), (42) and (43) with τ_* is the unique root of (38). Moreover the equilibrium (T, J_1, J_2) has the following form:*

(a) If $\nu_*^1 \geq \nu_*^2$ then

$$J_i^1 = \begin{cases} \frac{\alpha_i}{\beta_i} \left(\frac{\gamma + (1-\gamma)\sqrt{\nu_*^2/\nu_*^1}}{\omega_*} - \frac{N^0}{\alpha_i} \right), & i \in I_{11}, \\ 0, & \text{otherwise,} \end{cases}$$

$$J_i^2 = \begin{cases} \frac{\alpha_i}{\beta_i} \left(\frac{\gamma\sqrt{\nu_*^1/\nu_*^2} + 1 - \gamma}{\omega_*} - \frac{N^0}{\alpha_i} \right), & i \in I_{11}, \\ \frac{\alpha_i}{\beta_i} \left(\frac{1-\gamma}{\omega - \gamma\alpha_i/N^0} - \frac{N^0}{\alpha_i} \right) & i \in I_{01}, \\ 0, & \text{otherwise,} \end{cases}$$

$$T_i = \begin{cases} \left(\frac{\gamma\sqrt{\nu_*^1} + (1-\gamma)\sqrt{\nu_*^2}}{\omega_*} \right)^2, & i \in I_{11}, \\ \left(\frac{1-\gamma}{\omega - \gamma\alpha_i/N^0} \right)^2 \frac{\alpha_i \nu_*^2}{\beta_i} & i \in I_{01}, \\ 0, & \text{otherwise,} \end{cases}$$

with

$$I_{01} = \left\{ i \in [1, n] : \left(\gamma + (1-\gamma)\sqrt{\frac{\nu_*^2}{\nu_*^1}} \right) \frac{\alpha_i}{N^0} \leq \omega < \frac{\alpha_i}{N^0} \right\}$$

and

$$I_{11} = \left\{ i \in [1, n] : \omega < \left(\gamma + (1-\gamma)\sqrt{\frac{\nu_*^2}{\nu_*^1}} \right) \frac{\alpha_i}{N^0} \right\}.$$

(b) If $\nu_*^1 < \nu_*^2$ then

$$J_i^1 = \begin{cases} \frac{\alpha_i}{\beta_i} \left(\frac{\gamma + (1-\gamma)\sqrt{\nu_*^2/\nu_*^1}}{\omega_*} - \frac{N^0}{\alpha_i} \right), & i \in I_{11}, \\ \frac{\alpha_i}{\beta_i} \left(\frac{\gamma}{\omega - (1-\gamma)\alpha_i/N^0} - \frac{N^0}{\alpha_i} \right) & i \in I_{10}, \\ 0, & \text{otherwise,} \end{cases}$$

$$J_i^2 = \begin{cases} \frac{\alpha_i}{\beta_i} \left(\frac{\gamma\sqrt{\nu_*^1/\nu_*^2} + 1 - \gamma}{\omega_*} - \frac{N^0}{\alpha_i} \right), & i \in I_{11}, \\ 0, & \text{otherwise,} \end{cases}$$

$$T_i = \begin{cases} \left(\frac{\gamma\sqrt{\nu_*^1} + (1-\gamma)\sqrt{\nu_*^2}}{\omega_*} \right)^2, & i \in I_{11}, \\ \left(\frac{\gamma}{\omega - (1-\gamma)\alpha_i/N^0} \right)^2 \frac{\alpha_i \nu_*^1}{\beta_i} & i \in I_{10}, \\ 0, & \text{otherwise,} \end{cases}$$

with

$$I_{10} = \left\{ i \in [1, n] : \left(\gamma\sqrt{\frac{\nu_*^1}{\nu_*^2}} + 1 - \gamma \right) \frac{\alpha_i}{N^0} \leq \omega < \frac{\alpha_i}{N^0} \right\}$$

and

$$I_{11} = \left\{ i \in [1, n] : \omega < \left(\gamma\sqrt{\frac{\nu_*^1}{\nu_*^2}} + 1 - \gamma \right) \frac{\alpha_i}{N^0} \right\}.$$

5. ALGORITHM

In this section we present an algorithm based on the bisection method and Theorem 6 to find the optimal values of ω , ν^1 , ν^2 and the corresponding optimal solution. Here we assume that ϵ is the tolerance of computation and $\alpha_{\max} = \max_i \alpha_i$.

Algorithm: (Return ω_* , ν_*^1 , ν_*^2 and the optimal strategies defined by them)

S1: Find $\bar{\omega}^1$:

$$\bar{\omega}^1 = \text{Bisec}(\epsilon, b, R^1 - \bar{J}^1)$$

where

$$b = \alpha_{\max} \gamma / N^0.$$

S2: Find $\bar{\omega}^2$:

$$\bar{\omega}^2 = \text{Bisec}(\epsilon, b, R^2 - \bar{J}^2)$$

where

$$b = \max_i \alpha_{\max} (1 - \gamma) / N^0.$$

S3: Find τ_* :

S3a: If $\bar{\omega}^1 \leq \bar{\omega}^2$ then

Sa1: Set $a = \epsilon$, b is big enough.

Sa2: $c = (a + b) / 2$.

Sa3: If $b - a \leq \epsilon$ then return $\tau_* = (a + b) / 2$ and go to step S4.

Sa4: Set

$$F_a^1 = \text{Bisec}(\epsilon, B, H_J^1(\cdot, a) - \bar{J}^1)$$

where

$$B = \alpha_{\max} (\gamma + (1 - \gamma) \sqrt{a}) / N^0.$$

Sa5: Set

$$F_a^2 = \text{Bisec}(\epsilon, B, H_J^2(\cdot, 1/a) - \bar{J}^2)$$

where

$$B = \alpha_{\max} (\gamma \sqrt{1/a} + 1 - \gamma) / N^0.$$

Sa6: Set

$$F_c^1 = \text{Bisec}(\epsilon, B, H_J^1(\cdot, c) - \bar{J}^1)$$

where

$$B = \alpha_{\max} (\gamma + (1 - \gamma) \sqrt{c}) / N^0.$$

Sa7: Set

$$F_c^2 = \text{Bisec}(\epsilon, B, H_J^2(\cdot, 1/c) - \bar{J}^2)$$

where

$$B = \alpha_{\max} (\gamma \sqrt{1/c} + 1 - \gamma) / N^0.$$

Sa8: If $b - a > \epsilon$ then if $(F_a^1 - F_a^2)(F_c^1 - F_c^2) \leq 0$ then $b = c$ else $a = c$.

Sa9: Return $\tau_* = (a + b) / 2$ and go to Step 4.

S3b: If $\bar{\omega}^1 > \bar{\omega}^2$ then

Sb1: Set $a = \epsilon$, b is big enough.

Sb2: $c = (a + b) / 2$.

Sb3: If $b - a \leq \epsilon$ then return $\tau_* = (a + b) / 2$ and go to step S4.

Sb4: Set

$$F_a^1 = \text{Bisec}(\epsilon, B, H_J^1(\cdot, 1/a) - \bar{J}^1)$$

where

$$B = \alpha_{\max} (\gamma + (1 - \gamma) \sqrt{1/a}) / N^0.$$

Sb5: Set

$$F_a^2 = \text{Bisec}(\epsilon, B, H_J^2(\cdot, a) - \bar{J}^2)$$

where

$$B = \alpha_{\max} (\gamma \sqrt{a} + 1 - \gamma) / N^0.$$

Sb6: Set

$$F_c^1 = \text{Bisec}(\epsilon, B, H_J^1(\cdot, 1/c) - \bar{J}^1)$$

where

$$B = \alpha_{\max} (\gamma + (1 - \gamma) \sqrt{1/c}) / N^0.$$

Sb7: Set

$$F_c^2 = \text{Bisec}(\epsilon, B, H_J^2(\cdot, c) - \bar{J}^2)$$

where

$$B = \alpha_{\max} (\gamma \sqrt{c} + 1 - \gamma) / N^0.$$

Sb8: If $b - a > \epsilon$ then

if $(F_a^1 - F_a^2)(F_c^1 - F_c^2) \leq 0$ then $b = c$ else $a = c$.

Sb9: Return $\tau_* = (a + b) / 2$ and go to step S4.

S4: Return $\nu_*^1 = \bar{T} / H_T(\omega_*, \tau_*)$.

S5: Return $\nu_*^2 = \nu_*^1 \tau_*$.

S6: Return the optimal strategies J^1 , J^2 and T .

This function finds by bisection method the root of the equation $F(x) = 0$ in the interval $[a, b]$.

Function Bisec(a, b, F)

S0: If $F(a)F(b) > 0$ then the algorithm is terminated requesting the others a and b .

S1: Set $c = (a + b) / 2$.

S2: If $b - a \leq \epsilon$ then return $(a + b) / 2$.

S3: If $b - a > \epsilon$ then

if $F(a)F(c) \leq 0$ then $b = c$ else $a = c$.

S4: Return $(a + b) / 2$.

6. BOTH TYPES OF JAMMER JAM THE SAME CHANNELS

Note that in general case both jamming strategies do not act simultaneously on all the channels employed by user. For example, for five channels ($n = 5$) scenario with $\alpha = (1, 2, 3, 4, 5)$, $\beta = (5, 4, 3, 2, 1)$, $N^0 = 1$ and $\bar{T} = 10$, $\bar{J}_1 = 5$, $\bar{J}_2 = 10$ and $\gamma = 0.3$ the equilibrium strategies are

$$J_1 = (0.00, 0.17, 0.50, 1.17, 3.17),$$

$$J_2 = (0.05, 0.46, 1.08, 2.33, 6.08)$$

and

$$T = (0.01, 0.04, 0.08, 0.16, 0.39).$$

Thus, the first channel is harmed only by the jammer with bigger jamming power. That is why it is interesting to produce condition under which both jamming strategies applies efforts in all the channels employed by user.

Without loss of generality we can assume that the channels are arranged by the user fading channel gains in decreasing order, namely, the following inequalities hold:

$$\alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_n.$$

Then there is a $k \in [1, n]$ such that $T_i > 0$ for $i \in [1, k]$ and $T_i = 0$ for $i > k$.

Consider the situation where both jamming strategies J_i^1 and J_i^2 are positive for any i such that $T_i > 0$. So, $J_i^1 > 0$ and $J_i^2 > 0$ for $k \in [1, n]$. Then, by (13) and (14),

$$\frac{N^0 + \beta_i J_i^2}{N^0 + \beta_i J_i^1} = R := \sqrt{\frac{\nu^1}{\nu^2}} \text{ for } i \in [1, k].$$

It implies from (12) that

$$(\gamma + (1 - \gamma)R) \frac{\alpha_i}{N^0 + \beta_i J_i^1} = \omega \text{ for } i \in [1, k]$$

and

$$\frac{\gamma + (1 - \gamma)R}{R} \frac{\alpha_i}{N^0 + \beta_i J_i^2} = \omega \text{ for } i \in [1, k]$$

So,

$$J_i^1 = \left(\frac{\gamma + (1 - \gamma)R}{\omega} - \frac{N^0}{\alpha_i} \right) \frac{\alpha_i}{\beta_i}$$

and

$$J_i^2 = \left(\frac{\gamma + (1 - \gamma)R}{R\omega} - \frac{N^0}{\alpha_i} \right) \frac{\alpha_i}{\beta_i}$$

Since $\sum_{r=1}^k J_r^m = \bar{J}^m$ we have that

$$\frac{\gamma + (1 - \gamma)R}{\omega} = \frac{\bar{J}^1 + \sum_{r=1}^k (N^0/\beta_r)}{\sum_{r=1}^k (\alpha_r/\beta_r)}$$

and

$$\frac{\gamma + (1 - \gamma)R}{\omega R} = \frac{\bar{J}^2 + \sum_{r=1}^k (N^0/\beta_r)}{\sum_{r=1}^k (\alpha_r/\beta_r)}.$$

Thus,

$$R = \frac{\bar{J}^1 + \sum_{r=1}^k (N^0/\beta_r)}{\bar{J}^2 + \sum_{r=1}^k (N^0/\beta_r)}$$

and

$$\omega = \frac{\gamma \sum_{r=1}^k (\alpha_r/\beta_r)}{\bar{J}^1 + \sum_{r=1}^k (N^0/\beta_r)} + \frac{(1 - \gamma) \sum_{r=1}^k (\alpha_r/\beta_r)}{\bar{J}^2 + \sum_{r=1}^k (N^0/\beta_r)}.$$

The condition that gives k is the following:

$$\frac{\alpha_k}{N^0} \geq \omega > \frac{\alpha_{k+1}}{N^0}$$

or after taking into account the form for ω

$$\frac{\alpha_k}{N^0} \geq \frac{\gamma \sum_{r=1}^k (\alpha_r/\beta_r)}{\bar{J}^1 + \sum_{r=1}^k (N^0/\beta_r)} + \frac{(1 - \gamma) \sum_{r=1}^k (\alpha_r/\beta_r)}{\bar{J}^2 + \sum_{r=1}^k (N^0/\beta_r)} > \frac{\alpha_{k+1}}{N^0} \quad (44)$$

and

$$\frac{\alpha_k}{N^0} \geq \frac{\sum_{r=1}^k (\alpha_r/\beta_r)}{\bar{J}^m + \sum_{r=1}^k (N^0/\beta_r)} \text{ for } m = 1, 2. \quad (45)$$

So, (45) and (44) are condition telling J_i^1 and J_i^2 are positive for any i such that $T_i > 0$, and k is a such one that $T_i > 0$ for $i \in [1, k]$. Of course, if (45) holds then the left side inequality in (44) also holds.

7. CONCLUSIONS

In the present work we have dealt with the concept of jamming with incomplete information about the jammer. We considered two scenarios. In the first one a jammer could be either present in the environment bringing extra background noise to the natural one or absent. The user has only statistical knowledge about either presence or absence of the jammer. Namely, it knows that in the environment only a natural background noise could be with probability γ meanwhile with probability $1 - \gamma$ a jammer could come into the action distributing an extra noise of the total power \bar{J} among the channels. In the second scenario the user does not know exactly the total jamming power. This situation is described as if the user plays with a jammer of unknown type. For both scenarios the equilibrium are found.

Finally it is worth to mention that the jamming game principally differs with the game of several cooperative jammers [6]. In the several jammers game their optimal strategies are time-shared meanwhile in the considered game the optimal jammers strategies tell to jam the same set of the channels besides an extra set of the channels which are jammed by bigger expected jamming power.

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