**EAI Endorsed Transactions**
**on Cognitive Communications**
Research Article EAI.EU

# Dual-feature spectrum sensing exploiting eigenvalue and eigenvector of the sampled covariance matrix

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## Abstract

The signal can be charactered by both eigenvalues and eigenvectors of covariance matrix. However, the existing detection methods only exploit the eigenvalue or eigenvector. In this paper, we utilize the both eigenvalues and eigenvectors of the sampled covariance matrix to perform spectrum sensing for improving the detection performance. The features of eigenvalues and eigenvectors are considered integratedly and the relationship between the false-alarm probability and the decision threshold is offered. To testify this method, some simulations are carried out. The results demonstrate that the method shows some advantages in the detection performance over the conventional method only adapting eigenvalues or eigenvectors.

Received on 27 March 2018; accepted 29 March 2018; published on 05 May 2018

**Keywords:** dual-feature, spectrum sensing, cognitive radio,eigenvalue and eigenvector



doi:10.4108/eai.11-5-2018.154702

## 1. Introduction

At present, wireless spectrum resources are assigned statically, and the given spectrum bands are authorized to some communication systems. Simultaneously, other systems cannot occupy these spectrum bands. For the limited wireless resources, the static style of working results in a precipitous decline in available spectrum resources. However, some of these licensed frequency bands are underutilized[1–3]. To solve the unbalanced utilization of spectrum resources, cognitive radio is presented. Many institutions and scholars put more attentions on key issues of cognitive radio [4–7], such as spectrum sensing, spectrum sharing.

Nowadays, conventional spectrum sensing methods are composed of matched filter detection, energy detection, likelihood ratio test detection and cyclostationary detection [8–11]. When the signal is corrupted by the Gaussian white noise, matched filter detection has optimal detection performance. Unfortunately, some prior information about licensed users must be known. More difficultly, precise synchronization between licensed users and secondary users must also be achieved. It is mostly impossible for secondary users to acquire these prior knowledge. Energy detection is most commonly used in practice due to its simplicity and low computational complexity. No additional prior information about licensed users are required for energy detection. However, energy detection suffers from noise uncertainty and SNR wall, which degrade the detection probability and the false-alarm probability severely. Likelihood ratio test based detection method has optimal detection performance under the Neyman-Pearson criterion. This method detect the signal by virtue of the difference of probability density function between the licensed user and noise. Obviously, the algorithm requires the corresponding prior knowledge. For cyclostationary detection method, the inherent features of the licensed user, which arise from modulation style, signal rate or other parameters, are exploited. This method can overcome the effect of fading and shadow and has the better performance than other detection methods in the low SNR region. But, the computational complexity is unaffordable for the practical application at most cases.

From the previous analysis, we can see that the mentioned-above methods have advantages and disadvantages. So the detection method based on random matrix was first introduced by Cardoso to deal with these problems. This method can cope with

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the noise uncertainty of energy detection and require no prior information about the licensed user. So this method attract more attentions. In [12], the ratio of the maximum eigenvalue and the minimum eigenvalue is adapted as the test statistic to perform spectrum sensing, the corresponding threshold are calculated according to MP law of random matrix. But the requirement of too many samples is unsatisfactory. Aiming to solve this problem, the authors in [13] exploit the Tracy-Widom distribution of maximum eigenvalue to modify the threshold and propose the Maximum Minimum Eigenvalue (MME) method. In the light of these work, many modifications were made to improve the sensing performance.

In fact, it is well known that eigenvectors also contain related information about the presence of signal. Compared to the method exploiting the eigenvalue, only a few works about the utilization of eigenvector are reported. In terms of principle component analysis, eigenvector-based feature template matching (FTM) method was proposed [14]. This method utilizes the eigenvector of sampled covariance matrix as the test statistic to carry out spectrum sensing. Additionally, literature [15] combine kernel function of machine learning with FTM to propose the kernel feature template matching method. It is pointed that feature template matching (FTM) method is a special case of kernel feature template matching method. The authors of the literature also derived the false-alarm probability and decision threshold and analyzed the factors of affecting the detection performance.

Stimulated by the eigenvalue-based method and FTM algorithm, we combine the eigenvalue and eigenvector to carry out spectrum sensing. In the porposed method, the product of main eigenvector and the ratio of maximum and minimum eigenvalue is utilized as the test statistic. In terms of the concept of random matrix, the false alarm probability is derived by approximating some results. Finally, the decision threshold is calculated.

## 2. System model and algorithm description

We assume that there are K sensing nodes. For the ith node, the binary hypothesis test can be expressed as

$$y_i(n) = \begin{cases} \omega_i(n), & H_0 \\ h_i(n)x_i(n) + \omega_i(n), & H_1 \end{cases} \tag{1}$$

Where $\omega_i(n)$, $x_i(n)$ and $h_i(n)$ denote the additive noise, the licensed user signal and channel gain respectively. Two different segments signals are exploited to construct the received signal matrix $\mathbf{Y}_1$ and $\mathbf{Y}_2$

$$\mathbf{Y}_1 = \begin{bmatrix} y_1(1)\, y_1(2) \ldots y_1(N) \\ y_2(1)\, y_2(2) \ldots y_2(N) \\ \vdots \quad \ldots \quad \vdots \\ y_K(1)\, y_K(2) \ldots y_K(N) \end{bmatrix} \tag{2}$$

$$\mathbf{Y}_2 = \begin{bmatrix} y_1(N+1)\, y_1(N+2) \ldots y_1(2N) \\ y_2(N+1)\, y_2(N+2) \ldots y_2(2N) \\ \vdots \quad \ldots \quad \vdots \\ y_K(N+1)\, y_K(N+2) \ldots y_K(2N) \end{bmatrix} \tag{3}$$

Where $N$ is the number of signal samples. Their corresponding covariance matrix $\mathbf{R}_1$ and $\mathbf{R}_2$ are expressed as

$$\mathbf{R}_1 = \frac{1}{N}\mathbf{Y}_1{\mathbf{Y}_1}^T \tag{4}$$

$$\mathbf{R}_2 = \frac{1}{N}\mathbf{Y}_2{\mathbf{Y}_2}^T \tag{5}$$

If the only the noise exists, the maximum eigenvalue $\lambda_{\max}$ and minimum eigenvalue $\lambda_{\min}$ of the sampled covariance matrix has the following relation.

$$\lambda_{\max} = \lambda_{\min} = \sigma^2 \tag{6}$$

But, when the signal is present, the maximum eigenvalue and minimum eigenvalue differs

$$\lambda_{\max} > \lambda_{\min} = \sigma^2 \tag{7}$$

Therefore, by combining (6) and (7), we have

$$\left(\frac{\lambda_{\max}}{\lambda_{\min}}\right)_{H_0} < \left(\frac{\lambda_{\max}}{\lambda_{\min}}\right)_{H_1} \tag{8}$$

Analogously, for the main eigenvector of $H_0$ and $H_1$ case, the following relation holds

$$(|\langle \mathbf{a}_1, \mathbf{b}_1 \rangle|)_{H_0} < (|\langle \mathbf{a}_1, \mathbf{b}_1 \rangle|)_{H_1} \tag{9}$$

where $\mathbf{a}_1$ and $\mathbf{b}_1$ are the corresponding main eigenvectors of the sampled covariance matrix $\mathbf{R}_1$ and $\mathbf{R}_2$. Jointly considering (8) and (9), we can obtain

$$\left(\frac{\lambda_{\max}}{\lambda_{\min}}\right)_{H_0} (|\langle \mathbf{a}_1, \mathbf{b}_1 \rangle|)_{H_0} < \left(\frac{\lambda_{\max}}{\lambda_{\min}}\right)_{H_1} (|\langle \mathbf{a}_1, \mathbf{b}_1 \rangle|)_{H_1} \tag{10}$$

We can observe that the product of the ratio of the maximum eigenvalue and minimum eigenvalue and the correlation of main eigenvectors for the $H_0$ and $H_1$ case differ obviously. Compared to the method only employing eigenvalue or eigenvector, more obvious difference can be obtained for the product. Thus, we select the product as the test statistic of spectrum sensing. The proposed method is summarized as follows.

1)The received signal is sampled to get two different signal matrix $\mathbf{Y}_1$ and $\mathbf{Y}_2$, And then the corresponding sampled covariance matrices $\mathbf{R}_1$ and $\mathbf{R}_2$ are calculated.

2)The covariance matrices $\mathbf{R}_1$ and $\mathbf{R}_2$ are decomposed to obtain main eigenvectors $\mathbf{a}_1$ and $\mathbf{b}_1$ , the maximum eigenvalue $\lambda_{\max 1}$ and $\lambda_{\max 2}$, the minimum eigenvalue $\lambda_{\min 1}$ and $\lambda_{\min 2}$ .

3)The product is calculated as follows

$$T = \rho_1 \rho_2 \left|\langle \mathbf{a}_1, \mathbf{b}_1 \rangle\right| \tag{11}$$

where $\rho_1 = \frac{\lambda_{\max 1}}{\lambda_{\min 1}}$ and $\rho_2 = \frac{\lambda_{\max 2}}{\lambda_{\min 2}}$.

4) Based on the preset threshold $\gamma$, the final decision is made to perform spectrum sensing. If $T > \gamma$ , the signal exists, otherwise, only the noise is present.

## 3. Solving the false-alarm probability and the decision threshold

From the definition of false-alarm probability, we have

$$\begin{aligned} P_{fa} &= P(D_1|H_0) \\ &= P(T > \gamma|H_0) \\ &= P\left(\rho_1 \rho_2 \left|\langle \mathbf{a}_1, \mathbf{b}_1 \rangle\right| > \gamma|H_0\right) \end{aligned} \tag{12}$$

When only the noise exists, the covariance matrix of the received signal is Wishart matrix. From the MP law, the maximum eigenvalue and minimum eigenvalue in the $H_0$ case are

$$\lambda_{\max} = \sigma^2 \left(1 + \sqrt{c}\right)^2 \tag{13}$$

$$\lambda_{\min} = \sigma^2 \left(1 - \sqrt{c}\right)^2 \tag{14}$$

where $c$ is the ratio of the number of nodes and the number of samples. The ratio of the maximum eigenvalue and the minimum eigenvalue is

$$\eta = \frac{\lambda_{\max}}{\lambda_{\min}} = \frac{\left(1 + \sqrt{c}\right)^2}{\left(1 - \sqrt{c}\right)^2} \tag{15}$$

For the matrix $\mathbf{R}_1$ and $\mathbf{R}_2$, we can calculate the corresponding $\eta_1$ and $\eta_2$.

It is pointed by the literature [16] that the eigenvalue and eigenvector of Wishart matrix are independent to each other. Let $\gamma = \varepsilon\eta^2$ . Coupled with (15), the false-alarm probability can be expressed in the form

$$\begin{aligned} P_{fa} &= P\left(\rho_1 \rho_2 \left|\langle \mathbf{a}_1, \mathbf{b}_1 \rangle\right| > \gamma|H_0\right) \\ &= P\left(\rho_1 \rho_2 \left|\langle \mathbf{a}_1, \mathbf{b}_1 \rangle\right| > \varepsilon\eta_1\eta_2|H_0\right) \\ &\approx P\left(\left|\langle \mathbf{a}_1, \mathbf{b}_1 \rangle\right| > \varepsilon|H_0\right) \end{aligned} \tag{16}$$

To calculate the false-alarm probability, we derive the probability density function of $\left|\langle \mathbf{a}_1, \mathbf{b}_1 \rangle\right|$. In terms of the concept of eigenvalue decomposition, we can get

$$\mathbf{R}_1 = \mathbf{A}\mathbf{\Lambda}_1\mathbf{A}^T \tag{17}$$

$$\mathbf{R}_2 = \mathbf{B}\mathbf{\Lambda}_2\mathbf{B}^T \tag{18}$$

where $\mathbf{\Lambda}_1$ and $\mathbf{\Lambda}_2$ are the diagonal matrix containing eigenvalues of $\mathbf{R}_1$ and $\mathbf{R}_2$. The columun vectors of and correspond to eigenvectors. And the main eigenvectors $\mathbf{a}_1$ and $\mathbf{b}_1$ of the sampled covariance matrix $\mathbf{R}_1$ and $\mathbf{R}_2$ are the first column of matrix $\mathbf{A}$ and $\mathbf{B}$. Because $\mathbf{A}$ and $\mathbf{B}$ are unitary matrix, we have

$$f\left(\mathbf{A}^T\mathbf{B}\right) = f\left(\mathbf{B}\right) \tag{19}$$

We can say that the elements of $\mathbf{A}^T\mathbf{B}$ and $\mathbf{B}$ in the same position follow the same distribution

$$f\left(\langle \mathbf{a}_1, \mathbf{b}_1 \rangle\right) = f\left(b_{11}\right) \tag{20}$$

From the properties of unitary matrix, $b_{11}$ obeys the Beta distribution with parameters $\alpha = \frac{1}{2}$ and $\beta = \frac{N-1}{2}$

$$f\left(x\right) = \frac{\left(1 - x^2\right)^{(n-1)/2-1}}{B\left(\frac{1}{2}, \frac{N-1}{2}\right)} \tag{21}$$

Because the distribution are computationally complex, we approximate it with the Gaussian distribution for the large $N$ [15]. According to the result derived in literature [15], $f\left(x\right)$ can be replaced by $h\left(x\right)$

$$h\left(x\right) = \frac{\sqrt{N}}{\sqrt{2\pi}} e^{-\frac{Nx^2}{2}} \tag{22}$$

Substituting (22) into (16) yields to

$$\begin{aligned} P_{fa} &\approx P\left(\left|\langle \mathbf{a}_1, \mathbf{b}_1 \rangle\right| > \varepsilon|H_0\right) \\ &= P\left(\langle \mathbf{a}_1, \mathbf{b}_1 \rangle > \varepsilon, \langle \mathbf{a}_1, \mathbf{b}_1 \rangle < -\varepsilon|H_0\right) \\ &= 2P\left(\langle \mathbf{a}_1, \mathbf{b}_1 \rangle > \varepsilon|H_0\right) \\ &= 2\int_{\varepsilon}^{\infty} \frac{\sqrt{N}}{\sqrt{2\pi}} e^{-\frac{Nx^2}{2}} dx \\ &= 2Q\left(\sqrt{N}\varepsilon\right) \end{aligned} \tag{23}$$

When the false-alarm probability is preset, we get the parameter $\varepsilon$

$$\varepsilon = Q^{-1}\left(\frac{P_{fa}}{2}\right)\frac{1}{\sqrt{N}} \tag{24}$$

So, the corresponding threshold can be calculated as

$$\gamma = \varepsilon\eta^2 = \frac{\left(1 + \sqrt{c}\right)^4}{\left(1 - \sqrt{c}\right)^4} Q^{-1}\left(\frac{P_{fa}}{2}\right)\frac{1}{\sqrt{N}} \tag{25}$$

## 4. Numerical simulation and analysis

We first carry out simulations to testify the correctness of approximated probability density function in (22). Fig.1 plots the $f\left(x\right)$ and $h\left(x\right)$ when $N$=20,40 and 80. We observe from Fig.1 that the difference between $f\left(x\right)$ and $h\left(x\right)$ becomes gradually small with the increasing of $N$. When $N$=80, no obvious difference is found. Simulation results prove the approximation of probability density function

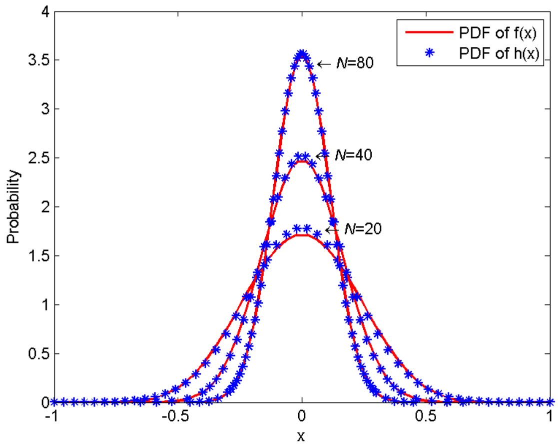


**Figure 1.** Comparison of $f(x)$ and $h(x)$

In the following, we compare the estimated probability density function and theoretical result for $\langle \mathbf{a}_1, \mathbf{b}_1 \rangle$. The simulated parameters are as follows. The number of Monte Carlo simulation is 2000, $K$=10, $N$=500. Fig.2 demonstrates the results. We can see that the estimated probability density function fits perfectly the theoretical result.

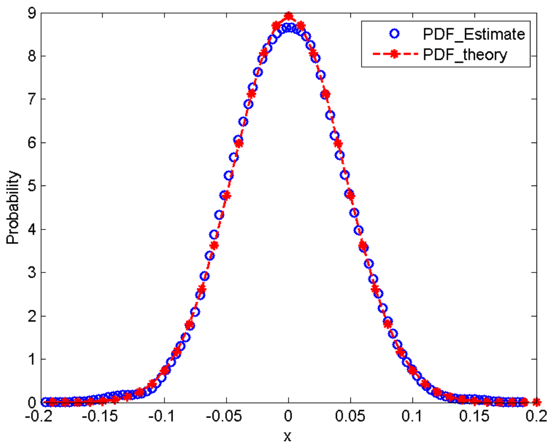


**Figure 2.** The estimated pdf and theoretical result of the main eigenvector for $H_0$

Next, we verify the rationality of the decision threshold. We offer the relationship among the test statistic of $H_0$ case, test statistic of H1case and the decision threshold in Fig.3 when $K$=10čňand $SNR$=-10dB. It is demonstrated that the threshold varies dynamically with the number of samples. Additionally, we can also observe that the difference of the test statistic for H0 and H1 case is obvious for all the samples and the decision threshold is between the test statistics for $H_0$ and $H_1$ case. All these results guarantee the proper threshold for obtaining the satisfactory performance.

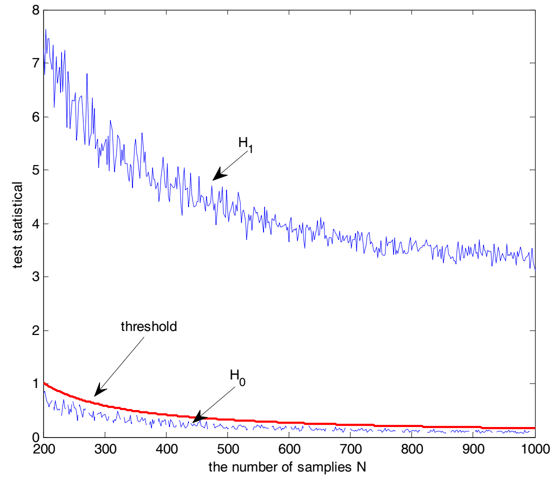


**Figure 3.** The estimated pdf and theoretical result of the main eigenvector for $H_0$

We now verify the detection performance of the proposed method. The simulated parameters are set as follows. $K$=10čň$N$=200ąć500ąć1000. The number of Monte Carlo simulation is 2000. Fig.4 plots the detection probability for the different SNR. We can draw three conclusions. 1) The detection probability is improved with the increasing of samples for the low $SNR$ region. For example, the detection probability is 0.5 and 1 for $N$=200 and 1000 when $SNR$=-15dB. 2) When SNR is more than -10dB, the detection probability remains a constant 1 for the different samples. 3) when the samples remains unchanged, the detection probability increases proportionately with $SNR$.

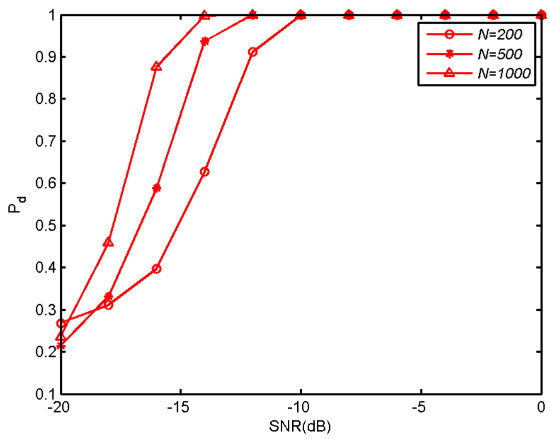


**Figure 4.** the detection probability for the different samples and SNR

To testify the superiority of the proposed method over other algorithms, we show the detection probability of FFM, MME and the proposed method in Fig.5. The simulated parameters are set as follows. $K$=10, $N$=500.

The number of Monte Carlo simulation is 2000. It is noted that the samples are divided into two segments for FTM and the proposed method, and each segment has $N/2$ samples. We can observe that the proposed method has good detection performance over other two algorithms.

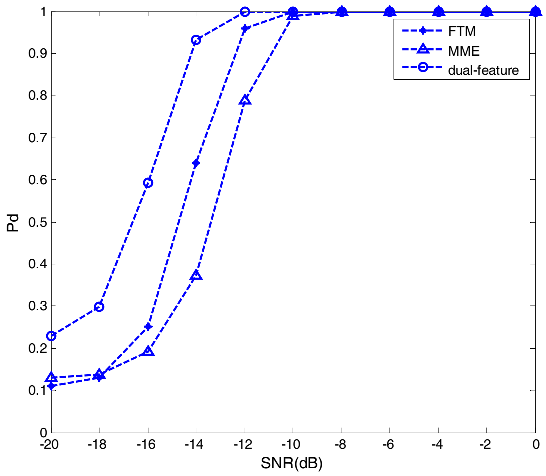


**Figure 5.** the detection probability for the different samples and SNR

## 5. Conclusion

Inspired by the FTM and eigenvalue-based spectrum sensing, we introduced the spectrum sensing algorithm based on dual-feature consisting of eigenvalues and eigenvector. We defined the product of the ratio of the maximum eigenvalue and the minimum eigenvalue and main eigenvector as the test statistic. This test statistic consider the eigenvalue and eigenvector comprehensively. Based on the test statistic, we derived the false-alarm probability and offer the closed form of threshold under Newman Pearson criterion. In simulations, we testified the correctness of Gaussian approximation of probability density function and compared the proposed method with FTM and MME algorithms. The simulation results showed that the proposed method outperforms the FTM and MME algorithms with lower computational complexity.

**Acknowledgement.** This work is supported by National Natural Science Foundation of China(NSFC)(Grant No.61671176)

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