Optimal Monotone Forwarding Policies in Delay Tolerant Mobile Ad-Hoc Networks

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ABSTRACT

We study in this paper fluid approximations for a class of monotone relay policies in delay tolerant ad-hoc networks. This class includes the epidemic routing and the two-hops routing protocols. We enhance the relay policies with a probabilistic forwarding feature where a message is forwarded to a relay with some probability p. We formulate an optimal control problem where a tradeoff between delay and energy consumption is captured and optimized. We compute both the optimal static value of p as well as the optimal time dependent value of p. We show that the time dependent problem is optimized by threshold type policies and we compute explicitly the value of the optimal threshold for some special cases of relay policies.

Categories and Subject Descriptors

C.2 [**C.2.1**]: Wireless communication

General Terms

Performance, algorithms, design

Keywords

Delay tolerant networks, fluid models, optimal control

1. INTRODUCTION

In delay tolerant mobile ad-hoc networks, instantaneous connectivity is not needed any more and messages can arrive at their destination thanks to the mobility of some subset of nodes that carry copies of the message. A naive approach to forward a message to the destination is by epidemic routing in which any mobile that has the message keeps on relaying it to any other mobile that arrives within its transmission range and who does not have the message yet. This would minimize the delivery delay at a cost of inefficient use of network resources (in terms of memory used in the relaying mobiles and in terms of the energy used for flooding the network).

The need for more efficient use of network resources motivated the use of more economic forwarding such as the two hop routing protocols in which the source transmits copies of its message to all mobiles it encounters, but where the latter relay the message only if they come in contact with the destination. Furthermore, timers have been proposed to be associated with messages when stored at relay mobiles, so that after some threshold (possibly random) the message is discarded. The performance of the two hop forwarding protocol along with the effect of the timers have been evaluated in [1], the framework of which allows for optimization of the choice of the average timer duration.

In this paper we analyze two alternative approaches for optimizing forwarding protocols. The first approach consists of forwarding a message to another relay with some probability p, where p can be optimized to meet some tradeoff between delay and resource utilization. The second optimization approach we introduce is based on further allowing this probability p to vary in time. These two approaches are studied in this paper in conjunction with a wide class of monotone relaying schemes of which the epidemic routing and the two-hops routing are special cases (a precise definition of monotone relaying policies is postponed until Section 3).

In order to optimize p in the context of a general monotone relaying policy, we first introduce fluid approximations of the system dynamics. We then use tools from optimization and optimal control theory to come up with optimal static and optimal dynamic choices for the parameter p. In the dynamic case, we establish the optimality of threshold type policies that use $p = 1$ up to some time t and then switch to the smallest possible value of p. We validate the fluid approximation through simulations, and compare it with the

Inter-Perf 2008 Athens, October 24, 2008.
Copyright 2008 ICST ISBN 978-963-9799-31-8.

Digital Object Identifier: 10.4108/ICST.VALUETOOLS2008.4430
http://dx.doi.org/10.4108/ICST.VALUETOOLS2008.4430

original discrete model. We illustrate through extensive numerical experimentations the benefits of our optimization approaches.

For the special cases of epidemic routing and two-hops routing, we obtain explicit expressions for the performance measures for both the optimal static as well as the optimal dynamic optimization problems. We compute in particular the optimal probability of successful delivery of the message by some time t for these forwarding schemes under the constraint that the energy consumption till time t is bounded by some constant $\mathcal{E}$. We show that when restricted to static policies, for the same t and $\mathcal{E}$, the optimal probability of successful delivery in the two-hops routing may be either larger than the one in the epidemic routing or smaller, depending on the values of the parameters. We then show that the same conclusion holds for the comparison between optimal dynamic policies. We then propose a new hybrid policy that performs better than the optimal dynamic 2-hops routing and better than the optimal epidemic routing policy.

The structure of the paper is as follows. The next section presents related work, followed by a section that introduces the model. We then solve in Section 4 the static control problems and establish the structure of optimal dynamic control policies. We then specialize in Section 5 to the two-hops routing and the epidemic routing, and compute the optimal policies in these two cases. The performances of these policies in the original discrete setting are then provided in Section 6. Section 7 provides a validation of the fluid model through simulations. It further provides an extensive numerical investigation that illustrates the usefulness and power of our optimal control approach. Section 8 provides some results for the case of timers and Section 9 concludes the paper.

2. RELATED WORKS

Delay Tolerant Networks (DTNs) have recently attracted attention from the research community. The literature reports several results of real experiments on DTNs [2, 3, 4]. In [2], the DieselNet network was deployed over a wide urban area, using buses as mobiles. Also, authors of [3] describe the use of human mobility to diffuse information through portable devices. In the above mentioned works, authors discuss several technical problems; in particular, due to lack of persistent connectivity, one central issue of DTNs is routing, and applicable techniques depend on the knowledge on input variables such as contact times, traffic demands or memory occupation [5].

In particular, in the case of routing with zero knowledge [6, 7], the problem is to deliver messages to destinations with high probability despite nonexistence of any *a priori* information on the encounter pattern of mobile devices.

To this respect, due to the simplicity of implementation and robustness, one of the first proposed forwarding algorithms has been epidemic routing [8, 9], also named controlled flooding [10]. Another proposed routing mechanism is the two-hops algorithm. Grossglauser and Tse proposed the two-hops routing algorithm in [11], and the main goal there was to characterize the capacity of mobile ad-hoc networks. The aim was to indicate a direction to overcome the severe limitations in static networks capacity [12]. The standard reference work for the analysis of the two-hops relaying protocol is [13]. Fluid approximations and infection spreading models are used in [14].

One *leitmotiv* of message diffusion algorithms is how to trade off message delay for energy consumption, i.e. number of copies per delivered message. In this respect epidemic routing and two-hops stand at opposite ends: epidemic routing has high delivery probability while flooding the system, whereas two-hops routing spreads messages at a much smaller pace, but the price paid is a lower delivery probability. Other variants include the use of TTL counters, probabilistic forwarding [10] and K-limited forwarding [1, 16].

The control of forwarding has been addressed in the ad-hoc networks literature, e.g in [17] and [18]. In [17], the authors describe an epidemic forwarding protocol based on the *susceptible-infected-removed* (SIR) model [14]. Authors of [17] show that it is possible to increase the message delivery probability by tuning the parameters of the underlying SIR model. In [18] a detailed general framework is proposed in order to capture the relative performances of different self-limiting strategies.

Finally, authors of [19] argue that even in sparse DTNs, finite bandwidth, scheduling and interference affect the performances of epidemic routing; in this respect, the performance figures of this work should be referred to as upper bounds.

Novel contributions

As indicated before, we consider in this paper sparse mobile ad-hoc networks. In these networks, mobility is the engine that permits network-wide information diffusion. With respect to the existing literature, this work makes several original contributions. Typically, the objective is to maximize the fraction of delivered messages and to minimize the latency between source and destination. In the paper we provide a formulation rooted in optimization, which relates explicitly the energy expenditure, i.e. the number of copies, with the delivery probability within a given deadline.

From an algorithmic standpoint, the fundamental result derived in this work, using tools from optimal control theory, is that *optimal forwarding policies are threshold-type policies* (see Definition 4.1). It then follows that static control policies, i.e. probabilistic forwarding with constant probability, are suboptimal.

We remark that the above mentioned results apply to all monotone forwarding strategies, which include several interesting cases for sparse ad-hoc networks, such as epidemic routing and two-hops routing.

The practical implications of this result are apparent, since the implementation of threshold policies is rather straightforward.

3. THE MODEL

Consider a network that contains N mobile nodes. The time between contacts of any two nodes is assumed to be exponentially distributed with parameter λ. The validity of this model has been discussed in [13], and its accuracy has been shown for a number of mobility models (Random Walker, Random Direction, Random Waypoint).

We assume that the message that is transmitted is relevant for some time τ. We do not assume any feedback that allows the source or other mobiles to know whether the message has made it successfully to the destination within the time τ.

A mobile terminal is assumed to have a message to send to a destination node. We focus in this paper on a class of

Digital Object Identifier: 10.4108/ICST.VALUETOOLS2008.4430
http://dx.doi.org/10.4108/ICST.VALUETOOLS2008.4430

so called *monotone relay strategies*. A relay strategy is said to belong to this class if the following holds:

- The number of nodes that contain the message does not decrease in time during the time τ,
- The number $\widetilde{X}(t)$ of nodes, not including the destination, that contain the message at time t is a Markov chain.

Example 1: Epidemic Routing.
At each encounter between a mobile that has the message and another one that does not, the message is relayed to the one that does not have it. This is a monotone relay strategy.

Example 2: Two-Hops Routing.
At each encounter between the source and a mobile that does not have the message, the message is relayed to that mobile. If a mobile that is not the source has the message and it is in contact with another mobile then it transfers the message if and only if the other mobile is the destination node. This is a monotone relay strategy.

Example 3: Adding timers.
Consider either the epidemic routing or the two-hops routing. In order to avoid saturation of the buffers of relay mobiles, a relay mobile that receives a message activates a Time To Live (TTL) timer which is exponentially distributed with parameter μ. $\widetilde{X}(t)$ is indeed a Markov chain but it is not necessarily monotone non-decreasing. Thus strategies that integrate TTL timers need not be monotone relay strategies. We shall explain the implication of that in Remark 3.1.

3.1 The control

Let $\{T_n\}$ be the sequence of instants where an encounter occurs between two mobiles. Only at these times the state $\widetilde{X}$ may change.

The actual system dynamics is obtained by combining the relay strategy with the control. Assume that at a given time T_n, one of the mobile nodes has a copy of the message and some other one does not, and under the relay strategy the message is transmitted to the other mobile. Then T_n is called a forwarding opportunity.

A natural way to optimize the system (with respect to some objectives that will be introduced shortly) is to control the forwarding probabilities of messages. We assume that each message contains a time stamp that shows when it was generated (so that it can be deleted at all nodes that have it when it becomes irrelevant, τ time units later). We shall consider two optimization approaches:

- *Static Approach:* Each time there is a forwarding opportunity, forwarding of the message is done with a constant probability c.
- *Dynamic Approach:* Each time a mobile has a forwarding opportunity, it checks the time t that elapsed since the message generating time and it forwards the message with some probability $u(t)$.

In both cases we shall assume that the forwarding probabilities can take any value within an interval $[u_{\min}, 1]$, where $u_{\min} > 0$.

3.2 Fluid Approximations

Uncontrolled Dynamics Let $\overline{X}(t)$ be the fraction of the mobile nodes that have at time t a copy of the message. (It includes the source node and thus $\overline{X}(0) \geq 1/N$.) We assume that $\overline{X}(t)$ grows at a rate given by the following differential equation:

$$\frac{d\overline{X}(t)}{dt} = f(\overline{X}(t))$$

where f is assumed to be strictly positive.

Controlled Dynamics Let $X(t)$ be the fraction of the mobile nodes that have at time t a copy of the message. We assume that $X(t)$ grows at a rate given by the following differential equation:

$$\frac{dX(t)}{dt} = u(t)f(X(t)) \tag{1}$$

where $u(t) \in [u_{\min}, 1]$, $u_{\min} > 0$. u has the meaning of a control that is used to slow down the growth rate of the number of copies of the message in the network. We will sometimes denote the solution to (1) by $X(t; u)$ to emphasize its dependence on u.

$X(t)$ coincides with $\overline{X}(t)$ when $u(t) = 1$; $\overline{X}(t)$ is the system's state in the case of no control. In the special case where $u(t)$ is a constant, $u(t) = c$, the solution of (1) satisfies:

$$X(t) = \overline{X}(ct). \tag{2}$$

$X(t)$ has thus the same dynamics as $\overline{X}(t)$ but at a slowed time-scale.

Next, we write the fluid approximation for the probability distribution of the delay T_d, denoted by $D(t) := P(T_d < t)$. Based on [20, Appendix A], we have

$$D(t) = 1 - (1 - D(0)) \exp\Big(- N\lambda \int_{s=0}^{t} X(s)ds\Big), \tag{3}$$

where $D(0) = z$ accounts for the probability that the destination is not among the nodes that possess the message at time 0. The controlled version reported in (3) derives from the differential equation in the form

$$\begin{aligned} \frac{d}{dt}D(t) &= -\lim_{h\to 0} \frac{\mathbb{P}[T_d > t+h] - \mathbb{P}[T_d > t]}{h} \\ &= \lim_{h\to 0} \frac{\mathbb{P}[T_d > t] - \mathbb{P}[T_d > t+h]}{h} \\ &= N\lambda\, X(t)\,[1 - D(t)]\,, \end{aligned} \tag{4}$$

which is separable and integrates as

$$\int_{D(0)}^{D(t)} \frac{dD}{1-D} = N\lambda \int_0^t X(s)ds, \tag{5}$$

from which (3) follows directly.

Denote by $\mathcal{E}(t)$ the energy consumed by the whole network for transmission during the time $[0, t]$. It is proportional to $X(t)-X(0)$ since we assume that the message is transmitted only to mobiles that do not have the message, and thus the number of transmissions of the message during $[0, t]$ plus the number of mobiles that had it at time zero equals to the number of mobiles that have it. We thus have $\mathcal{E}(t) = \varepsilon(X(t) - X(0))$

REMARK 3.1. *If TTL timers are added as discussed in Section 3, or more generally, when the nodes that have a copy of the message may lose it and may thus later receive*

Digital Object Identifier: 10.4108/ICST.VALUETOOLS2008.4430
http://dx.doi.org/10.4108/ICST.VALUETOOLS2008.4430

it again, the energy spent for transmission till time t can be larger than $\varepsilon(X(t) - X(0))$. Thus a constrained optimization problem where we place a constraint on the transmission energy does not translate anymore into a constraint on the final state. Such cases will therefore be introduced later in Section 8.

4. OPTIMAL CONTROL FOR THE FLUID MODEL

Our goal is to maximize $D(\tau)$. In view of (3), this is equivalent to maximizing $\int_0^\tau X(r;u)dr$.

On the other hand, we also would like to keep $\mathcal{E}(\tau)$ small.

Define $\sigma(x,z) := \overline{X}^{-1}(x+z)$ given $\overline{X}(0) = z$, which is the time elapsed until x extra nodes (in addition to the initial z ones) receive the message in the uncontrolled system. Notice that $\sigma(x,z)$ is a function of both x and z; in the following, for the sake of notation, unless misleading, we will refer simply to $\sigma(z)$.

Further define

$$J(z,u) = \int_0^\tau X(r;u)dr$$

for an initial state z, where $X(r;u)$ is the state trajectory under a control u (dependence on u will be suppressed in most of the development below).

4.1 Optimal static control

Theorem 4.1. *Consider the problem of maximizing $D(\tau)$ subject to a constraint on the energy $\mathcal{E}(\tau) \leq \varepsilon x$.*
(i) If $\overline{X}(\tau) \leq x+z$ (or equivalently, $\tau \leq \sigma(z)$), then a control policy u is optimal if and only if $u(t) = 1$ for $t \in [0,\tau]$ a.e.
(ii) If $\overline{X}(u_{\min}\tau) > z + x$ (or equivalently, $u_{\min}\tau > \sigma(z)$), then there is no feasible control strategy.
(iii) If $\overline{X}(\tau) > z + x > \overline{X}(u_{\min}\tau)$ (or equivalently, $\tau > \sigma(z) > u_{\min}\tau$), then the best static control policy $u_c(t) = c$ is given by the constant

$$c = \frac{\sigma(z)}{\tau}$$

and the optimal value among static policies is given by

$$J_c^*(z) = \frac{\tau}{\sigma(z)} \int_0^{\sigma(z)} \overline{X}(r)dr$$

Proof. Parts (i) and (ii) are obvious, and part (iii) follows directly from (2). ◇

4.2 Optimal dynamic control

Definition 4.1. *A policy u is called a threshold policy with parameter h if $u(t) = 1$ for $t \leq h$ a.e. and $u(t) = u_{\min}$ for $t > h$ a.e.*

Theorem 4.2. *Consider the problem of maximizing $D(\tau)$ subject to a constraint on the energy $\mathcal{E}(\tau) \leq \varepsilon x$.*
(i) If $\overline{X}(\tau) \leq x + z$ (or equivalently, $\tau \leq \sigma(z)$), then a control policy u is optimal if and only if $u(t) = 1$ for $t \in [0,\tau]$ a.e.
(ii) If $\overline{X}(u_{\min}\tau) > z + x$ (or equivalently, $u_{\min}\tau > \sigma(z)$), then there is no feasible control strategy.
(iii) If $\overline{X}(\tau) > z + x > \overline{X}(u_{\min}\tau)$ (or equivalently, $\tau > \sigma(z) > u_{\min}\tau$), then an optimal policy is necessarily a threshold one.

Proof. Parts (i) and (ii) are obvious. We thus proceed with proof of part (iii), working under the assumption $\overline{X}(\tau) > z + x > \overline{X}(u_{\min}\tau)$. We use the Maximum Principle [15]. The Hamiltonian is

$$H(X,u;p) = X - p\,u\,f(X)$$

where p is the co-state variable, which is a continuous, piecewise continuously differentiable function of t. If u is an optimal solution for the original problem, then it maximizes the Hamiltonian, and with the latter being linear in the former, the optimal control takes the two extreme values $u_{\min}$ and 1, depending on whether the product of p and f, $p(t)f(X)$, is positive or negative. But since $f(X)$ is positive for all values of X of interest, we arrive at the simple optimality condition:

$$u(t) = \begin{cases} u_{\min} & \text{if } p(t) > 0 \\ 1 & \text{if } p(t) < 0 \end{cases} \tag{6}$$

Furthermore, the co-state $p(\cdot)$ is obtained as the solution of the adjoint differential equation

$$\frac{dp(t)}{dt} = H_X = 1 - p(t)u(t)f_X(X(t)) \tag{7}$$

where subscript X denotes differentiation with respect to X.

We first make the assumption that $f_X(X) < 0$ for all X in the interval $[z, z+x]$. This is in fact a property that holds for most of the models of interest. Now note also that since by the condition of this case (iii) neither $u = u_{\min}$ nor $u = 1$ can be optimal policies, we either have the static constant policy derived earlier to be optimal or have a switching policy. The former can immediately be ruled out because p cannot be zero except at isolated points (note that from the co-state equation, if $p(t) = 0$ for some t, then $p(t^+) > 0$, and from the optimality condition (6) u immediately takes one of the extreme values, in this case $u_{\min}$). Hence, optimal u has to take on its extreme values, and since it cannot take on only one of them for the entire interval, as argued earlier, it has to switch between the two. Clearly, which of the two values it takes for a particular t depends solely on the sign of $p(t)$, as dictated by (6). We now consider separately the two possible choices for $p(0)$, positive and negative: If $p(0) > 0$, then it follows from (7) and from the negativity of f_X that $p(t)$ is increasing and hence remains positive. Hence by (6), $u(t) = u_{\min}$ for all t, and this is in contradiction with the hypothesis of case (iii). Thus we have to have $p(0) < 0$, and initially $u = 1$. But we know that this cannot be sustained during the entire interval because it violates the hypothesis of case (iii). Then, at some point u has to switch to $u_{\min}$. But for this switch to take place, we have to have p positive, and once it is positive it remains positive for the remaining time, and hence optimum u does not switch back. Thus there is a threshold h such that $p(t) < 0$ and $u(t) = 1$ for $t < h$ and $p(t) > 0$ and $u(t) = u_{\min}$ for $t > h$.

Now we assume that $f_X(X) > 0$ for all X in the interval $[z, z+x]$. An analysis similar to the one in the previous paragraph leads to the conclusion that it is not possible for p to be positive at $t = 0$, since then it always remains positive (here the argument is somewhat more detailed than the one above, since it is possible for p to decrease initially, but when it hits zero, it immediately starts increasing at a rate 1, and hence never goes negative), and hence $u(t) = u_{\min}$ for all t, a contradiction. If $p(0) < 0$, then initially $u = 1$, and p grows and at some point hits zero and turns positive

Digital Object Identifier: 10.4108/ICST.VALUETOOLS2008.4430
http://dx.doi.org/10.4108/ICST.VALUETOOLS2008.4430

since the time-derivative of p at zero is positive (which is 1), and hence u switches to its lower value. We know from the previous argument that p cannot become negative after this point, and hence we again have a switching policy with only one switch.

The case when f_X is not sign-definite can be handled along the same lines of the two cases above. We thus conclude that an optimal dynamic policy has to be a threshold one. ⋄

The way to implement a threshold policy is quite simple. When a mobile has a forwarding opportunity, it checks the time-stamp on the message which indicates its generation time. It then computes the t that has elapsed since the generation time. If t is smaller than the threshold then the message is forwarded, otherwise it is transmitted with probability $u_{\min}$.

4.3 Time change and the optimal threshold

Let

$$ds = u(t)dt, \quad s(0) = 0$$

Then

$$\frac{d\overline{X}(s(t))}{dt} = \frac{d\overline{X}(s)}{ds} \times \frac{ds(t)}{dt} = f(\overline{X}(t))\,u(t)$$

We thus conclude that

$$X(t) = \overline{X}(s(t)).$$

Thus the controlled state evolves as a slower version of the uncontrolled one, and the control u can be interpreted as the slowing factor.

Remark 4.1. *In the special case that a static policy $u(t) = p$ is used then we have $s(t) = pt$ so that $X(t) = \overline{X}(pt)$.*

Theorem 4.3. *If $\overline{X}(\tau) > z + x > \overline{X}(u_{\min}\tau)$ (or equivalently, $\tau > \sigma(z) > u_{\min}\tau$), then the threshold value h^* of the optimal policy is given by*

$$h^* = \frac{\sigma(z) - \tau u_{\min}}{1 - u_{\min}}. \tag{8}$$

The optimal value is given by

$$J^*(z) = \int_0^{h^*} \overline{X}(r)dr + \frac{1}{u_{\min}} \int_{h^*}^{\sigma(z)} \overline{X}(r)dr$$

Proof. Consider any control policy u satisfying $X(\tau) \le x + z$. Then, notice that

$$\sigma(z) \ge s(\tau) = \int_0^\tau u(r)dr$$

where equality holds iff $X(\tau) = x+z = \overline{X}(s(\tau)) = \overline{X}(\sigma(z))$.

Since u is positive, s is invertible and we let $t = \eta(s)$. Hence, it follows

$$J(u) = \int_0^\tau X(t)dt = \int_0^\tau \overline{X}(s(t))dt = \int_0^{s(\tau)} \overline{X}(r)d\eta(r)$$

$$\le \int_0^\sigma \overline{X}(r)d\eta(r) = \int_0^\sigma \frac{\overline{X}(r)}{u(\eta(r))}dr$$

where we used the fact that $\eta'(s) = \frac{1}{u(\eta(s))}$: equality is obtained if and only if $X(\tau) = x + z$.

Among the threshold control policies, the optimal one is thus the one for which h satisfies the constraint $X(\tau) = x+z$, and hence for which h satisfies

$$\sigma(z) = h \cdot 1 + (\tau - h) \cdot u_{\min}$$

which yields (8). The expression for $J^*(z)$ follows accordingly. ⋄

Let u^* be a threshold policy with parameter h^*. The difference between the value function under the optimal dynamic policy and the optimal static one is given by

$$J^*(z) - J_c^*(z) = -\left(\frac{\tau}{\sigma(z)} - 1\right) \int_0^{h^*} \overline{X}(r)dr$$

$$+ \left(\frac{1}{u_{\min}} - \frac{\tau}{\sigma(z)}\right) \int_{h^*}^{\sigma(z)} \overline{X}(r)dr$$

Since the static policies are a subset of the dynamic policies, the latter is positive.

Remark 4.2. *So far we have formulated the control problem as a constrained maximization one: Maximize $D(\tau)$, (or equivalently maximize $\int_0^\tau X_s ds$) subject to a constraint on the energy $\mathcal{E}(\tau) \le \varepsilon x$. We could also study the unconstrained version, i.e. the problem where we do not consider the energy constraint any more but where we add instead a term to the objective function related to the energy:*

$$\text{Maximize} \quad \int_0^\tau \mathrm{X_s ds} + \zeta\mathcal{E} \tag{9}$$

where $\zeta \le 0$ determines the tradeoff between the transfer probability and the energy. This maximization problem can be interpreted as the Lagrangian relaxation of the original constrained problem, where ζ is the Lagrange multiplier.

We note that in the two-hops policy, $X(t)$ is affine in $u(t)$, and thus both constraints as well as the objective are convex. Assume that the Slater condition holds. We can use Karush-Kuhn-Tucker Theorem to conclude that there exists a ζ such that an optimal policy for the optimization of the Lagrangian is also optimal for the constrained problem. This implies that the unconstrained formulation also admits optimal dynamic policies with a threshold structure.

5. SOME EXAMPLES

5.1 Epidemic routing

Here f is given by

$$f(x) = \rho x(1 - x),$$

where $\rho = N\lambda$. The solution of $\frac{dx}{dt} = f(x(t))$ is

$$\overline{X}(t) = \frac{1}{1 + \left(\frac{1}{z} - 1\right)e^{-\rho t}}$$

We have

$$\sigma(z) = -\frac{1}{\rho}\log\left(\frac{z(1-x-z)}{(x+z)(1-z)}\right)$$

We note that

$$\frac{d^2\overline{X}(t)}{dt^2} = \frac{c\rho^2\exp(-\rho t)(c\exp(-\rho t) - 1)}{(1 + c\exp(-\rho t))^3}.$$

where $c = 1/z - 1$.

Digital Object Identifier: 10.4108/ICST.VALUETOOLS2008.4430
http://dx.doi.org/10.4108/ICST.VALUETOOLS2008.4430

$\frac{d^2\overline{X}(t)}{dt^2}$ is seen to be positive for $t \le S_0$ and negative for $t \ge S_0$ where $S_0 = \log(c)/\rho$. Hence $\overline{X}(t)$ has a sigmoid form: it is convex for $t \le S_0$ and concave for $t \ge S_0$.

We also note that

$$\frac{d\overline{X}(t)}{dt}\bigg|_{t=0}^{(epid-routing)} = z(1-z)\rho \qquad (10)$$

Using Remark 4.1 we have for a static policy $u(t) = p$:

$$X(t) = \frac{1}{1 + \left(\frac{1}{z} - 1\right) e^{-\rho p t}}$$

5.2 Two-hops routing

Here$f$ is given by

$$f(x) = \lambda(1-x)$$

Then

$$\overline{X}(t) = 1 + (z-1)e^{-\lambda t}$$

which is a concave function of t.

Note that

$$\frac{d\overline{X}(t)}{dt}\bigg|_{t=0}^{(2-hops)} = (1-z)\lambda = z(1-z)\rho/Nz$$

$$= \frac{1}{Nz}\,\frac{d\overline{X}(t)}{dt}\bigg|_{t=0}^{(epid-routing)} \qquad (11)$$

Since $z = \overline{X}(0) \ge 1/N$ we get

$$\frac{d\overline{X}(t)}{dt}\bigg|_{t=0}^{(2-hops)} \le \frac{d\overline{X}(t)}{dt}\bigg|_{t=0}^{(epid-routing)},$$

with equality for $z = 1/N$.

We have

$$\sigma(z) = -\frac{1}{\lambda}\log\left(\frac{1-x-z}{1-z}\right)$$

and further

$$\overline{J}(z) = \int_0^{\sigma(z)} \overline{X}(r)dr = \frac{-1}{\lambda}\left[\log\left(\frac{1-x-z}{1-z}\right) + x\right]$$

Optimizing over all static policies

For a static policy $u_c(t) = c$ we have

$$J(z, u_c) = \frac{\overline{J}(z)}{c}$$

which is decreasing in c. The optimal value is given for $c = \frac{\sigma(z)}{\tau}$ which gives

$$J_x^*(z) := \sup_c J(z, u_c) = \tau\left[1 + \frac{x}{\log\left(\frac{1-x-z}{1-z}\right)}\right]$$

5.3 Comparison

The uncontrolled epidemic routing maximizes the number of nodes that possess the message at any time and thus maximizes the delivery probability at any time. Yet alternative policies have been proposed in order to reduce the number of "infected" nodes as the propagation and storing of copies requires resources that might be quite precious (such as energy or memory). We now have several alternatives to reduce the infection rate: we could either keep using epidemic routing but control the probability of forwarding a message to another node, or we could switch to the two-hops (controlled or uncontrolled). Which one is better?

We shall first compare the optimal static epidemic routing with the optimal static two-hops routing. We shall then compare the optimal dynamic epidemic routing to the optimal dynamic two-hops routing.

THEOREM 5.1. *Consider $z = 1/N$. Let $u(t) = p$ be the optimal static policy under the epidemic routing. Assume that $\tau \le S_0/p$ where*

$$S_0 = \frac{\log(N-1)}{N\lambda}.$$

Then the optimal static policy under the two-hops routing has larger probability of success delivery by time τ than the optimal static policy under the epidemic routing.

Proof. It suffices to show that $\int_0^\tau X(s)ds$ is larger under the optimal static two-hops policy, But this is a direct consequence of the fact that

- $\overline{X}(t)$ is convex in t for $t \le S_0$, under the epidemic routing uncontrolled policy.
- Thus $X(t)$ is convex in t for $t \le S_0/p$ under the epidemic routing policy with $u(t) = p$.
- $\overline{X}(t)$ is concave in t for all $t > 0$ under the two-hops policy, and hence also $X(t)$ under any static policy.
- $X(\tau)$ corresponding to the epidemic routing and to the two-hops routing coincide at time 0 and time $t = \tau$.

$\diamond$

In the case of optimal policies, the comparison is a bit more involved. But, we can relate the dynamics of optimal policies for the epidemic routing and the two-hops routing. First, observe that in general

$$\sigma_t(z) = \sigma(z)|^{(two-hops)} \ge \sigma(z)|^{(epid-routing)} = \sigma_e(z).$$

THEOREM 5.2. *Let $\tau > \sigma_t(z) \ge \sigma_e(z) > u_{\min}\tau$, then there exist a threshold $0 < T^* \le \tau$ such that for the dynamics of the optimal dynamic policies it holds*

$$\begin{cases} X(t)|^{(epid-routing)} \ge X(t)|^{(two-hops)} & 0 < t \le T^* \\ X(t)|^{(epid-routing)} \le X(t)|^{(two-hops)} & T^* < t \le \tau \end{cases}$$

where equality holds at $t = \tau$ and $t = T^*$ only.

Proof. Let $0 < h^* < \tau$ be the optimal threshold for the epidemic routing. It holds

$$X(h^*)|^{(epid-routing)} > X(h^*)|^{(two-hops)}$$

Also, by construction of optimal policies it holds

$$X(\tau)|^{(epid-routing)} = X(\tau)|^{(two-hops)}$$

Notice that $X(t)$ is concave for the two-hops routing and it is convex in $[h^*, \tau]$ for the epidemic routing: since they coincide in τ, the two dynamics can intersect in at most one point in (h^*, τ).

If they do not intersect in (h^*, τ), then the statement is clearly true since $T^* = \tau$. Otherwise, if $0 < T^* < \tau$, consider that

$$X(T^*)|^{(epid-routing)} = X(T^*)|^{(two-hops)}$$

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The inequality for $h^* < t < T^*$ is obvious. Also, let

$$y(t) = X(T^*) + (t - T^*)\frac{X(\tau) - X(T^*)}{\tau - T^*}$$

From the convexity properties of the two dynamics

$$X(T^*)|^{(epid-routing)} < y(t) < X(T^*)|^{(two-hops)}$$

for $T^* < t < \tau$. ◇

Notice that in the case $u_{\min} = 0$ it holds indeed $T^* = \tau$.

5.4 New Hybrid Policy

We saw that the optimal dynamic two-hops policy may perform better or worse than the optimal dynamic epidemic routing policy. We now propose a hybrid policy that always does better than both.

DEFINITION 5.1. *Define a hybrid policy with parameter T^* to be one that first forwards according to the optimal epidemic routing up to time T^* and from T^* up to time τ according to the optimal two-hops forwarding.*

From the implementation standpoint this is rather immediate, since from T^* on, the required operation is to inhibit relays to forward to nodes which are not the destination. Such a hybrid policy is monotone, and, from Theorem 5.2, the integral $\int_0^\tau X(s)ds$ under such a policy (with T^* chosen as in that Theorem) is either equal or larger than that under the optimal dynamic two-hops and the optimal dynamic epidemics routing policies. The general theory developed in the previous sections guarantees that the delivery probability is thus larger.

In order to construct such a policy explicitly, the value T^* is needed. We show how to compute it in closed form in a specific case; the general case is more involved and it is not detailed here. Let $\tau = \sigma_t(z) \geq \sigma_e(z) > u_{\min}\tau$: this is the case when the optimal policy under the two-hops routing is the uncontrolled one.

In order to simplify the notation, let $\zeta = X(\tau)$ and $\Gamma = (1/\zeta - 1)$. We use the equation $f(x) = u_{min}\rho x(1-x)$ for the epidemic routing when $h^* < t \leq \tau$. It follows

$$X(T^*)|^{(epid-routing)} = \frac{1}{1 + \Gamma\, e^{u_{\min}\rho(\tau - T^*)}}$$

In the same way, for the two-hops routing we obtain

$$X(T^*)|^{(two-hops)} = 1 - (1 - \zeta)\, e^{\lambda(\tau - T^*)}$$

Equating the two expressions we obtain the equation

$$H(\theta) = 0 \quad \text{where} \quad H(\theta) = (1 - \theta) - \zeta\,(1 + \theta^{N u_{\min}}) \quad (12)$$

and $\theta = e^{-\lambda(\tau - T^*)}$, which can be solved numerically in order to obtain T^*.

Note that $H(\theta)$ is monotone decreasing in θ on the $\theta \in [0, 1]$, $H(0) = 1 - \zeta > 0$ and $H(1) = -2\zeta < 0$. Hence there is a unique solution on that interval that can be computed using bisection. (Any solution for $\theta > 1$ is not of interest since it corresponds to $T^* > \tau$.)

5.5 Numerical examples

To illustrate a situation where the optimal static two-hops forwarding policy does better than the epidemic routing we consider $N = 200$ nodes with initially only the source node infected (i.e. $z = 1/200$). We set $\rho = 1$ and as time horizon we take $\tau = 26.467$ sec. The energy constraint is taken so as to correspond $X(\tau) = 0.128$. The parameters were chosen such that (i) among the static two-hops routing policies, the uncontrolled one is optimal (ii) the optimal static epidemic routing policy is obtained with $p = 0.123$. With these parameters, $S_0 = \log(c)/\rho = \log(199) = 5.700$.

Figure 1a) shows the fraction of nodes that have a copy of the message as a function of the time. It contains two curves: one is concave as a function of time, which corresponds to the fraction of infected nodes under the two-hops routing, and the other one is convex as a function of the time and corresponds to the fraction of infected nodes under the epidemic routing as a function of time. The convexity of the latter curve follows from the fact that the choice $\tau < S_0$ implies that we operate at the convex part of the sigmoid shape of the epidemic routing. We see in the figure that indeed, the surface below the curve of the epidemic routing (restricting to $t \leq \tau$) is smaller than the curve of the two-hops routing, which confirms Theorem 1a).

At a coarser time scale, the epidemic routing curve is sigmoid, as already mentioned, which is seen in Figure 1b) (this Figure corresponds to the non-controlled case). The breaking point $S_0 = 5.30$ in which the curve starts becoming concave is well seen in that figure.

Finally, Figure 1c) and 1d) report on two cases where Theorem 5.2 applies.

Figure 1c) in particular depicts the case when $0 < h^* < \tau$, $T^* = \tau$ and $\sigma_t(z) \geq \tau \geq \sigma_e(z) > u_{\min}\tau$: this is the degenerate case when the optimal policy under the two-hops routing is the uncontrolled one and the optimal solution is seen to be the optimal dynamic policy under epidemic routing.

Figure 1d) depicts the case when both thresholds are smaller than τ, obtained for the energy constraint corresponding to $X(\tau) = 0.064$: as it can be seen there, the switching time $T^* = 4.645$ occurs at the intersection of the optimal dynamics. In particular, we can recognize easily the dynamics of the hybrid policy, which would be the envelope of the two curves; apparently, the area of the envelope of such an hybrid policy is larger so that the deliver probability is higher than the two optimal policies. In particular, in the depicted case, it holds $D(\tau) = 0.652$ for the epidemic routing and $D(\tau) = 0.734$ for the two-hops routing. The hybrid policy attains $D(\tau) = 0.74$.

6. THE DISCRETE MODELING

We consider now the original discrete setting. We define X_n to be the number of mobiles among N which have a copy of the message at time $n\Delta$. For simplicity we consider discrete time where a time slot is of duration Δ. We assume that a mobile that receives a copy during a time slot can forward it starting from the following time slot.

Let $\xi_n^{(i)}$ be the indicator that the ith mobile among the $N - X_n$ mobiles that do not have the message at time $n\Delta$, receives the message during $(n\Delta, (n+1)\Delta]$. Then we have

$$X_{n+1} = X_n + \sum_{i=1}^{N-X_n} \xi_n^{(i)}$$

The forwarding probability during $(n\Delta, (n+1)\Delta]$ is assumed to be constant and is denoted by u_n. Denote by q_n the probability that a mobile does not receive the message in time slot n.

As we had in the case of fluid approximations, we would like to minimize the expected energy used till some time n,

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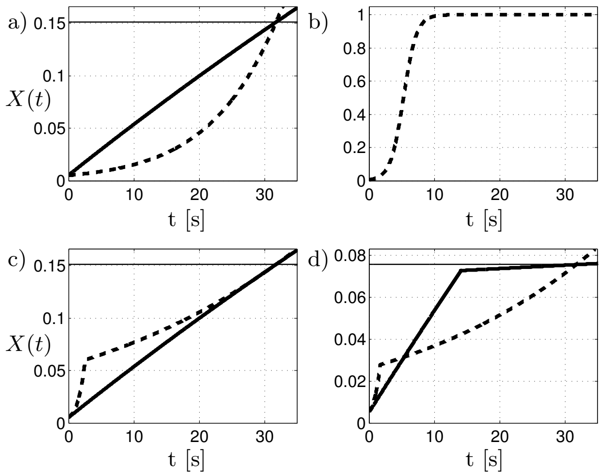

Figure 1: a) Fraction of infected nodes as function of time, $N = 200$ b) Evolution of the fraction of infected nodes in the uncontrolled epidemic routing c) Comparison of the optimal forwarding policies, degenerate case for the two-hops policy d) Comparison of the optimal forwarding policies. Dashed lines reports on the epidemic routing, continuous lines report on the two-hops routing; horizontal lines refer to the target $x + z$.

which is proportional to $E[X_n]$, and on the other hand maximize $D(n\Delta)$. We shall provide below an explicit expression for $E[X_n]$ and a simple recursive way to compute $D(n\Delta)$.

We shall assume in the sequel that q_n does not depend on X_n. This is the case in the two hop routing where we have

$$q_n = \exp(-\lambda u_n \Delta)$$

We then have:

$$E[X_{n+1}] = E[X_n] + (N - E[X_n])(1 - q_n) = N(1 - q_n) + E[X_n]q_n$$

Hence

$$E[X_n] = x_0 \prod_{i=0}^{n-1} q_i + N \sum_{j=1}^{n-1} \prod_{i=j}^{n-1} q_i$$

Define $G_n^*(Z) := E[Z^{X_n}]$ the probability generating function (PGF) of X_n and let $X^*(s) := E[\exp(-sX_n)]$ be the Laplace Stieltjes Transform of X_n.

$X_n^*(s)$ is useful since it follows from (3) that

$$D(n\Delta) = 1 - (1 - D(0)) \sum_{i=1}^{n} X_n^*(\lambda) \qquad (13)$$

We have the relation $X_n^*(s) = G(\exp(-s))$.

Define

$$\gamma_n(Z) := E[Z^{\xi_n^{(1)}}] = q_n + (1 - q_n)Z$$

Then

$$\begin{aligned} G_{n+1}^*(Z) &= E\left[Z^{\left(X_n + \sum_{i=1}^{N-X_n} \xi_n^{(i)}\right)}\right] \\ &= E\left[E\left\{ Z^{\left(X_n + \sum_{i=1}^{N-X_n} \xi_n^{(i)}\right)} \middle| X_n \right\}\right] \\ &= E\left[Z^{X_n} \gamma_n(Z)^{(N-X_n)}\right] \\ &= \gamma_n(Z)^N G_n^*\left(\frac{Z}{\gamma_n(Z)}\right) \end{aligned}$$

We thus have a recursive formula to compute $G_n^*(Z)$.

7. NUMERICAL RESULTS

This section complements the previous sections with both simulations and with numerical studies. The objective of the simulations is to check the validity of our modeling and solution approaches: we validate the basic model, study the fluid approximation and get an insight on when it performs well. We then perform a numerical study and compute optimal policies based on the theory that we had developed in previous sections in order to get insight on the value of static and of dynamic control in DTNs.

The simulation set up is as follows. First, we generated several contact traces based on different mobility patterns, which collect sequences of pairs of nodes coming into radio range, and the time when such contacts occurred (specific settings are reported below). In particular, we used two synthetic mobility models, mobility traces which were generated using Omnet++ according to the Random Waypoint (RWP) mobility model [21], and traces of random contacts occurring according to a marked Poisson process with i.i.d. marks (poissonian traces). Then, we considered uniform i.i.d. pairs of source-destination nodes, and simulated the (un)controlled forwarding process based on the contact patterns derived from the mobility traces, using Matlab.

7.1 Uncontrolled forwarding ($u(t) = 1$)

First, we conducted some experiments aimed at validating the delay formula (3) in the case of uncontrolled forwarding. The first set of numerical examples reported below are referred to the case where pairs of nodes meet with frequency $\lambda = 1 \times 10^{-5} s^{-1}$; values are averaged over 10^4 samples (confidence intervals were calculated within 95% accuracy).

In both cases of two-hops routing and of epidemic routing, as depicted in Fig 2, the fluid model follows quite accurately the dynamics of the fraction of infected nodes.

As a further step, we tested the validity of the fluid approximation when changing the number of nodes in the system. In particular, as in Fig 3, in the case of the epidemic routing, and for the settings considered here, the fraction of infected nodes is approximated accurate by the fluid model as soon as N is on the order of 30 of nodes.

We further validated the model by collecting the statistics for the message delay, namely $D(t)$. As reported in Fig 4, in particular, the experimental CDF shows a tight match with the theoretical prediction. Notice that, according to eq. (3), the good fit of the delay CDF confirms the match for the dynamics of $X(t)$.

From these preliminary results we can already characterize some typical features of the two representative routing techniques considered in this paper, namely two-hops routing and epidemic routing. In fact, if we compare Fig 2 and

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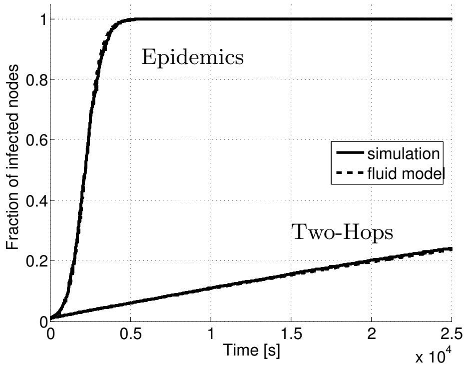

Figure 2: Fraction of infected nodes, uncontrolled case, $N = 200$.

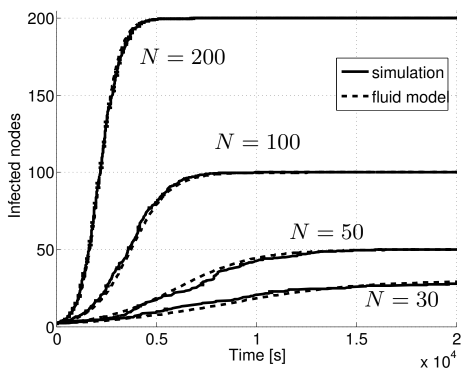

Figure 3: Fraction of infected nodes, Epidemic Routing, uncontrolled case, various number of nodes $N = 30, 50, 100, 200$.

Fig 4, we notice that in the case of the two-hops relaying protocol, it takes around four times longer than epidemic routing in order to deliver a message with high probability. Of course this was expected, since epidemic routing minimizes the delay at the cost of message overhead. Conversely, the price paid by epidemic routing for such a gain is apparent, since the corresponding fraction of infected nodes, i.e. the energy expenditure, in order to achieve 5 times larger than in the case of two-hops relaying.

Finally, we repeated the experiments on the dynamics of infected nodes and delay using the Omnet++ generated RWP mobility traces. In this setting there are $N = 200$ nodes in the system, which move at $v = 5$ m/s in a square area with side length $L = 5000$ m and have $R = 15$ m radio range. The initial distribution of nodes is drawn from the stationary distribution according to the RWP mobility model [22] in order to avoid transitory effects. Notice that the equivalent value of λ for this case coincides with the value used to generate poissonian contact traces, according to [13]. As a general remark, the numerical results were derived under the conditions $R \ll L$, and moderate speed; this choice of the parameters is aimed to mimic conditions for the network to be *sparse*, which is in line with a DTN scenario.

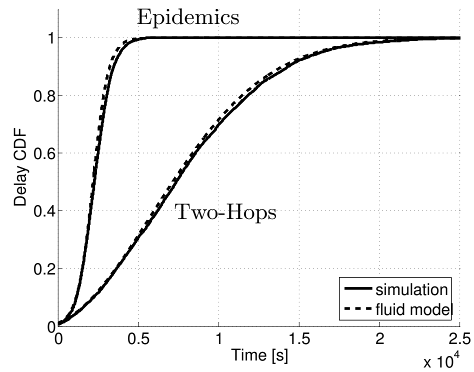

Figure 4: Delay CDF, uncontrolled case, $N = 200$.

As it can be appreciated in Fig. 5 the dynamics of the number of nodes is captured by the model, and even in the case of Fig. 6 the fit of the CDF shows a very tight match with the theoretical prediction.

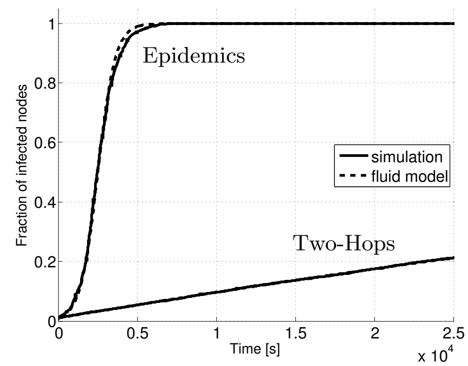

Figure 5: Fraction of infected nodes, uncontrolled case, RWP mobility, $N = 200$, $L = 5000$ m, $v = 5$ m/s.

Also, as seen in Fig. 7, the fluid approximation starts applying when the number of nodes is on the order of some tenth of nodes (30 with the given settings), as already verified in the previous experiments.

In the following we verify the performances of the control policies described in previous sections; the following measurements were obtained using poissonian traces.

7.2 Static control policies ($u(t) = c$)

After verifying the match of the model with the experimental data, we simulated static control policies under the same settings described before.

In particular, under the two-hops routing policy, we tested two reference values of the maximum permitted delay, $\tau = 10000$ and $\tau = 18000$, respectively. We then considered an increasing normalized energy constraint and, in particular, the corresponding optimal static control is reported in Fig 8. The reference performance figure tested is the outage probability, i.e., $1 - D(\tau)$. As depicted in the figure, the change of slope of the control corresponds to the point beyond which it is optimal to forward messages with unit probability. We

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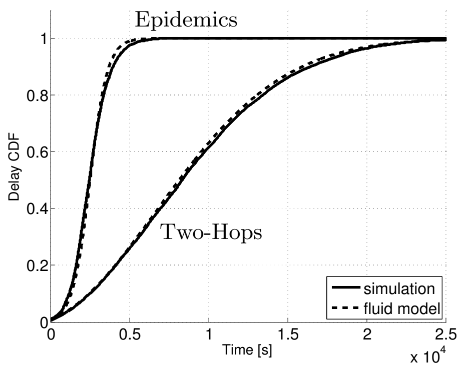

Figure 6: Delay CDF, uncontrolled case, RWP mobility, $N = 200$, $L = 5000$ m, $v = 5$ m/s.

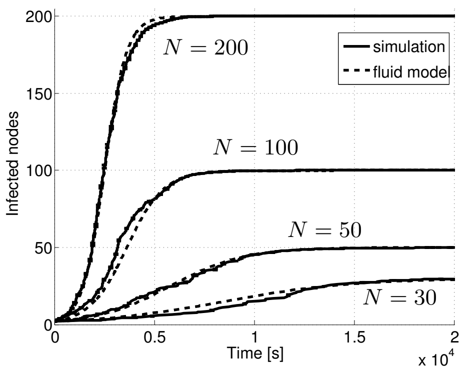

Figure 7: Fraction of infected nodes, Epidemic Routing, uncontrolled case, RWP mobility, $L = 5000$ m, $v = 5$ m/s, various number of nodes $N = 20, 50, 100, 200$.

also remark that, for the settings considered in this series of experiments, the optimal static control increases almost linearly in the energy constraint.

But, as reported in Fig 9, while the forwarding probability increases linearly, the outage probability decreases exponentially towards the minimum allowed outage probability, i.e., the value corresponding to uncontrolled forwarding. This suggests that a moderate increase in the forwarding probability has a sensible impact in the delivery probability.

7.3 Dynamic control policies

In the next set of simulations, we verified the performances of the optimal dynamic policies at the increase of the normalized energy constraint, under the same simulation settings discussed before. We arbitrarily chose $u_{\min} = 0.1$; we remark, though, that according to the optimality of the dynamic policies, within the same feasibility region, the outage probability does not change with the choice of $u_{\min}$ (whereas the control actually does).

In the case of the two-hops routing policy, as reported in Fig 10, the optimal threshold h^* increases almost linearly towards the maximum value, i.e., τ, which corresponds to uncontrolled forwarding.

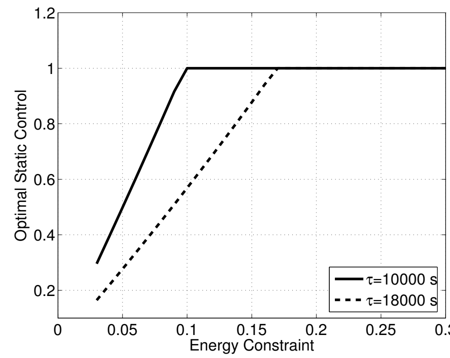

Figure 8: Optimal static control, $\tau = 10000, 18000$ s, two-hops routing.

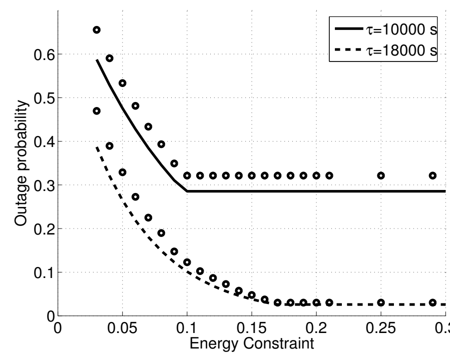

Figure 9: Outage probability at time $\tau = 10000, 18000$ at the increase of the energy constraint, static control case, two-hops routing. Markers superimpose simulation outcomes.

The corresponding behavior of the outage probability is reported in Fig 11. For the sake of clearness, we also reported the behavior of optimal static policies in the same region: the gain obtained by optimal dynamic policies is quite apparent compared to optimal static control, with a decrease of outage probability of 0.1 below $x = 0.1$.

Incidentally, we notice that in the leftmost regions of Fig 10 and Fig 11, the graphs do not cover values of the energy constraint such that $x < \tau u_{\min}$; as discussed before, in fact, below such values of the fraction of infected nodes, there exists no feasible forwarding policy.

In order to complete the performance characterization of optimal policies, we depicted the probability distribution function (PDF) of the number of infected nodes under the condition that the message is received by time τ. Basically, this represents the energy expenditure at reception time when $\varepsilon = 1$. As depicted in Fig 12, when the energy constraint is tight, the probability distribution function is concentrated around the reference value ($x = 0.05$ in the figure), whereas for a loose bound, practically the uncontrolled case, ($x = 0.12$ in the figure) the PDF is smoother. The shape of the PDF is rather insensitive to the value of τ.

Finally, it is interesting to compare the dynamics of the

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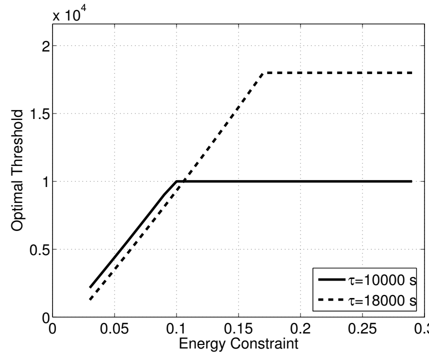

Figure 10: **Optimal threshold, $\tau = 10000, 18000$ s; two-hops routing.**

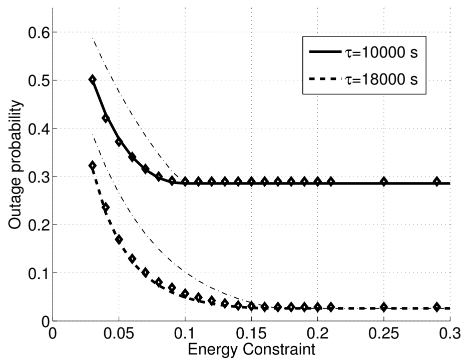

Figure 11: Outage probability at time $\tau = 10000, 18000$, at the increase of the energy constraint; optimal dynamic control, two-hops routing. Markers superimpose simulation outcomes, thin dashed lines report optimal static policies for comparison

fraction of infected nodes as reported in Fig 13 for the case of epidemic forwarding and in Fig 14 for two-hops routing, under different relaying controls. In particular, with respect to epidemic routing, and with $\tau = 2000$ s and $x = 0.14$, $u_{min} = 0.1$: with this choice in the case of the static control $u = 0.68$, whereas in the case of optimal control, $h^* = 1296$ s. Conversely, in the case of two-hops routing, $\tau = 10000$ s, $x = 0.07$, $u_{min} = 0.1$, and in this case static control $u = 0.7$, whereas in the case of optimal control, $h^* = 6675$ s.

It is quite apparent that the static control policy generates a delayed version of the uncontrolled dynamics. In the case of the optimal dynamic control, instead, around the threshold, the fraction of infected nodes starts increasing at a slower pace. We notice that the two controlled dynamics intersect at time τ and the intersect corresponds to $x + z$.

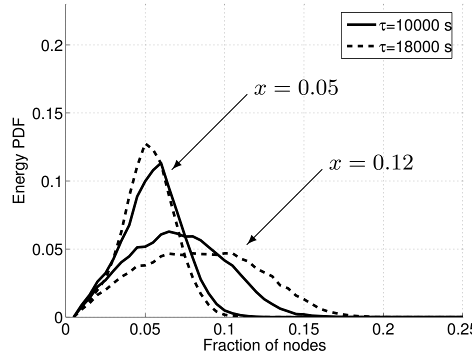

Figure 12: Conditional PDF of energy expenditure at the reception time $\tau = 10000, 18000$; energy constraints are $x = 0.05$ and $x = 0.12$, respectively.

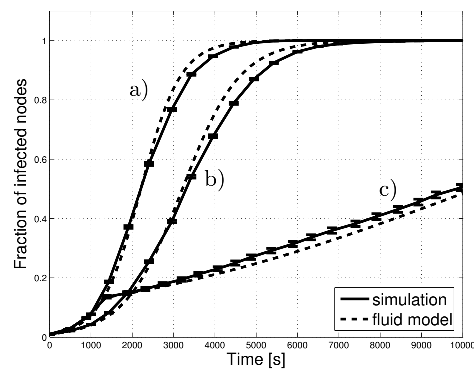

Figure 13: Dynamics of the fraction of infected nodes under uncontrolled a), static b) and optimal c) forwarding policies; Epidemic routing.

7.4 Discrete model

Consider now the discrete model as introduced in Section 6: due to the relative simplicity of the models developed before, we are interested in the limit of validity of the fluid approximation with respect to the discrete system. In this experiment, in particular, we verified numerically whether, in the case of the two-hops routing static policies, the optimal forwarding probability in the discrete case is actually close to the value predicted by static policies. In order to do so we used the optimal static control and we verified the fit of the experimental CDF as given by the fluid model, for different values of the time-slot Δ.

Fig. 15a) depicts the measured value of $D(n\Delta)$ versus increasing values of the forwarding probability; we used the same setting as for the fluid model, with $\tau = n\Delta = 18000$ s and $x + z = 0.11$. In Fig. 15b) we reported on the corresponding expected value of the fraction of infected nodes: as seen there, the intercept corresponds the optimal forwarding probability as predicted by the fluid model. In particular, for the case at hand, the optimal static control predicted by the fluid model is $u = 0.566$. The indication is that the expected number of infected nodes at time τ, as given considering the (deterministic) fluid approximation, matches very

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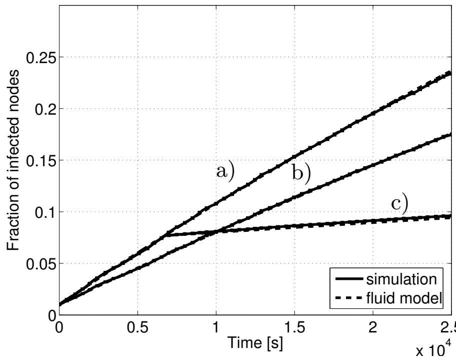

Figure 14: Dynamics of the fraction of infected nodes under uncontrolled a), static b) and optimal c) forwarding policies; two-hops routing.

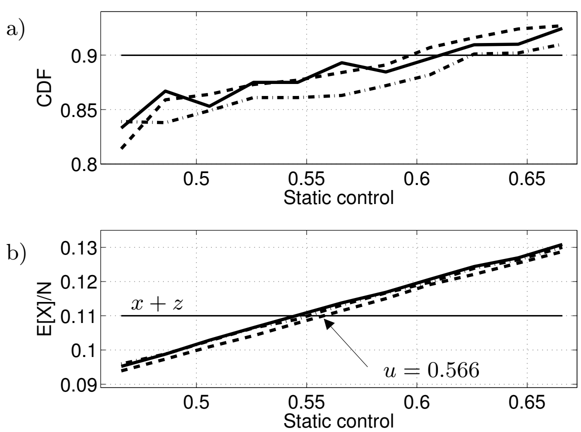

Figure 15: Discrete time model a) Delay CDF, $D(n\Delta)$; b) $E[X_n]/N$; two-hops routing, $N = 200$, $\tau = n\Delta = 18000$. Horizontal lines: a) $D(\tau)$ from the fluid model and b) $x + z$; solid line $\Delta = 0.1$ s, dash-dotted line $\Delta = 1$ s, dashed line $\Delta = 10$ s.

well the average number of nodes experienced in the discrete system. The values found for the CDF are within 5% relative error compared to the value predicted by the fluid model, represented by the horizontal line in Fig. 15a). The prediction is accurate in a range of values of the time-slot $\Delta = 0.1, 1, 10$ s; we then conclude that, within this range, the fluid model, and the policies derived before, represent a valid approximation even for the discrete system.

8. EXTENSIONS TO NON-MONOTONE FORWARDING POLICIES

Consider a monotone policy for which the fluid dynamics is given by (1). Now add to each node a timer; it starts counting whenever the corresponding node receives a copy of the message and expires some exponentially distributed time later. At that time the message is discarded.

The evolution of the state is now modified to

$$\frac{dX(t)}{dt} = u(t)\overline{f}(X(t)) \text{ where } \overline{f}(x) = f(x) - \eta x \qquad (14)$$

and where $u(t) \in [u_{\min}, 1]$, $u_{\min} > 0$.

Example In particular, for the uncontrolled two-hops routing this gives

$$\frac{dX(t)}{dt} = \lambda(1 - X(t)) - \eta X(t)$$

It has a stationary point

$$X^* = \frac{\lambda}{\lambda + \eta}$$

We shall assume throughout that we start initially at $z < x^*$. Then $X(t)$ will converge by monotonically increasing to x^* and will not exceed this value. Thus if we define $y = x/x^*$ to be the normalized fraction of nodes that have a copy of the message, then it indeed satisfies the same constraints $0 \leq y \leq 1$ and $y \geq 0$, and its dynamics is given by

$$\frac{dY(t)}{dt} = u(t)\overline{f}(Y(t))$$

where $\overline{f}(Y(t)) = -(\lambda + \eta)(1 - Y(t))$. Thus the normalized state dynamics in presence of timers is an accelerated version of the one without timers.

THEOREM 8.1. *Consider the two-hops forwarding policy with exponential timers with parameter ζ. Consider the relaxed problem*

$$\text{Maximize} \quad \int_0^\tau \mathrm{X_s ds} + \zeta\mathcal{E} \qquad (15)$$

with $\zeta \leq 0$, and assume that $|\varepsilon\zeta| < 1$. Then it is maximized by the policy that solves the problem

$$\text{Maximize} \quad \int_0^\tau \mathrm{X_s ds}(1 + \epsilon\zeta) + \zeta(\mathrm{X}(\tau) - \mathrm{X}(0)) \qquad (16)$$

and which does not have timers. Hence the optimal policy is of a threshold type.

Proof. The energy is not any more proportional to $X(\tau) - X(0)$, but instead to $X(\tau) - X(0) + R(\tau)$, where $R(\tau)$ is the total number of deletions of the message due to timers timeout. It is given by $R(\tau) = \int_0^\tau X(s)ds$. Thus (15) is equivalent to (16). ◇

9. CONCLUDING COMMENTS

We have studied in this paper the question of how to control efficiently message forwarding in delay tolerant networks. To achieve a desirable tradeoff between a large probability of successful transmission within a given time and the desire to manage well resources (in particular energy), we formulated a constrained optimal control problem based on a fluid model of the system's dynamics. The quantity that we proposed to control was the probability of forwarding a message to another mobile when two mobiles come into each other's transmission range. This control is an additional feature that can be combined with any type of forwarding policy: we have studied in particular its use in conjunction with the two-hops and the epidemic routing policies. We considered both static policies as well as dynamic policies for

Digital Object Identifier: 10.4108/ICST.VALUETOOLS2008.4430
http://dx.doi.org/10.4108/ICST.VALUETOOLS2008.4430

choosing the message. We identified a threshold structure of the optimal dynamic policies and computed the optimal threshold for the two-hops routing as well as the epidemic routing. Our modeling assumptions and the use of the fluid model were validated through simulations.

Explicit expressions for the performance measures for the fluid model have been obtained which enabled us to study the performance of optimal policies and the dependence on system's parameters.

10. ACKNOWLEDGMENTS

This work has been partially supported by the European Commission within the framework of the BIONETS project IST-FET-SAC-FP6-027748, `www.bionets.eu`. Research reported here has also been facilitated by a UIUC-INRIA Collaborative Research Grant jointly from the University of Illinois at Urbana-Champaign and INRIA, France.

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Digital Object Identifier: 10.4108/ICST.VALUETOOLS2008.4430
http://dx.doi.org/10.4108/ICST.VALUETOOLS2008.4430