Effects of Parameters on Nash Games with OSNR Target

Yan Pan
Department of Electrical and Computer Engineering
University of Toronto
Toronto, ON M5S 3G4, Canada
yanpan@control.utoronto.ca

Tansu Alpcan
Deutsche Telekom Labs
Ernst-Reuter-Platz 7, D-10587
Berlin, Germany
tansu.alpcan@telekom.de

Lacra Pavel
Department of Electrical and Computer Engineering
University of Toronto
Toronto, ON M5S 3G4, Canada
pavel@control.toronto.edu

ABSTRACT

This paper studies efficiency in a Nash game with optical signal-to-noise ratio (OSNR) target. Instead of looking from the view point of degree of efficiency ("price of anarchy"), we investigate the effects of parameters in individual cost functions. We show that the aggregate cost function in the game-theoretic formulation is not automatically convex and the optimal solution of the associated constrained optimization problem is not immediate. Then we build the relation between these two formulations by indicating that the individual cost function $C_i(u_i)$ in the system optimization formulation has an approximate interpretation with the one $J_i(u)$ in the game-theoretic formulation. We compare simulation results from both a system optimization and a user optimization (game-theoretic) approach for a single optical link. It is well known that the Nash equilibria of a game may not achieve full efficiency. We show the effects of pricing mechanisms on system performance. We also show that OSNR target can be achieved and efficiency can be possibly improved by appropriate selection of parameters.

1. INTRODUCTION

In optical communication networks, OSNR is considered as the dominant performance parameter in link optimization, with dispersion and nonlinearity being limited with proper link design, [7]. A signal over the same optical link can be regarded as an interfering noise for other channels (signals), which leads to degradation in the quality of service, i.e., OSNR degradation. Regulating the input optical power per channel at *Source* (Tx) aims to achieve a satisfactory OSNR level at *Destination* (Rx). Particularly in optical networks, because all wavelength-multiplexed channels in a link share the optical fiber, the total input power in a link has to be below the nonlinearity threshold, which can be regarded as the *link capacity constraint.* The OSNR optimization problem in optical networks belongs to a subclass of resource allocation in general communication networks [2, 4, 18].

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Since each channel belonging to different carriers is operating at its own carrier wavelength and no communication among channels exists, the OSNR optimization problem is formulated as a noncooperative (Nash) game in [13], where channels are the players. In the game, each channel wishes to optimize its OSNR independently. A solution to the Nash game is called a Nash equilibrium (NE) solution. Moreover, the Nash game immediately leads to distributed iterative algorithms towards finding the NE solution. In the game-theoretical formulation, channel OSNR target is not considered. It is possible that the game settles down at an NE solution where channels do not reach their OSNR target. A system optimization approach is considered in [12] and the OSNR optimization problem is formulated as one of utility maximization problems. Each channel obtains a minimum OSNR level that is slightly greater than the desired one. Meanwhile, it optimizes its input optical power considering target OSNR levels of all other channels and link capacity constraint.

In this paper we compare by simulation the two approaches, i.e., a system optimization approach and a user optimization (game-theoretic) approach, for a single optical link. The numerical results are developed based on our previous theoretic work [12–14]. It is well known that the Nash equilibria of a game may not achieve full efficiency. The inefficiency of the NE solution has been studied extensively [1, 5, 8, 15, 16, 18]. The resulting degree of the efficiency loss is known as the "price of anarchy" [11]. There has been an increasing literature in recent years trying to quantify the efficiency loss under separable costs [8] and non-separable costs [15], respectively. In particular, results suggest that the selfish behavior of players in a Nash game may not degrade the system performance arbitrarily, provided a pricing mechanism is chosen properly [8, 18]. In this paper, we study a Nash game with non-separable cost functions and pricing mechanism is applied to improve the system performance. Instead of looking from the view point of degree of efficiency, we investigate the effects of parameters in individual cost functions. Moreover, we select the system optimization formulation as the system framework to measure the efficiency of the NE solution. We show that the aggregate cost function in the game-theoretic formulation is not automatically strictly convex and the optimal solution of the associated constrained optimization problem is not immediate. We indicate that an individual cost function $C_i(u_i)$ in the system optimization formulation has an approximate interpretation with the cost function $J_i(u)$ in the game-theoretic formulation and is

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uncoupled in u, that is, it is defined with respect to u_i only. We build the relation between these two formulations and compare the results based on this. We show the effects of pricing mechanisms. We also show that OSNR target can be achieved and the efficiency can be possibly improved by appropriate selection of parameters.

The paper is organized as follows. In Section 2, we first review a link OSNR model. The OSNR optimization problem is formulated and solved via game-theoretic approach [13,14] and via system optimization approach [12], respectively. In Section 3, we compare the cost functions in the two formulations. We show that an individual cost function in the system optimization approach can be approximately interpreted with the individual cost function in the game-theoretic approach. Then we present parameter selection strategy. By adjusting the parameters in individual cost functions, the game-theoretic approach leads to an NE solution that satisfies the OSNR constraint. After that in Section 4, we compare by simulation the results from both approaches and show that the efficiency can be improved by appropriate selection of parameters. Section 5 gives conclusions and directions for future work.

2. BACKGROUND

2.1 Link OSNR Model

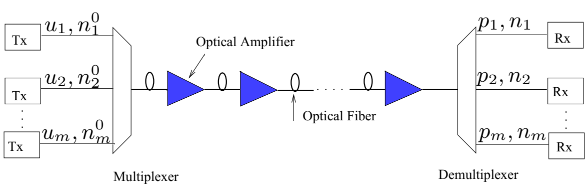

Figure 1: Point-to-point optical link

We study a point-to-point optical link case (Fig. 1). A set $\mathcal{M} = \{1, \dots, m\}$ of channels are transmitted over the link. The link consists of N cascaded spans of optical fiber each followed by an optical amplifier. All optical amplifiers have the same gain spectral shape and the gain value for channel i is G_i.

We denote u_i and n_i^0 the signal optical power and noise optical power of channel $i \in \mathcal{M}$ at Tx, respectively. Similarly, we denote p_i and n_i the signal optical power and noise optical power of channel $i \in \mathcal{M}$ at Rx, respectively. Let $u = [u_1, \dots, u_m]^T$ denote the vector form. Equivalently, we write (u_{-i}, u_i) with $u_{-i} = [u_1, \dots, u_{i-1}, u_{i+1}, \dots, u_m]^T$ in some context to represent the same vector u. The signal optical power at Tx is typically bounded for every channel. That is, u_i is in a bounded set $\Omega_i = [0, u_{max}]$ where u_{max} is sufficiently large. The bounded set Ω_i is identical for all channels. We use $\Omega = \Omega_1 \times \cdots \times \Omega_m$ to represent the Cartesian product.

The OSNR of channel i at Rx is given by

$$OSNR_i = \frac{u_i}{n_i^0 + \sum_{j \in \mathcal{M}} \Gamma_{i,j} u_j}, \tag{1}$$

$$\Gamma_{i,j} = \sum_{s=1}^{N} \frac{(G_j)^s}{(G_i)^s} \frac{ASE_{s,i}}{P^0}, \tag{2}$$

where $ASE_{s,i}$ is the ASE noise power of channel i after span s and P^0 is the total power target for the link.

Equivalently, (1) can be rewritten as

$$OSNR_i = \frac{u_i}{X_{-i} + \Gamma_{i,i} u_i}, \tag{3}$$

where

$$X_{-i} = n_i^0 + \sum_{j \in \mathcal{M}, j \neq i} \Gamma_{i,j} u_j. \tag{4}$$

The OSNR optimization problem in optical networks belongs to a subclass of resource allocation in general communication networks [2,18]. The OSNR model reflects that the optical power of one channel can be regarded as an interfering noise for others, which leads to the OSNR degradation. Regulating the optical powers at Tx, i.e., allocating optical resource among channels, aims to achieve a satisfied OSNR level for each channel at Rx.

2.2 User Optimization Formulation

Next from the user optimization point of view, we use the game theoretic approach to study the OSNR optimization problem. We formulate it as a noncooperative (Nash) game, where channels are the players. In the game, each channel wishes to optimize its OSNR independently, i.e., to minimize its individual cost function by adjusting its input optical power at Tx, given the input optical powers of all other channels.

Considering the link capacity constraint,

$$\sum_{j \in \mathcal{M}} u_j - P^0 \leq 0.$$

The action space is coupled:

$$\bar{\Omega} = \left\{ u \in \Omega \mid \sum_{j \in \mathcal{M}} u_j - P^0 \leq 0 \right\}. \tag{5}$$

And the action set for each channel i is the projection set

$$\hat{\Omega}_i(u_{-i}) = \left\{ \xi \in \Omega_i \mid \sum_{j \in \mathcal{M}, j \neq i} u_j + \xi - P^0 \leq 0 \right\}. \tag{6}$$

An individual cost function J_i assigned to each channel $i \in \mathcal{M}$ is defined as the difference between a *pricing function* P_i and a *utility function* U_i:

$$J_i = P_i - U_i. \tag{7}$$

Here the utility function U_i is chosen to be a logarithmic function of the associated channel's OSNR:

$$U_i = \beta_i \ln\left(1 + \frac{a_i}{\frac{1}{OSNR_i} - \Gamma_{i,i}}\right), \ \forall\, i \in \mathcal{M} \tag{8}$$

where $\beta_i > 0$ is a channel-defined parameter indicating the strength of the channel's desire to maximize its OSNR and $a_i > 0$ is a scaling parameter for flexibility.

From (8) we can see that U_i is monotonically increasing in $OSNR_i$. Hence, maximizing the utility function U_i is related to maximizing $OSNR_i$. From (3), the utility function can be equivalently written in terms of u,

$$U_i(u_{-i}, u_i) = \beta_i \ln\left(1 + a_i \frac{u_i}{X_{-i}}\right), \ \forall\, i \in \mathcal{M}. \tag{9}$$

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It can be checked that the utility function $U_i(u_{-i}, u_i)$ is twice continuously differentiable, monotonically increasing and strictly concave with respect to u_i.

The reasons for the choice of a logarithmic function (9) as follows. A logarithmic function is analytically useful and is widely used as a utility function in flow control [9, 19] and power control [3] for general communication networks. In some cases, the logarithmic utility function is intimately associated with the concept of proportional fairness [9]. More specifically, since $OSNR_i$ is strictly increasing with respect to u_i and tends to $\frac{1}{\Gamma_{i,i}}$ as u_i tends to infinity. The OSNR model has a striking similarity with the wireless SIR model [3], but with a more general full system matrix Γ. Moreover, OSNR is no longer a linear function of its associated channel power. A direct logarithmic utility function of associated SIR in the wireless case can not be applied.

The pricing function consists of two terms: a linear pricing term and a regulation (penalty) term,

$$P_i(u_{-i}, u_i) = \alpha_i u_i + \frac{1}{P^0 - \sum_{j\in\mathcal{M}} u_j},\ \forall\, i \in \mathcal{M}, \tag{10}$$

where $\alpha_i > 0$ is a pricing factor determined by the system.

The linear pricing term is a linear function of the individual input power. It can be interpreted as the price a channel pays for using the system resources [17]. The linear pricing term reflects the fact that increasing one channel's power degrades the OSNR of all other channels. From the system performance point of view, the linear pricing term here is to limit the interferences of other channels caused by this channel and hence it improves the overall system performance [18].

The regulation term is constructed by considering the link capacity constraint. It penalizes any violation of the constraint in the following way: The regulation term tends to infinity when the total power approaches the total power target P^0, so the pricing function $P_i(u_{-i}, u_i)$ increases without bound. Hence the system resource is preserved by forcing all channels to decrease their input powers and indirectly satisfies the link capacity constraint.

Thus the m-player Nash game is defined in terms of the cost functions $J_i(u_{-i}, u_i)$, $i \in \mathcal{M}$, played within the action space $\bar{\Omega}$. This game is called an *OSNR Nash game* and denoted by $GAME(\mathcal{M}, \hat{\Omega}_i, J_i)$. A vector $u = (u_{-i}, u_i)$ is called *feasible* if $u \in \bar{\Omega}$. The concept of an NE solution is well defined.

Definition 1. *A vector $u^* \in \Omega$ is called an* NE solution *of $GAME(\mathcal{M}, \hat{\Omega}_i, J_i)$ if for all $i \in \mathcal{M}$ and for every given u^*_{-i},*

$$J_i(u^*_{-i}, u^*_i) \leq J_i(u^*_{-i}, u_i),\ \ \forall\, u_i \in \hat{\Omega}_i(u^*_{-i}). \tag{11}$$

If in addition, the solution is not on the boundary of the action space $\bar{\Omega}$, it is called an *inner NE solution*.

There exists an NE solution for this m-player OSNR Nash game. Furthermore, the NE solution is inner. The following result is adapted from [13] which provides sufficient conditions for existence and uniqueness of of an inner NE solution. We restate here for completeness.

Theorem 1. *[13] $GAME(\mathcal{M}, \hat{\Omega}_i, J_i)$ admits a unique inner NE solution u^* if α_i, β_i and a_i satisfy*

$$a_i > \sum_{j\in\mathcal{M}, j\neq i} \Gamma_{i,j} \tag{12}$$

$$\frac{\beta_{min}}{\sum_{j\in\mathcal{M}, j\neq i} \frac{\Gamma_{j,i}}{a_j}} > \beta_i \geq \beta_{min} \tag{13}$$

$$\alpha_{max} \geq \alpha_i > \alpha_{max} \sqrt{\sum_{j\in\mathcal{M}, j\neq i} \frac{\beta_i}{\beta_j} \frac{\Gamma_{j,i}}{a_j}} \tag{14}$$

where $\beta_{min} = \min_{j\in\mathcal{M}} \beta_j$ and $\alpha_{max} = \max_{j\in\mathcal{M}} \alpha_j$.

2.3 System Optimization Formulation

Next we study the constrained OSNR optimization problem from the system optimization point of view. A system optimization problem formulated as one of utility maximization problems is considered. The system optimization problem is subject to two specific constraints: The target OSNR constraint and the link capacity constraint. Let $\widehat{\gamma}_i$ be the i^{th} channel's target OSNR and $\widehat{\gamma} = [\widehat{\gamma}_1, \ldots, \widehat{\gamma}_m]^T$. The *target OSNR constraint* can be written as

$$OSNR_i \geq \widehat{\gamma}_i,\ \forall\, i \in \mathcal{M}. \tag{15}$$

Another constraint is the *link capacity constraint*

$$\sum_{j\in\mathcal{M}} u_j - P^0 \leq 0.$$

The system cost function is composed of individual channel cost functions. Let $C_i(u_i)$ be channel i's individual cost function. It satisfies the following assumption:

Assumption 1. *$C_i(u_i)$ is strictly convex, continuously differentiable and*

$$\lim_{u_i\to 0} C_i(u_i) = +\infty. \tag{16}$$

The system cost function is the sum of all individual cost functions. Thus the system optimization problem is formulated as

$$\begin{aligned} \min \sum_{i\in\mathcal{M}} C_i(u_i) \\ \text{subject to}\ \ OSNR_i \geq \widehat{\gamma}_i,\ \forall\, i \in \mathcal{M} \\ \sum_{j\in\mathcal{M}} u_j - P^0 \leq 0 \\ u_i \geq 0,\ \forall\, i \in \mathcal{M}. \end{aligned}$$

By using the OSNR model, (15) can be rewritten as

$$u_i + \sum_{j\in\mathcal{M}} (-\widehat{\gamma}_i\, \Gamma_{i,j})\, u_j \geq n_i^0\, \widehat{\gamma}_i.$$

The associated vector form is

$$T\, u \geq b, \tag{17}$$

where

$$T = \begin{bmatrix} 1-\widehat{\gamma}_1\Gamma_{1,1} & -\widehat{\gamma}_1\Gamma_{1,2} & \cdots & -\widehat{\gamma}_1\Gamma_{1,m} \\ -\widehat{\gamma}_2\Gamma_{2,1} & 1-\widehat{\gamma}_2\Gamma_{2,2} & \cdots & -\widehat{\gamma}_2\Gamma_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ -\widehat{\gamma}_m\Gamma_{m,1} & -\widehat{\gamma}_m\Gamma_{m,2} & \cdots & 1-\widehat{\gamma}_m\Gamma_{m,m} \end{bmatrix}$$

and

$$b = \begin{bmatrix} n_1^0\widehat{\gamma}_1 & n_2^0\widehat{\gamma}_2 & \cdots & n_m^0\widehat{\gamma}_m \end{bmatrix}^T.$$

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The link capacity constraint can also be written in a vector form,

$$\mathbf{1}^T u \leq P^0, \tag{18}$$

where $\mathbf{1}$ is the $m \times 1$ all ones vector.

Thus the system optimization problem is formulated by

$$\min_{u \in \widetilde{\Omega}} C(u), \tag{19}$$

where the system cost function is

$$C(u) = \sum_{i \in \mathcal{M}} C_i(u_i)$$

and the constraint set $\widetilde{\Omega}$ is

$$\widetilde{\Omega} := \{u \in \mathbb{R}^m \mid T\, u \geq b, \mathbf{1}^T u \leq P^0 \text{ and } u_i \geq 0, \forall\, i \in \mathcal{M}\}.$$

We denote the system optimization problem by $OPT(\widetilde{\Omega}, C)$.

The condition (16) in Assumption 1 ensures that the solution to $OPT(\widetilde{\Omega}, C)$ does not hit $u_i = 0$, $\forall\, i \in \mathcal{M}$. The following result adapted from [12] characterizes the unique solution of $OPT(\widetilde{\Omega}, C)$.

THEOREM 2. *[12] If the following conditions on $\widehat{\gamma}$ hold:*

$$\widehat{\gamma}_i < \frac{1}{\sum_{j \in \mathcal{M}} \Gamma_{i,j}}, \ \forall\, i \in \mathcal{M}, \tag{20}$$

where $\Gamma = [\Gamma_{i,j}]$ is the system matrix defined in (2) and

$$\mathbf{1}^T \cdot \widetilde{T}(\widehat{\gamma}) \cdot b(\widehat{\gamma}) \leq P^0, \tag{21}$$

with $b(\widehat{\gamma}) = [n_1^0 \widehat{\gamma}_1, \ldots, n_m^0 \widehat{\gamma}_m]^T$ and $\widetilde{T}(\widehat{\gamma}) = T^{-1}(\widehat{\gamma})$, then the constraint set $\widetilde{\Omega}$ is non-empty and $OPT(\widetilde{\Omega}, C)$ has a unique positive solution u^{opt}.

REMARK 1. *In the Nash game formulation, the OSNR target is not considered. Theorem 1 provides sufficient conditions for the existence and uniqueness of an NE solution for $GAME(\mathcal{M}, \hat{\Omega}_i, J_i)$. We notice from (13) that the ratio of β_i to β_{min} is bounded in*

$$[1, \frac{1}{\sum_{j \in \mathcal{M}, j \neq i} \frac{\Gamma_{j,i}}{a_j}}),$$

which implies that channels are not allowed to ask unilaterally for a much higher OSNR target than others. In the system optimization formulation, the sufficient condition for the existence of a unique optimal solution for $OPT(\widetilde{\Omega}, C)$ provides explicitly the bound on channel OSNR targets.

3. COMPARISON OF OPTIMIZATION AND GAME FORMULATIONS

In this section, we compare the two approaches presented in Section 2 for a single point-to-point optical link. The comparison is motivated by the fact that an individual cost function $C_i(u_i)$ in $OPT(\widetilde{\Omega}, C)$ has an approximate interpretation similar to the one of the cost function $J_i(u)$ in $GAME(\mathcal{M}, \hat{\Omega}_i, J_i)$. It is well known that there is no full efficiency in the Nash game. Instead of looking from the view point of degree of efficiency, our study focuses on the effects of parameters, which is somewhat related to pricing mechanism, [8, 10].

3.1 Comparison of cost functions

Recall in Section 2.2, the individual cost function is selected such that it indicates each channel's preference for a better OSNR. This is realized by defining a utility function with respect to channel's OSNR. However, generally speaking, the convexity of the aggregate game cost function defined as

$$J(u) := \sum_{i \in \mathcal{M}} J_i(u) \tag{22}$$

is not longer guaranteed. The following simple example illuminates this, in which we omit the penalty term and noise in the OSNR model for simplicity.

EXAMPLE 1. *Consider a Nash game with 3 players ($m = 3$) with individual cost functions,*

$$J_i(u) = u_i - \ln \frac{u_i}{\sum_{j \neq i} u_j}, \ i = 1, 2, 3.$$

It follows that

$$\frac{\partial^2 J_1}{\partial u_1^2} = \frac{u_2 + u_3}{u_1} > 0$$
$$\frac{\partial^2 J_2}{\partial u_1^2} = -\frac{1}{(u_1 + u_3)^2} < 0$$
$$\frac{\partial^2 J_3}{\partial u_1^2} = -\frac{1}{(u_1 + u_2)^2} < 0$$

Therefore,

$$\frac{\partial^2 J}{\partial u_1^2} = \frac{u_2 + u_3}{u_1} - \frac{1}{(u_1 + u_3)^2} - \frac{1}{(u_1 + u_2)^2}$$

The sign of $\frac{\partial^2 J}{\partial u_1^2}$ is uncertain. Thus $J(u)$ is not always convex with respect to u_1, even though $J_1(u)$ is strictly convex with respect to u_1. □

Thus the constrained optimization problem associated with an aggregate cost function is not always a convex optimization problem and the optimal solutions are not immediate.

In fact the individual cost function C_i in the system optimization problem $OPT(\widetilde{\Omega}, C)$ in Section 2.3 can be defined similarly in the form of (7), $C_i = P_i - U_i$. The pricing function P_i is a linear function of u_i. The utility function U_i is a logarithmic function of u_i, which quantifies approximately the link's demand or channel's willingness to pay for a certain level of $OSNR_i$ based on the relationship between $OSNR_i$ and u_i. The relationship is illustrated approximately in Fig.2.

By this approximate definition, the individual cost function C_i is uncoupled in u for a given set of other power u_{-i}. Furthermore, it has an approximate interpretation similar to the one of J_i. Thus we build the relation between these two formulations, i.e., $GAME(\mathcal{M}, \hat{\Omega}_i, J_i)$ and $OPT(\widetilde{\Omega}, C)$. We will compare simulation results based on this. Moreover, we select the system optimization formulation as the system framework to measure the efficiency of the NE solution.

3.2 Parameter selection strategy

The following parameter selection strategy provides a guideline for the selection of each β_i, given channel OSNR targets.

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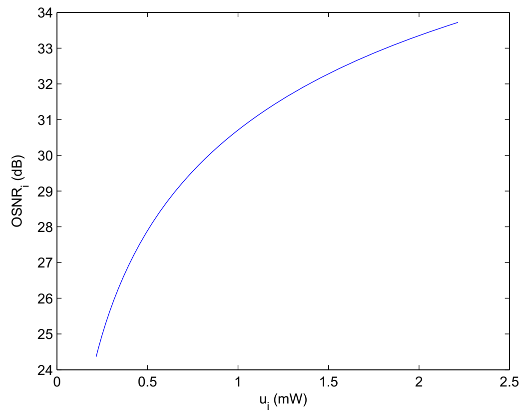

Figure 2: $OSNR_i$ vs u_i for a given set of other power u_i's

In $GAME(\mathcal{M}, \hat{\Omega}_i, J_i)$, the link sets fixed channel prices α_i and channels decide their willingness β_i to obtain higher OSNR levels. From the first-order necessary condition

$$\frac{\partial J_i(u)}{\partial u_i} = 0$$

we have

$$\beta_i(OSNR_i) = \frac{\alpha_i}{a_i}(X_{-i} + a_i u_i(OSNR_i)) + \frac{X_{-i} + a_i u_i(OSNR_i)}{a_i(P^0 - \sum_{j\in\mathcal{M}, j\neq i} u_j - u_i(OSNR_i))^2},$$

where

$$u_i(OSNR_i) = \frac{X_{-i}}{1/OSNR_i - \Gamma_{i,i}}.$$

For a given OSNR target $\widehat{\gamma}_i$ for each channel i, i.e, a lower OSNR bound, we can show that if β_i is adjusted to satisfy

$$\beta_i > \frac{\alpha_i}{a_i}\frac{1 + (a_i - \Gamma_{i,i})\widehat{\gamma}_i}{1 - \Gamma_{i,i}\widehat{\gamma}_i}X_{-i} + \frac{(1 - \Gamma_{i,i}\widehat{\gamma}_i)(1 + (a_i - \Gamma_{i,i})\widehat{\gamma}_i)}{a_i(P^0 - \sum\limits_{j\neq i} u_j - (P^0 - \sum\limits_{j\neq i} u_j\Gamma_{i,i} + X_{-i})\widehat{\gamma}_i)^2}X_{-i}, \quad (23)$$

each channel achieves at least the $\widehat{\gamma}_i$ level, i.e., $OSNR_i > \widehat{\gamma}_i$. Thus this result briefly provides an idea of how to achieve given OSNR targets by tuning the parameters in associated cost functions in the game theoretic framework. Intuitively, larger β_i possibly leads to increase the lower OSNR bound.

4. COMPARISON BY SIMULATIONS

Next we compare by simulation the two approaches for a single point-to-point optical link shown in Fig. 1. The link has six channels ($m = 6$) and the link capacity constraint is $P^0 = 2.5\,\text{mW}$ (3.98 dBm). The associated system matrix Γ is obtained as

$$\Gamma = \begin{bmatrix} 7.463 & 7.378 & 7.293 & 7.210 & 7.127 & 6.965 \\ 7.451 & 7.365 & 7.281 & 7.198 & 7.115 & 6.953 \\ 7.438 & 7.353 & 7.269 & 7.186 & 7.103 & 6.942 \\ 7.427 & 7.342 & 7.258 & 7.175 & 7.093 & 6.931 \\ 7.409 & 7.324 & 7.240 & 7.157 & 7.075 & 6.914 \\ 7.387 & 7.303 & 7.219 & 7.136 & 7.055 & 6.894 \end{bmatrix} \times 10^{-5}.$$

Within the set of six channels, there are two levels of target OSNR, a 26 dB level desired on the first three channels and a 22 dB OSNR level on the next three channels.

In $GAME(\mathcal{M}, \hat{\Omega}_i, J_i)$, the individual game cost function defined in (7) is restated here,

$$J_i(u) = \alpha_i u_i + \frac{1}{P^0 - \sum\limits_{j\in\mathcal{M}} u_j} - \beta_i \ln(1 + \frac{a_i u_i}{n_i^0 + \sum_{j\neq i}\Gamma_{i,j}u_j}). \quad (24)$$

The NE solution is denoted by u^*. The following gradient iterative scheme [14]

$$\dot{u}_i(t) = -\mu\left(\frac{\partial J_i(u_{-i}, u_i)}{\partial u_i(t)}\right) = -\mu\left(\alpha_i + \frac{1}{(P^0 - \sum\limits_{j\in\mathcal{M}} u_j(t))^2} - \frac{\beta_i a_i/u_i(t)}{(\frac{1}{OSNR_i(t)} + a_i - \Gamma_{i,i})}\right)$$

where (3) is used and the step-size is selected as $\mu = 0.01$.

Remark 2. *Theorem 2 in [14] states that the gradient scheme converges to the NE solution if*

$$\min_{j\in\mathcal{M}} a_j > \left(\frac{2P^0}{u_{min}}\right)^2 \max_{i\in\mathcal{M}} \sum_{j\in\mathcal{M},\neq i}\Gamma_{j,i},$$

$$\beta_i > \left(\frac{2P^0}{u_{min}}\right)^2 \frac{\max_{i\in\mathcal{M}}\sum_{j\in\mathcal{M},j\neq i}\Gamma_{j,i}}{\min_{j\in\mathcal{M}} a_j} \cdot \max_{j\in\mathcal{M}} \beta_j,$$

where $u_{min} > 0$ is a positive lower bound on each u_i. The second condition sets a lower bound on β_i. Since each channel attempts to select larger β_i for the purpose of higher OSNR, the lower bound does not affect the results of the efficiency study.

In $OPT(\widetilde{\Omega}, C)$, instead of a generic $C_i(u_i)$, we use a specific individual cost function, i.e.,

$$C_i(u_i) = \alpha_i u_i - \beta_i \ln u_i \quad (25)$$

as indicated in Section 3.1. The system cost function is $C(u) := \sum_{i\in\mathcal{M}} C_i(u_i)$. The optimal solution is denoted by u^{opt}. A dual algorithm presented in [12]

$$\dot{\lambda}_i(t) = \frac{d}{dt}\lambda_i(t) = k_i\left(\widehat{b}_i - row_i(\widehat{T})u(t)\right)$$

where $u_i(t) = C_i'^{-1}(row_i(\widehat{T}^T)\lambda(t))$ with $\widehat{T} = \begin{bmatrix} T \\ -\mathbf{1}^T \end{bmatrix}$, is used. We implement it in a distributed way:

$$\textbf{Link:}\ \dot{\lambda}_i(t) = k_i\left(\widehat{b}_i - row_i(\widehat{T})u(t)\right)$$

$$q_i(t) = row_i(\widehat{T}^T)\lambda(t),\ i\in\mathcal{M}$$

$$\textbf{Channel:}\ u_i(t) = C_i'^{-1}(q_i(t)),\ i\in\mathcal{M}.$$

The step-size is fixed for each channel with $k_i = 0.01$, $i = 1, \cdots, 6$.

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REMARK 3. *Note that the two individual cost functions (24) and (25), have a similar structure. They both have a price term $\alpha_i u_i$. The utility term in (24) is in fact a function of OSNR, while the utility term in (25) reflects indirectly the preference for better OSNR based on the relationship between channel input power and OSNR (Fig.2). The link capacity constraint is considered indirectly as a penalty term in (24).*

We use the system optimization formulation $C(u)$ with $C_i(u_i)$ defined as in (25) as the system framework to measure the efficiency of the NE solution u^*. Furthermore, the parameters are selected to satisfy the conditions for the existence and uniqueness of a solution and for the convergence of the algorithms. Specifically, the parameters in $OPT(\widetilde{\Omega}, C)$ are selected as $\alpha_i = 1$, $i = 1, \ldots, 6$ and

$$\beta = [0.5\ 0.51\ 0.52\ 0.3\ 0.31\ 0.32].$$

The total power vs iteration and channel OSNR vs channel number are shown in Fig 3.

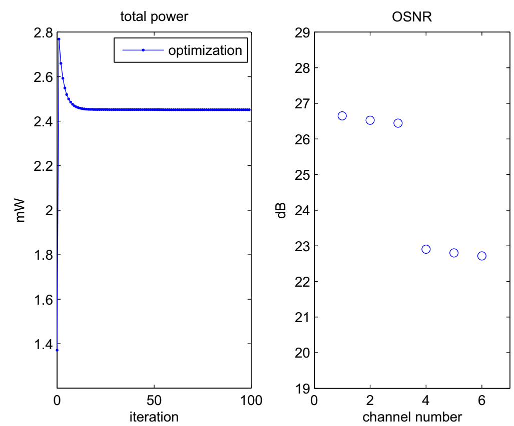

Figure 3: Total power and $OSNR_i$ in OPT

We note that both total power constraint and link capacity constraint are satisfied. The optical channel powers in $OPT(\widetilde{\Omega}, C)$ are achieved as

$$u^{opt} = [0.4022\ 0.4876\ 0.5730\ 0.2955\ 0.3295\ 0.3633].$$

The system cost values with respect to u^{opt} is

$$C(u^{opt}) = 4.5963.$$

4.1 Games without OSNR target

We present three cases in which the parameter selection strategy is not used as a guideline. In all cases, the user-defined parameters β_i in $GAME(\mathcal{M}, \hat{\Omega}_i, J_i)$ are chosen as same as β_i in $OPT(\widetilde{\Omega}, C)$, i.e.,

$$\beta = [0.5\ 0.51\ 0.52\ 0.3\ 0.31\ 0.32].$$

The link sets fixed α_i at 0.001, 1 (same as in $OPT(\widetilde{\Omega}, C)$) and 20, respectively. With these pricing mechanisms, the total power (u_T) vs iteration and channel OSNR vs channel number are shown in Fig 4, Fig 5 and Fig 5 for three cases.

It is observed that without proper pricing mechanism, OSNR targets may not be achieved for some channels or

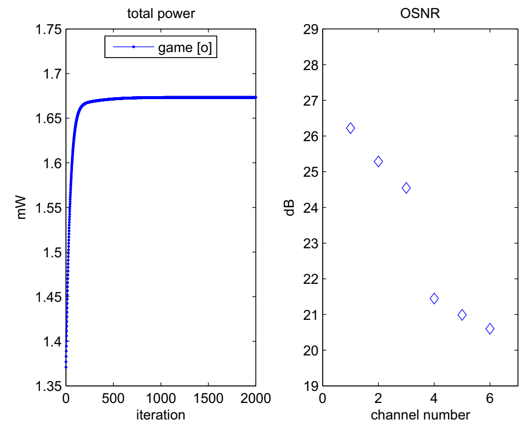

Figure 4: u_T and $OSNR_i$ in Game with $\alpha_i = 0.001$

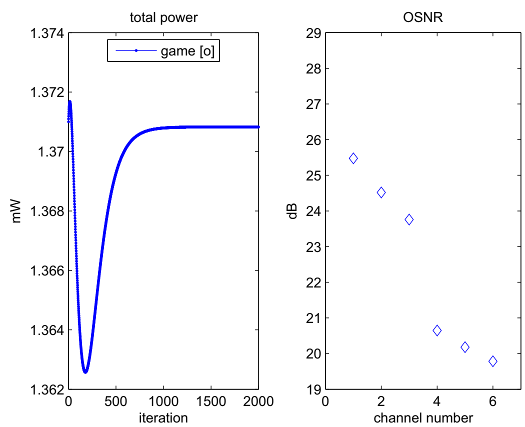

Figure 5: u_T and $OSNR_i$ in Game with $\alpha_i = 1$

all channels. The link capacity constraint is satisfied in all cases. Furthermore, we notice that the penalty term

$$\frac{1}{P^0 - \sum_{j \in \mathcal{M}} u_j}$$

in $GAME(\mathcal{M}, \hat{\Omega}_i, J_i)$ plays a key role with small α_i. In other words, with larger α_i (say, $\alpha_i = 20$ in the third case), total power is great smaller than the link capacity constraint. While with smaller α_i in the first two cases, total power is approaching the constraint and higher channel OSNR is possibly achieved.

Channel powers, u^{opt} and u^* in three games vs channel number are shown in Fig. 7. The system cost $C(u) = \sum_{i \in \mathcal{M}} C_i(u_i)$ is evaluated via u^{opt} and u^*, respectively and is shown in Table 1. Results imply that larger α_i degrades system performance and even violate the system constraints.

In the following section, we use smaller α_i. Specifically, we use $\alpha_i = 1$ in $GAME(\mathcal{M}, \hat{\Omega}_i, J_i)$ same as in $OPT(\widetilde{\Omega}, C)$ for comparison.

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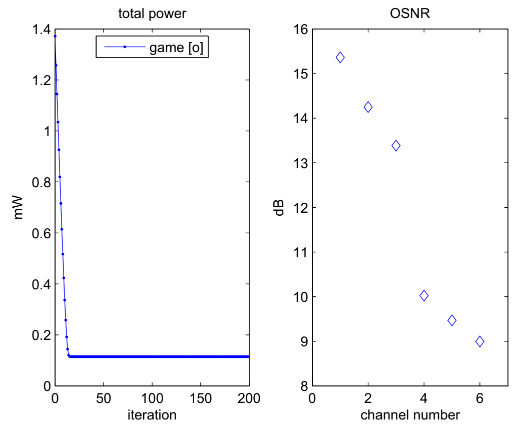

Figure 6: u_T and $OSNR_i$ in Game with $\alpha_i = 20$

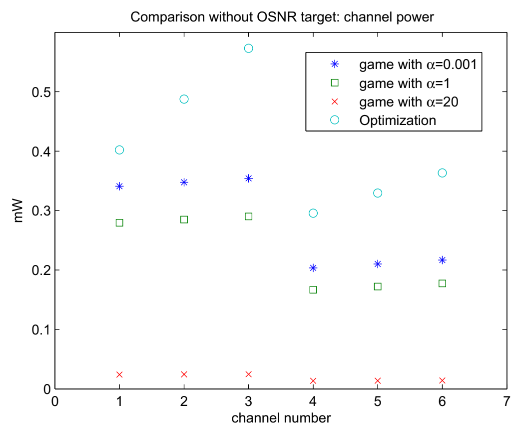

Figure 7: Comparison without OSNR target: channel power

4.2 Games with OSNR target

Next we present three other cases in which proper pricing mechanisms are chosen such that OSNR targets for all channels are achieved. Although the parameter selection strategy acts as a guideline for the selection of each β_i, it is practically intractable. Thus we choose proper pricing mechanism in simulation by trial and error.

The parameters α_i are set at 1 for all cases and β_i are selected as in Table 2 such that different pricing mechanisms are chosen for $GAME(\mathcal{M}, \hat{\Omega}_i, J_i)$ by selecting different β_i.

Since we do not use Monte Carlo method [6] to simulate, we select β_i in three games by using the following rules.

Table 1: System Cost Values

	$C(u^*)$	$C(u^{opt})$
$\alpha_i = 0.001$	4.7403	4.5963
$\alpha_i = 1$	4.9282	4.5963
$\alpha_i = 20$	9.7804	4.5963

Table 2: Parameters

	β_i
Game [a]	[3.8 4.8 5.8 2.6 3 3.5]
Game [b]	[5.5 7 9.4 4 4.5 5]
Game [c]	[10 12 15 8.4 8.5 8.3]

Firstly, β_i increases for each channel, i.e., Game [c] has the largest β_i compared to Game [a] and Game [b]. Secondly, Game [b] has the largest ratio of β_i to β_{min}.

The efficiency of these two solutions u^* and u^{opt}, is compared by evaluating the system cost $C(u)$ in Table 2. The corresponding system cost values are obtained and shown in Table 3.

Table 3: System Cost Values

	$C(u^*)$	$C(u^{opt})$
Game [a]	4.6171	4.5963
Game [b]	4.6216	4.5963
Game [c]	4.6056	4.5963

The results in Table 3 (compared with Table 1) verify that the efficiency in the solution of the Nash game (user optimization) can be improved by proper pricing mechanism. The fact that no full efficiency in the solution of the Nash game is a well-known fact in the literature of economics [5], transportation [16] and network resource allocation [8]. Moreover, the Nash game solution gets very close to the optimal solution for system optimization (see Table 3). Furthermore, we can see that the NE solution in Game [c] is most efficient among these three cases. It implies that the efficiency can be possibly improved by appropriate selection of parameters.

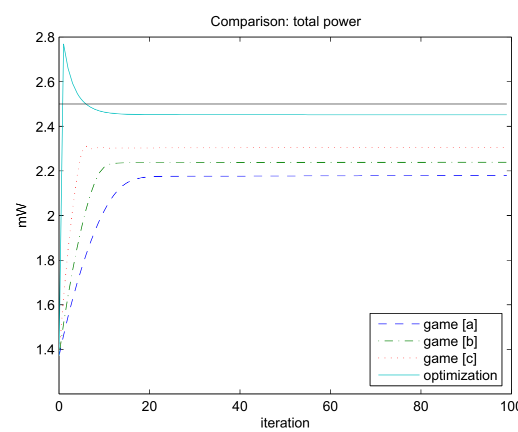

Figure 8: Comparison: total power

Fig. 8 shows the total power vs iteration. Channel power and channel OSNR vs channel number are shown Fig. 9 and Fig. 10, respectively. The constraints (link capacity constraint and channel OSNR target) are satisfied in all cases. The total power in Game [c] approaches P^0 more than others. Moreover, we can tell from Fig. 10 that among the three cases, channel final OSNR values in Game [c] approach the

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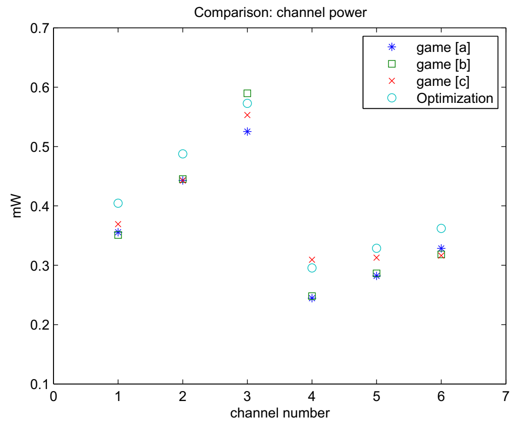

Figure 9: Comparison: channel power

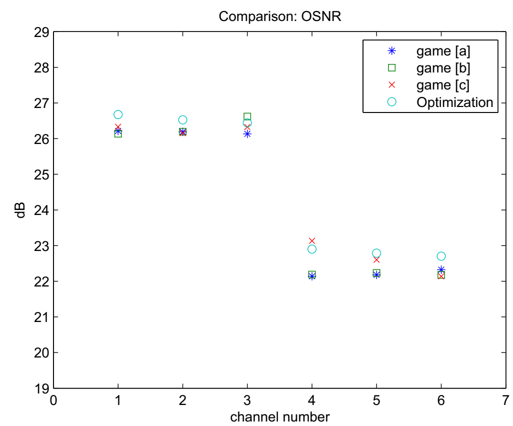

Figure 10: Comparison: channel OSNR

optimal solution of $OPT(\widetilde{\Omega}, C)$ most.

We recall that the parameters β_i in the Nash game are upper-bounded by the condition (13) in Theorem 1, or in other words, the ratio of β_i to β_{min} is upper bounded. The condition restricts each channel asking unilaterally for a much higher OSNR target than others. This phenomenon is also reflected in the selections of β_i in three cases. We take Game [b] for an example. In this case, $\beta_3 = 9.4$ which is greatly larger than other β_i, indicating that channel 3 asks for a highest OSNR level. Meanwhile, channel 4 has the smallest $\beta_4 = 3.9$. Thus the largest ratio of β_i to β_{min} in Game [b] is around 2.41, which is also the largest ratio among the three cases (2.23, 2.41, 1.81, respectively). Recall that Game [b] also has the largest system cost value $C(u^*)$, 4.6208. This phenomenon somewhat implies that the relative deviation of β_i from the average may cause the degradation of system performance and less efficiency of Nash equilibrium.

5. CONCLUSION

In this paper we have compared by simulation the two approaches, i.e., a system optimization approach and a user optimization (game-theoretic) approach. We first show that the aggregate cost function in the game-theoretic formulation is not automatically strictly convex and the optimal solution of the associated constrained optimization problem is not immediate. Then we build the relation between these two formulations by indicating that the individual cost function $C_i(u_i)$ in the system optimization formulation has an approximate interpretation with the cost function $J_i(u)$ in the game-theoretic formulation. After that we compare the results based on this. It is well known that the Nash equilibria of a game may not achieve full efficiency. The comparison results have verified the inefficiency. We also show that OSNR target can be achieved and the efficiency can be possibly improved by appropriate selection of parameters.

In this paper, we select the system optimization formulation as the system framework used to measure the efficiency of the NE solution. A different system framework can be developed to study the efficiency problem. Questions are still remaining for applying Monte Carlo method numerically and an extension of this work theoretically.

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