

Linear and Arithmetically-Progressive Taxation of Spectrum Resources in Cognitive OFDMA

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ABSTRACT

In this paper, we consider a Cognitive Radio (CR) OFDMA-based network, in which the nodes detect available spectrum resources, and adopt a subset of accessible spectrum units (subcarriers). The idea of our approach to the efficient spectrum utilization is to allow each node to optimize the resources acquisition, where the competition of nodes for available common resources is observed. This can be modeled as a multi-stage game in the sense of game theory. In our game model, the efficiency and rationality mechanisms have been introduced in the utility function, which reflect the throughput achieved by a single player and the network potential to serve multiple nodes. Moreover, taxation of resources is recommended to coerce desirable players' behavior. Simulation results show that there exist the trade-off values of the linear and arithmetically-progressive tax-rate parameters to approach high overall spectral efficiency, high perceived throughput and fairness in distributing available resources.

Categories and Subject Descriptors

C.2.1 [Network Architecture and Design]: Wireless Communication, Network communications

General Terms

Algorithms, Theory.

Keywords

cognitive radio, dynamic spectrum access, utility, game theory.

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1. INTRODUCTION

In the last years, the demand for ubiquitous mobile communications and efficient spectrum utilization is noticeably growing. Cost-effective Dynamic Spectrum Sharing (DSS) procedures are needed to increase the utilization of the scarce spectrum resources as well as to handle the fairness of the resources distribution among users. Such procedures have been presented in the literature (e.g. in [1]) however, most of them are the centralized ones, and require the Channel State Information (CSI) of all considered links. This necessitates the adequate amount of control traffic, which in turn occupies the spectrum resources. Moreover, a centralized approach is not appropriate for the Cognitive Radio (CR) concept [2], where the nodes are expected to be autonomously sensing, learning and decision-taking entities.

In this paper, we consider the cognitive Orthogonal Frequency Division Multiple Access (OFDMA), in which the secondary users are able to detect available spectrum resources, and to adopt a subset of accessible subcarriers (SCs), as well as the transmission power level at these SCs. The OFDMA is a promising technique, considered for modern high data rate wireless systems, e.g. for IEEE 802.16e, 3GPP LTE downlink, or IEEE 802.22 (WRANs). Most of the algorithms for flexible resource allocation in OFDMA presented in the literature so far are the centralized ones or assume some limited cooperation between the cells, and few of them employ the game theory [3][4][5].

Game theory applied in radio spectrum acquisition models behavior and strategic interactions among network nodes (the players) competing for the spectrum access [6]. Game-theoretic scheduling for OFDMA has been also considered in the literature in two major directions. The first one is the centralized SCs allocation, which allows for more efficient and fair spectrum utilization and applies cooperative game theory, Nash-bargaining and arbitrary Pareto-optimal solutions ([7][8]). The second one is the distributed decision making, which on the contrary, deploys non-cooperative games for multi-cell OFDMA systems and results in Nash equilibrium as a game solution for the base stations, which act as players ([9]). The drawbacks of the above solutions are either lack of optimality (Nash bargaining solution is usually not optimal), lack of rationality (the utilities are usually defined so as to maximize the total sum-throughput, and neglect the quality of experience and the power economy), lack of generality (Pareto-optimal solutions are found for two users only) or lack of

suitability for the CR networks (some near-optimal solutions can be found, as in [8], but they are centralized).

In this paper, we consider non-cooperative allocation of SCs for multiple CR nodes in an OFDMA-based network. The considered approach is based on non-cooperative game theory, and aims at high spectral efficiency and high energy-efficiency of the CR-nodes communication, as well as at the network ability to serve multiple users and resource-distribution fairness. This can be done based on the concept of coercive resource taxation and reduced-complexity non-cooperative multi-stage game model.

2. THE GAME-THEORETIC TAXATION-BASED RESOURCE ACQUISITION

In our OFDMA-based network scenario, multiple CR nodes are secondary users of the network. Each node is able to detect available spectrum, and its goal is to acquire radio resources, and maximize the throughput at a minimum cost, expressed in energy consumption. In our considerations we do not care how the achieved data rates are mapped to data streams. For a proof of concept we also assume that a CR-node can acquire any subset of available SCs, and that there exists a collision-avoidance mechanism, used to prevent nodes from trying to access the spectrum at the same time instant.

An intelligent Control Unit (CU) of a CR node makes decision concerning which SCs and how many of them it will use for the transmission. It can be considered as an element of the broader concept of the *cognitive engine* [2]. The CU acts as a player in our game for resources. In order to prevent a single player from occupying all resources, we limit the maximum number of SCs a node can acquire at a time, introduce the “social consciousness” mechanism such as network capacity factor in the utility function and employ resource taxation. Resulting throughput depends on the power levels allocated to acquired SCs, and we assume that this is done in an optimal way based on the so-called *water-pouring principle* [11]. After the application of water-pouring (calculation of the water-level), it may turn out that some SCs acquired cannot be used for the transmission with assumed reliability (due to their poor quality). In such a case, they are returned to the pool of available resources.

Let us define our game, which each CU plays against the other users. The elements of this definition are the players, the strategies, and the payoffs.

2.1 The players and their strategies

Following the spectrum-sensing algorithm, the CU of a CR-node can make decision on the number of acquired SCs. If there are N available SCs, K nodes, and each one can take any subset of these SCs (but no more than I SCs), the problem of finding the game solution becomes extremely complex. This is because the game is K -dimensional, and the number of possible outcomes of the game equals:

$$\Omega = \left[\sum_{i=0}^I \frac{N!}{i!(N-i)!} \right]^K \quad (1)$$

Thus, analyzing the existence of the Nash equilibriums becomes very complex particularly for high I , N and K values.

As shown above, it is crucial to narrow the space of our analysis. For this purpose we let each CU take decisions independently

while treating the rest of the players as one player (the CR-nodes community), and by eliminating its strategic choices which are disadvantageous. Naturally, a selfish player would occupy the maximum of available SCs, because such an action would maximize its throughput. However, from the fairness point of view, this behavior would decrease the number of users served. The problem is related to the classical common resource utilization dilemma known as the *Tragedy of Commons* ([10]), which shows that selfish behavior of the common-resource users leads to inefficient (even the poorest possible) utilization of these resources.

In order to eliminate disadvantageous strategic choices (eliminate dominated strategies [3]), we simply assume that, when a CU wants to use a number of i subcarriers, for sure these should be i strongest SCs (of the highest Carrier-gain-to-Interference-and-Noise Power Ratios (CINRs)), because making use of the strongest SCs results in highest spectral efficiency. A single player does not care, how efficient the other players are in utilizing their acquired resources, i.e. what are their CINRs at various SCs, but rather how many of these SCs the other players (all together) are going to use. The number of occupied SCs affects the network ability to serve the incoming users including the users who are already being served, but would like to acquire more resources in the future.

Thus, in our considered case, the game for each player becomes two dimensional only, and the number of possible game outcomes to be analyzed is significantly reduced: $\Omega_{\text{red}} = I \times N$.

To summarize, the strategies of a CU are all possible numbers of the strongest detected SCs (from 0 to I). The strategies of the rest of the CR-nodes community are the numbers of SCs this community may occupy all together. Moreover, the game is dynamic, i.e. the players take decisions subsequently thanks to the collision-avoidance procedure.

2.2 The payoffs

The payoff at each stage of the game is the value of the utility function and exhibits the game outcome, when the CU player chooses strategy i (i strongest SCs) while the rest of the community occupies j subcarriers. For our game, we suggest that the payoff reflected the normalized throughput (throughput divided by the subcarrier distance Δf) achieved by the CR-node and the network potential to serve other users (related to the notion of fairness). Thus, the proposed payoff function for player k (at the k th stage of the game), who acquires i SCs ($i \leq I$), while the rest of the CR-nodes community is expected to occupy j SCs equals:

$$\zeta_{ki,j} = \left\{ \frac{1}{N} \sum_{s \in \mathbf{S}_i} \log_2 [1 + \alpha \cdot P(f_s) \gamma(f_s)] \right\} \cdot \{N_k - i - j\}, \quad (2)$$

where $\gamma(f_s) = |H(f_s)|^2 / (\mathcal{N}_0 \Delta f + \mathcal{J}_0)$ is the CINR at the SC frequency f_s , whose index s belongs to the set \mathbf{S}_i of indices of i strongest SCs (the cardinality of \mathbf{S}_i is i), $H(f_s)$ and $P(f_s)$ are the channel characteristic and the power level allocated to this SC frequency f_s respectively, \mathcal{N}_0 is the noise power spectral density, \mathcal{J}_0 is the interference power, Δf is the SCs distance, α is the factor depending on the assumed Bit Error Probability (BEP) P_e , and N_k is the number of available SCs at the game stage k , i.e. for user k .

(Note that for M -QAM, α can be approximated by $\alpha = -1.5 / \ln(0.5P_e)$ or more precisely as in [11].) Let us note, that the optimal power level $P(f_s)$ allocated to SCs should be calculated according to the water-pouring principle, assuming that CINR values are perfectly known by the CU, i.e. the CSI is perfect, although in reality the player uses estimated CINR for each chosen subcarrier frequency f_s . The first factor in (2) represents the potential throughput of a single player at the k th stage of the game, while the second one represents the network potential to serve other players at subsequent game stages. This way, the players factor the *social aspect* of the network (to serve multiple users) in their decision-making.

Note, that for any j value (any strategy of the CR-nodes community), $\zeta_{kI,j} \geq \zeta_{ki,j}$ ($i = 0, \dots, I$), and thus, $\zeta_{kI,j}$ represents the highest payoff regardless of j . As a result, the dominating strategy for every considered player is to use the maximum allowable number of SCs I , and the common radio resources would not be utilized in a fair way. Therefore, the payoff (2) should be modified so as to force the more rational behavior of the players with respect to the available resources. Our proposal is to introduce taxation of resources, what results in the following payoff function:

$$\tilde{\zeta}_{ki,j} = \left\{ \frac{1}{N} \sum_{s \in S_i} \log_2 [1 + \alpha \cdot P(f_s) \gamma(f_s)] \right\} \cdot \{N_k - i - j\} - \tau_{ki}, \quad (3)$$

where τ_{ki} is the tax value, that a player k has to “pay” at the k th stage of the game. This tax value should depend on the amount of resources used by a player. $\tilde{\zeta}_{ki,j}$ represents the profit (revenue minus cost), and thus, if a single player faces poor channel-quality, and yet occupies a lot of SCs, its profit should be small or even negative due to poor spectral utilization and relatively high tax “to pay”. Taxes should bring pressure to bear upon desirable players’ behavior and the Pareto-optimal solution of the game. However, calculation of such an optimal tax is quite a difficult task, requires the knowledge of the CINRs at all SCs for all links, and results in many tax values specific for every player. These calculations can be done only centrally, e.g. at a base station, from where the tax values (and their updates every time the CINRs change) can be broadcasted. In our considered CR network, all decisions are expected from the intelligent CU. Therefore, we look at a simplified case of common (for all nodes) and invariant (with respect to CINRs changes) way of taxing the resources. Moreover, we concentrate on the linear taxation and arithmetically-progressive taxation. In both cases the tax can be defined as follows:

$$\forall k \in [1, K]: \tau_{ki} = \tau_i = i \cdot (\tau_0 + i \cdot \Delta_\tau), \quad (4)$$

where $(\tau_0 + i \cdot \Delta_\tau)$ is the tax rate determined by the arithmetic progression, with the first element τ_0 and the common difference Δ_τ . Both τ_0 and Δ_τ should be carefully adopted to the number of SCs, the number of players and their anticipated behavior. Let us note, that if $\Delta_\tau = 0$, the above formula defines the linear taxation, i.e. the tax value is a linear function of the number of acquired SCs. If $\Delta_\tau > 0$, the taxation is arithmetically-progressive. Both parameters τ_0 and Δ_τ should also depend on the average Signal-to-Interference-and-Noise ratio (SINR) (averaged over all SCs)

experienced by CR-nodes in the system. Too high taxes encourage every player to not use any subcarrier, and results in poor usage of available resources, if the nodes SINRs are low.

Finally, the payoff of the other player, here referred to as “the rest of the CR-nodes community” is defined as the number of SCs that can be potentially occupied by this player:

$$\psi_{ki,j} = j, \quad (5)$$

Let us note, that after we have introduced taxation of the occupied number of SCs, there may not be one dominating strategy of the single player. Although the values of payoffs decrease with increasing j :

$$\forall i \in [0, I] \forall j \in [0, N_k - I - 1]: \tilde{\zeta}_{ki,j} \geq \tilde{\zeta}_{ki,j+1} \quad (6)$$

(equality occurs for $i = 0$), it is not the case for increasing i :

$$\forall j \in [0, N_k - I] \exists i^*: \begin{cases} \tilde{\zeta}_{ki-1,j} \leq \tilde{\zeta}_{ki,j} & \text{for } 0 < i \leq i^* \\ \tilde{\zeta}_{ki,j} \geq \tilde{\zeta}_{ki+1,j} & \text{for } i^* \leq i < I \end{cases} \quad (7)$$

Moreover, there may still exist some dominated strategies after the tax has been introduced. If, for instance, the i th strongest subcarrier is a very bad one (has low CINR), an associated tax exceeds the benefit of using it for every considered strategy j of the rest of the community. In such a case, this dominated strategy i should be removed from the considerations of the rational player.

We may conclude the above considerations that there exist the saddle point or the Nash equilibrium for the above defined game, i.e. for strategies i^* defined by (7) and for $j = N_k - I$. It is well known however, that a Nash equilibrium may be far from optimality [4][5][6]. In our game, the player named as “the rest of the CR-nodes community” does not rationalize its behavior to meet this equilibrium. Although the behavior of individual players is hard to anticipate (because it depends on the channel quality they face), the behavior of the whole community of the CR nodes can be assessed. Its strategy j is the aggregated number of SCs this community occupies, and thus, the probability of this strategy approximates the Gaussian distribution. Thus individual players can make decisions depending on the anticipated behavior of the rest of the CR-nodes community. The procedure should apply mixed strategy reflecting probability of the CR-community strategies.

3. SIMULATION RESULTS

For the purpose of our game-model verification and simulations, we have considered the following setup: the number of available SCs at the beginning of the game $N = 256$, the maximum number of SCs each player can take I and the number of competing nodes K satisfy: $KI = N$ (potentially there is enough SCs to serve all nodes). Each node is allowed to transmit with the same power limit. For a given number of competing nodes K , the total power in the CR-nodes community (being the sum of the transmit powers of the CR nodes) has also been fixed, so that if some nodes do not use available SCs, the other nodes are allowed to increase their power. Thus, the interference level is constant. Moreover, two scenarios have been considered: equal average-SINR case (the ‘E’ case) when the average SINR $\bar{\gamma} = 30$ dB is

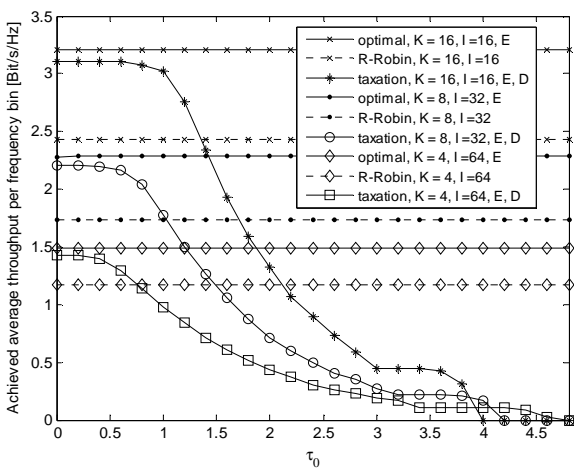
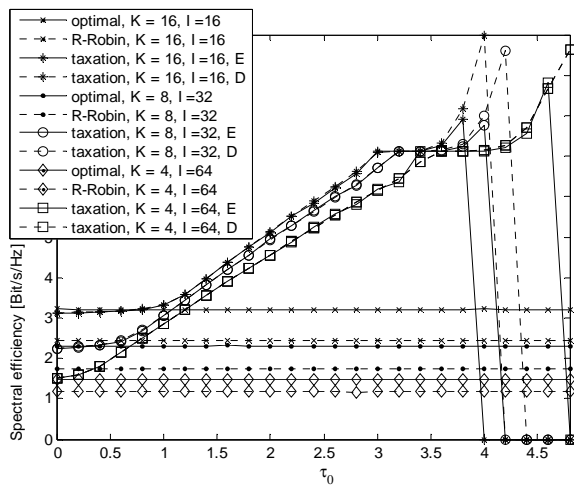


Figure 1. Spectral efficiency (a) and the overall throughput averaged over N available SCs (b) vs. the linear tax-rate parameter value τ_0 (E – equal average SINR for all nodes, D –diverse SINR with 1 dB max deviation from $\bar{\gamma}$).

the same for every node due to the Power-Control (PC) mechanism, and diverse average-SINR case ('D' case) when the PC mechanism has a tolerance of 1.5 dB, so that random deviation not exceeding 1 dB from the average SINR is possible for any node. For modern cellular systems the PC inaccuracy can be usually around 1.5 dB. This number has been chosen to observe differences in the obtained results for both scenarios.

The channel model considered is the two-paths Rayleigh-fading channel with the delay spread ranging from 0 to one fourth of the OFDM symbol, and the average power of the second path being – 3 dB relative to the average power of the first path. The simulation time covered the channel coherence time, and 1000 channel realizations. The assumed BEP for uncoded QAM modulation is $P_e = 10^{-3}$.

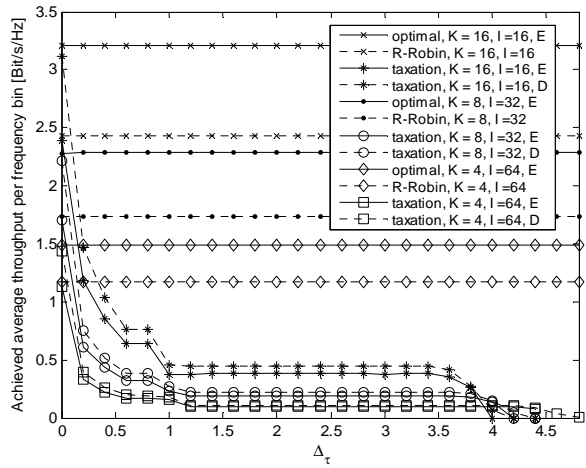
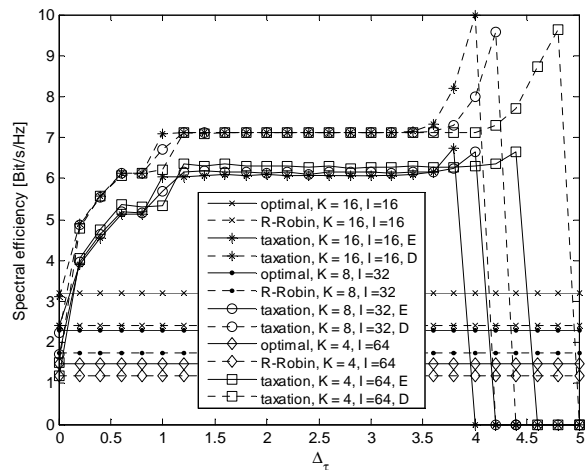


Figure 2. Spectral efficiency (a) and the throughput averaged over N available SCs (b) vs. the arithmetically-progressive tax-rate parameter Δ_τ (E – equal average SINR for nodes, D – diverse SINR with 1 dB max deviation from $\bar{\gamma}$).

The main challenge of our simulation experiments is to calibrate the tax value, i.e. the tax-rate parameters. As mentioned above, the optimal tax values calculation for distinct nodes resulting in either maximal spectral efficiency, fairness-based or Pareto-optimal solution can be implemented only centrally. Since our scenario is the CR network without the centralized management, we have tested common tax-rate values for all nodes, and searched for a near-optimum value. In Fig. 1, simulation results are presented for the case of linear taxation of resources, i.e. for $\Delta_\tau = 0$ and various K and l values. In Fig. 1.a, the spectral efficiency (sum-throughput averaged over the number of used SCs) is presented, while in Fig. 1.b the throughput averaged over the number of all available SCs N is plotted vs. the tax-rate parameter value τ_0 . Moreover, these results are compared with the Round-Robin (R-R) SCs allocation and with the *optimal* SCs allocation. The optimality here refers only to the spectral efficiency of all-resources distribution, which

means that the subsequent SCs are allocated to nodes, which experience the highest CINRs at these SCs.

In Fig. 2.a and 2.b, analogous results for the arithmetically-progressive taxation of resources are presented vs. the tax-rate parameter value Δ_τ ($\tau_0 = 0$). Let us note, that for the appropriately high tax values, the spectral efficiency is higher in our game model than even this optimal allocation of SCs, although the average throughput per SC decreases with an increase of the relevant tax-rate parameters: τ_0 and Δ_τ . Low average throughput for high tax-values occurs due to poor SCs utilization (Fig. 5), which become too “costly” (in terms of low payoff). Moreover, the increase of the spectral efficiency, the decrease of the average throughput, and the decrease of the percentage of utilized SCs is more gradual for linear taxation than for the arithmetically-progressive taxation (where more rapid reaction of the system-model performance to the Δ_τ parameter change is observed). Thus the calibration of Δ_τ can be relatively more sensitive to the CSI imperfections than τ_0 .

Moreover, as expected, the spectral efficiency for the ‘D’ case is slightly higher than for the ‘E’ case defined above. This is because in the diverse-average-SINR scenario, the nodes with higher average SINRs acquire more SCs than the nodes with lower average SINRs. In case of linear taxation of resources, the average throughput is approximately the same for both cases ‘E’ and ‘D’, while for arithmetically-progressive taxation the achieved average throughput for case ‘D’ is slightly higher than for case ‘E’.

Finally, because every node has the same transmission power limit, the higher number of players K , the lower number of SCs they can acquire and higher power is allocated for each SC, and thus, the higher spectral efficiency is obtained.

In Fig. 3 and 4, the Cumulative Distribution Function (CDF) of the nodes’ perceived throughput averaged over the node’s SCs share (perceived spectral efficiency) is presented for the optimal SCs allocation discussed above, for the R-R SCs allocation and for our linear and arithmetically-progressive resource-taxation model implemented with various tax-rate parameters (τ_0 and Δ_τ values). Let us note, that for the optimal SCs allocation, the probability that the perceived spectral efficiency is lower than even very low but positive value is quite high (e.g. approximately 0.5 for $K = 16$, $I = 16$ and 0.3 for $K = 8$, $I = 32$). This means that there is a high number of nodes, which are not served at all. In the R-R algorithm, every node receives approximately equal share of SCs, and the channel quality of each node is not accounted for. Therefore their perceived throughput does not drop below a certain level, however this level is rather low.

The CDF curves obtained for our game model show that depending on the type of taxation and the τ_0 or Δ_τ value, high perceived spectral efficiency can be approached with adequately high taxes. Too high taxes however, result in low percentage of the utilized SCs and served nodes, what can be observed in Fig. 5 and 6. More fair SCs allocation (resulting in 100% of served nodes), better SCs utilization (resulting in a decent percentage of used SCs), higher average throughput, lower perceived and overall spectral efficiency (although close to the spectral efficiency of optimal SCs allocation) can be obtained with lower taxes, e.g. with $\tau_0 = 1$ for $K = 16$ and $I = 16$ for linear taxation and with $\Delta_\tau = 0.1$ for $K = 16$ and $I = 16$ and arithmetically-progressive

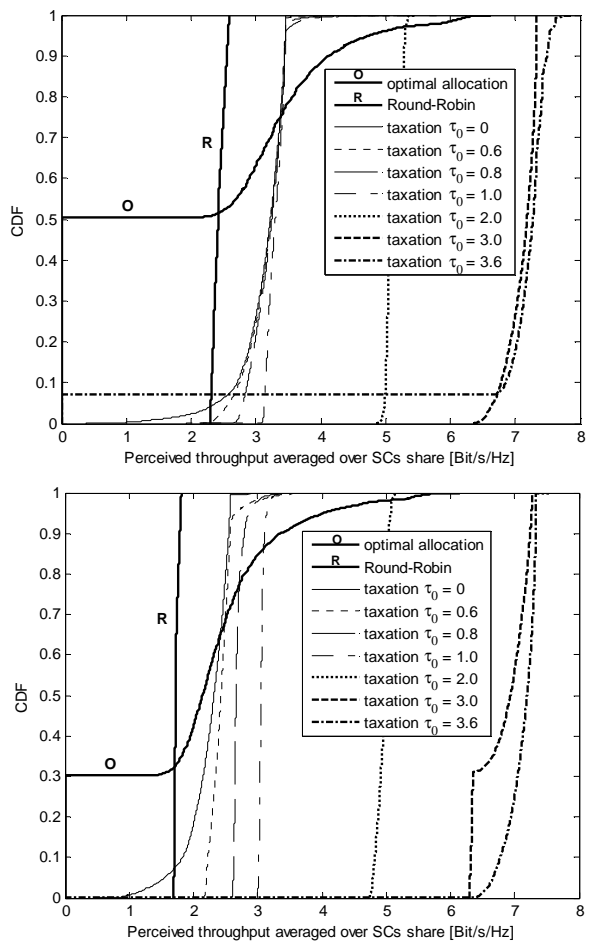


Figure 3. Cumulative Distribution Function (CDF) of the perceived throughput per frequency unit in the game with linear taxation for the ‘E’ case, $\bar{\gamma} = 30$ dB and a) $K = 16$, $I = 16$, b) $K = 8$, $I = 32$.

taxation. Adoption of these values results in a proper balance between the average throughput, percentage of served nodes, percentage of used SCs and overall spectral efficiency. Additionally, for these values of τ_0 or Δ_τ an improvement over the R-R SCs allocation is noticeable, i.e. cut-off value of the perceived nodes’ spectral efficiency below which the CDF equals 0 is much higher for our taxation-based approaches than for the R-R algorithm.

An interesting observation can also be made, that even with no taxes, i.e. $\tau_0 = 0$ or $\Delta_\tau = 0$, the perceived spectral efficiency CDF curves of our taxation models can be found below the respective CDF for the optimal SCs allocation for adequately low CDF argument values. This is because in our game model we have defined the payoff, which reflects not only the potential throughput, but also the fairness, i.e. network potential to serve multiple nodes.

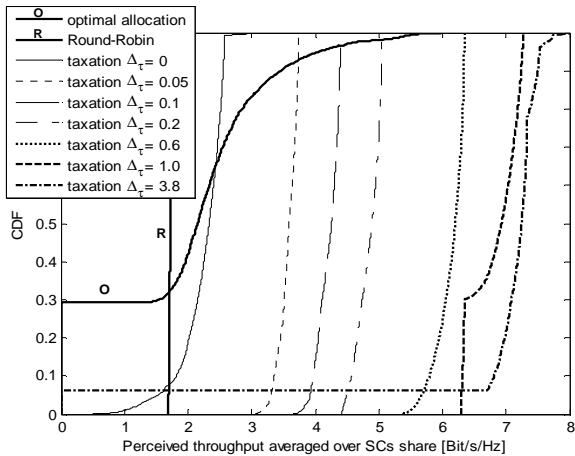
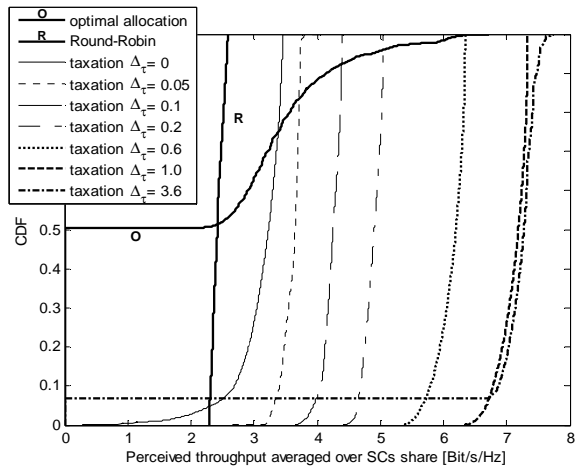


Figure 4. Cumulative Distribution Function (CDF) of the perceived throughput per frequency unit in the game with arithmetically-progressive taxation for the ‘E’ case, $\bar{\gamma} = 30$ dB and a) $K = 16, I = 16$, b) $K = 8, I = 32$.

In Fig. 6 one can observe, that for a given K and I there exist the values of the linear and arithmetically-progressive tax rate parameters τ_0 and Δ_τ , above which the percentage of served nodes dramatically drops. The so-called barrage taxes prevent the players from occupying even a single one SC. For example, in case ‘E’, for $K=16$ and $I=16$, the barrage linear-tax-rate parameter and the barrage arithmetically-progressive tax-rate parameter equal: $\tau_0 = 4$, $\Delta_\tau = 4$ respectively. The barrage tax-rate parameters are a little higher in case ‘D’, because in this scenario some nodes are in a better position (in terms of the average SINR) than others. For these nodes in the considered situation, it still pays off to acquire some SCs, even if taxes are as high as the barrage taxes in case ‘E’.

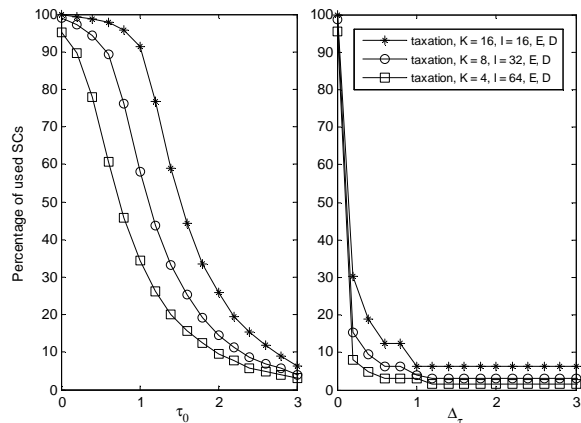


Figure 5. The percentage of used subcarriers vs. the tax-rate parameter values: a) τ_0 for linear taxation and b) Δ_τ for arithmetically-progressive taxation ($\bar{\gamma} = 30$ dB).

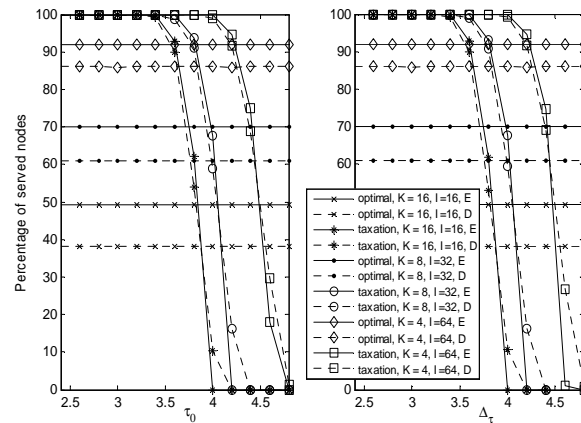


Figure 6. The percentage of served nodes vs. the tax-rate parameter values: a) τ_0 for linear taxation and b) Δ_τ for arithmetically-progressive taxation ($\bar{\gamma} = 30$ dB).

4. CONCLUSIONS

The taxation-based framework for the distributed, efficient, and rational resource acquisition in the cognitive OFDMA network has been presented. In our game-theoretic model, each player makes decisions independently based on the local CSI and the taxation parameters τ_0 and Δ_τ . For the sake of the algorithm efficiency, the payoff has been defined so as to reflect the throughput achieved by a CR-node and the network capacity of serving the CR-nodes community. For the sake of rationality, taxation of the SCs has been introduced. By treating the CR-nodes community as a single observed player, and by introducing the efficiency and rationality measures we can approach high overall and nodes’ perceived spectral efficiency, high usage of the available SCs and 100% of served nodes with low complexity and in the distributed manner as opposed to solutions described in the literature. The base station role is only to beacon the tax-rate parameter value in its service area.

Furthermore, in our simulated model, we observe that there is some degree of freedom in establishing the linear and arithmetically-progressive tax-rate parameters τ_0 and Δ_τ to approach high overall spectral efficiency and fairness in distributing available resources. The optimal values of τ_0 and Δ_τ can be chosen to compromise between these two goals.

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