Sparsity Adaptive Joint Greedy Algorithm for Dual Signal Estimation

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ABSTRACT

This paper presents two novel joint greedy algorithms for signal reconstruction in the case of impulsive noise. The performance of most existing greedy algorithms based on the assumption of Gaussian noise significantly deteriorates when the noise is impulsive noise that has heavy tails. To address the impulsive noise, in this work, it is modeled as α-stable distribution and formulated as a sparse signal in the time domain. Therefore, the noise suppression problem becomes an estimation one. With this reformulation, two methods by simultaneously estimating the signal of interest and noise are developed based on sparsity adaptive matching pursuit (SAMP). The numerical studies demonstrate that the proposed approaches provide excellent performance in both the signal recovery and noise suppression.p

KEYWORDS

joint greedy algorithm, signal reconstruction, dual signal estimation, impulsive noise

ACM Reference format:
Xin Xu, Hongqing Liu, Xiaohua Peng, and Xiaorong Jing. 2017. Sparsity Adaptive Joint Greedy Algorithm for Dual Signal Estimation . In *Proceedings of 10th EAI International Conference on Mobile Multimedia Communications, Chongqing, China, July 2017 (MOBIMEDIA'17),* 6 pages.
DOI: 10.1145/nnnnnnn.nnnnnnn

1 INTRODUCTION

In the past decade, compressive sampling (CS) has been a hot research topic because it provides a much lower sampling rate than traditional sampling methods [8]. As an important step of CS, sparse signal reconstruction is also widely used to recover the signal of interest (SOI) from a lower-dimension [1, 6].

The key idea behind sparse signal reconstruction is to exploit the spare feature of a signal in a certain domain [4]. For example, images are sparse in wavelet domain, which is also the core idea to many lossy compression techniques such as JPEG [8]. Once the domain is determined, the process of finding the sparse solution is nonlinear, where ℓ_0-norm is usually utilized to promote the sparsity. However, the ℓ_0-norm based formulation is a NP-hard problem, and to efficiently obtain the sparse solution, there are usually two types of solutions. The first type is to convert the sparse solution finding problem into a convex optimization by utilizing ℓ_1-norm, which is known as basis pursuit. Alternatively, by finding the support set from dictionary matrix, greedy algorithm are developed such as orthogonal matching pursuit (OMP), compressing sampling matching pursuit (CoSaMP), subspace pursuit (SP) [2], to name a few, to directly solve ℓ_0-norm based optimization. The consensus is that the greedy algorithm are more time efficient than the basis pursuit.

Generally speaking, the noise contained in the received signal is usually assumed to be Gaussian distributed for the most considerations. Therefore, the algorithm above-mentioned cannot recover the SOI that is corrupted by impulsive noise that is a more common noise source in practice. To suppress the impulsive noise, a joint algorithm for signal recovery in the case of impulsive noise was proposed in [7], where the impulsive noise is modeled as α-stable distribution that is sparse in the time domain. Based on the reformulation, the SOI and impulsive noise are estimated simultaneously by the proposed algorithm. Unfortunately, the problem is that the sparsity of the signal must be known beforehand. In

MOBIMEDIA 2017, July 13-14, Chongqing, People's Republic of China
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DOI 10.4108/eai.13-7-2017.2270043

practice, that information is very hard to obtain and if the wrong sparsity is assumed, the performance will be inevitably degraded.

In this paper, the objective is to perform joint SOI recovery and impulsive suppression without the prior knowledge of the signal sparsity. To achieve that goal, the impulsive noise is still formulated as a vector that is sparse in the time domain. Similar to [7], the joint optimization problem is developed to simultaneously obtain the SOI and to suppress the noise. To efficiently solve the optimization and to satisfy the requirement of the lack of the sparse degree, two algorithms are developed based on sparsity adaptive matching pursuit (SAMP).

The rest of the paper is organized as follows. In Section 2, the problem overview is provided that includes the descritions of the impulsive noise and dual reconstruction development. In Section 3, two solvers based on the SAMP are developed to perform simultaneously the SOI reconstruction and impulsive noise suppression. Simulations are presented in Section 4 to demonstrate the performance of the proposed approaches. Finally, this paper concludes with a brief summary in Section 5.

2 PROBLEM OVERVIEW

2.1 Impulsive Noise

Impulsive noise consists of sudden on\off noise pulses at relatively short duration [9], and it is at low energy in the most times and at relatively large energy at an sudden incident lasting very short period of time, demonstrated in Figure 1. In this paper, we consider the signal is only corrupted by impulsive noise, which is model by a-stable distribution [5], and its characteristic function is given by

$$\varphi(t) = \begin{cases} \exp\{j\mu - \gamma|t|^{\alpha}[1 + j\beta sign(t)\tan(\frac{\alpha\pi}{2})]\} & \alpha \neq 1 \\ \exp\{j\mu - \gamma|t|^{\alpha}[1 + j\beta sign(t)\frac{\pi}{2}\log|t|]\} & \alpha = 1, \end{cases} \quad (1)$$

where α, β, γ, μ respectively are the characteristic exponent, the symmetry parameter, the scale parameter, and the location parameter. When $\beta = 0.5$, $\gamma = 1$ and $\mu = 0$, Figure 2 depicts the probability density function (PDF) of α-stable distribution in terms of different values of α. It is seen when α becomes smaller, the tail of the PDF becomes heavier indicating it is highly possible that significantly large values are present in the amplitudes. This observation suggests that the impulsive noise indeed is sparse in time domain.

2.2 Dual signal reconstruction

Suppose impulsive noise is present, the received signal is

$$\mathbf{y} = \mathbf{s} + \mathbf{e}, \quad (2)$$

where $\mathbf{y}$ is observed signal, $\mathbf{s}$ is the SOI, and $\mathbf{e}$ is the impulsive noise. In this work, we also assume that the SOI has a sparse representation in a transform domain Φ, namely $\mathbf{s} = \Phi\mathbf{x}$ where $\mathbf{x}$ is the coefficient vector that is sparse in Φ. With the sparse property in $\mathbf{e}$, the received signal in (2) becomes

$$\mathbf{y} = \Phi\mathbf{x} + \mathbf{I}\mathbf{e}, \quad (3)$$

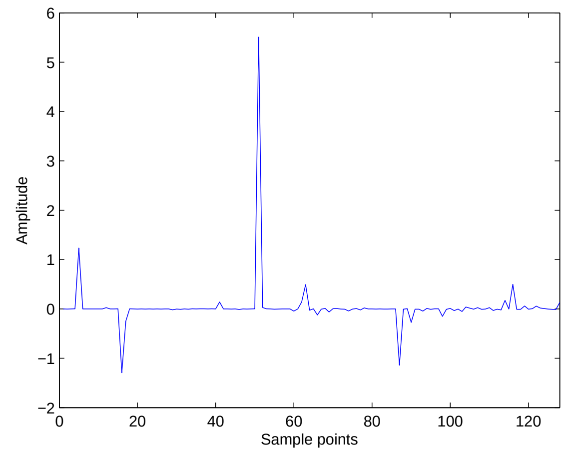


Figure 1: Example of impulsive noise when $\alpha = 0.6$

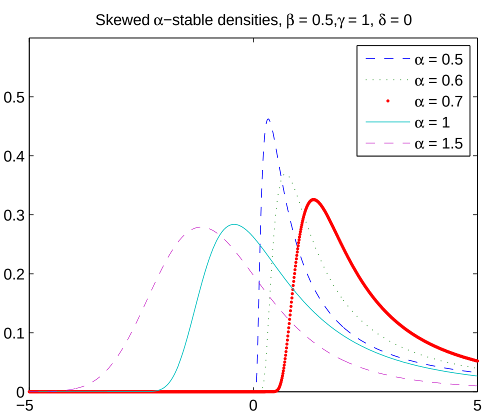


Figure 2: Probability density function for different values of α

where $\mathbf{I}$ is an identity matrix of proper size. The signal $\mathbf{s}$ and noise $\mathbf{e}$ are sparse separately in different domains, which provides one a chance to estimate them simultaneously utilizing the method of sparse representation. To obtain the solution in (3), the following optimization is formaulted by

$$\begin{aligned} &\text{minimize} \;\; \|\mathbf{x}\|_0 + \tau\|\mathbf{e}\|_0 \\ &\text{subject to} \;\; \|\mathbf{y} - \Phi\mathbf{x} - \mathbf{I}\mathbf{e}\|_2 < \epsilon, \end{aligned} \quad (4)$$

with variables $\mathbf{x}$ and $\mathbf{e}$. In (4), the τ and ϵ respectively are penalty and precision constants, and ℓ_0-norm is utilized to promote sparse solution.

3 JOINT ALGORITHM DESCRIPTION

Since both the SOI and impulsive noise have sparse representations in separate domains, it is feasible to estimate them in different dictionary matrices based on the iteration of residual. The remainder of this section will introduce two novel methods to estimate the signal whose sparsity is unknown.

3.1 Joint SAMP

The SAMP is developed for the reconstruction of the practical signal whose sparsity is unknown. The SAMP algorithm sets an iteration step to find the matched sparsity by comparing the present residual and the last residual at each step. The original SAMP algorithm [3] is provided as follows.

(1) Initialize the iteration step, sparsity variable L and residual
(2) Multiply the residual with the inverse of dictionary matrix to obtain the candidate list of columns in dictionary.
(3) Perform the least square solution to find the final list of column index (the L largest values in the estimated solution).
(4) Calculate the residual and compare the residual with the last residual to decide if the sparsity needs to be changed and if the residual needs to be updated.
(5) Check if the iteration criteria is satisfied.

As discussed early, the SAMP has the ability to adjust the sparsity level at each step to finally obtain the correct solution. Therefore, based on SAMP, a joint SAMP (JSAMP) algorithm is developed to recover the SOI and to suppress the noise. The core idea behind the JSAMP is that the estimates of SOI and noise are iteratively performed based on the separation property of the variables of $\mathbf{x}$ and $\mathbf{e}$. The steps of JSAMP are summarized in Table 1.

3.2 Sparsity Adaptive Dual Matching Pursuit (SADMP)

In the development of JSAMP, at each step the original SAMP is completely implemented. By doing so, the computational complexity can be high. A more economical way to realize joint estimation is to perform simultaneous updates for the SOI and the noise at each iteration step. Inspired by this thinking, an approach termed as sparsity adaptive dual matching pursuit (SADMP) is developed in which the SOI and noise are simultaneously updated. The implementation details of the SADMP are provided in Table 2.

4 SIMULATIONS

In order to access the performance of the proposed algorithms, simulations are conducted where the frequency estimation problem in case of impulsive noise is studied. For comparison purposes, the results from JCoSaMP [7] that requires the sparsity levels are also provided . In the simulations, the parameters of $T_1 = T_2 = 10$, $T = 5$ and $\delta = 10^{-6}$ for JSAMP are chosen, and also, $\delta = 10^{-5}$ is selected for SADMP. In Figure 3, impulsive noise is generated by $\alpha = 0.6$, $\beta = 1$, $\gamma = 2$, and $\delta = 0$. In figure 4, the value of γ is varied to produce different SNR levels. To generate the SOI, two sinusoidals of length $M = 128$ with $\omega_1 = 0.048\pi$, $\omega_2 = 0.108\pi$, and their amplitudes being unit are synthesized. To reveal the sparsity of SOI, the frequency domain as a sparse domain is utilized because the spectrum of sinusoidal signals is concentrated along the true frequencies. The dictionary required to represent the SOI is given by

$$\mathbf{\Phi} = \begin{bmatrix} \sin(\omega_0) & \sin((\omega_0 + d\omega)) & \cdots & \sin(\omega_N) \\ \sin(2\omega_0) & \sin(2(\omega_0 + d\omega)) & \cdots & \sin(2\omega_N) \\ \vdots & \vdots & \vdots & \vdots \\ \sin(M\omega_0) & \sin(M(\omega_0 + d\omega)) & \cdots & \sin(M\omega_N) \end{bmatrix}, \tag{5}$$

where $d\omega$ is frequency forward step.

To perform accurate assessment on the proposed approaches, the recovery ratio that is defined below is utilized as a performance measure.

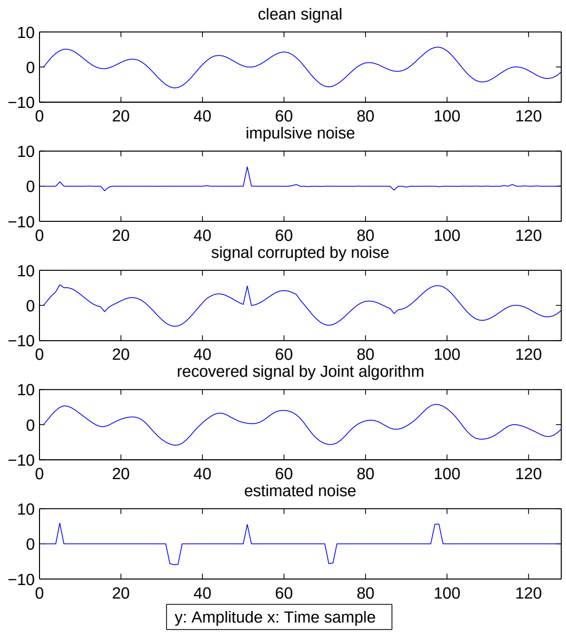
$$\text{recovery ratio:} \quad \kappa = \frac{\sum (x_e(i) - \overline{x_e})(x_0(i) - \overline{x_0})}{\sqrt{\sum (x_e(i) - \overline{x_e})^2 \sum (x_0(i) - \overline{x_0})^2}}, \tag{6}$$

where $x_e(i)$ and $x_0(i)$ respectively represent the reconstructed signal and the clean signal, $\overline{\cdot}$ performs the average operation. For a ideal signal reconstruction method, the recovery ratio κ should be close to 1 since the recovered signal should be close to the clean signal as much as possible.

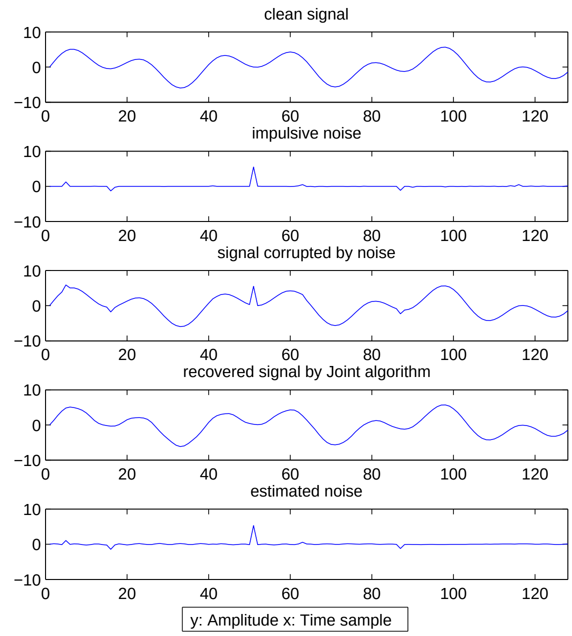
The simulation results obtained from JSAMP and SADMP are depicted in Figure 3 in which both JSAMP and SADMP have excellent ability to estimate SOI. For the noise, on the other hand, the JSAMP tends to overestimate the noise producing more spikes. Surprisingly, the SADMP performs more consistently than JSAMP in terms of noise suppression ability. The recovery ratio for SOI and noise are provided in Figure 4 versus different SNRs. Generally speaking, the SADMP produces better recovery ratio than the JSAMP. When SNR is low, the recovery ratio of the SADMP has better rate to approach unit than the JSAMP in terms of SOI recovery. When SNR is high meaning the noise is low, the recovery ratio of both approaches drops, but the SADMP still maintains a steady decline. For JCoSaMP approach, when the assumed the sparsity level is correct, the recovery ratio can be excellent, see the the green dash line in Figure 4a, denoted by JCoSaMP (K1=2), whereas when the assumed the sparsity level is wrong, the performance of JCoSaMP deteriorates significantly, see the pink solid line, denoted by JCoSaMP (K1=1). In Figure 4b, either $K2 = 2$ or $K2 = 6$ matches the true sparsity level for the noise, its performance in noise recovery is inferior to the other approaches.

5 CONCLUSION

In this work, two approaches of dual signal estimation based on SAMP are proposed without the prior information on the sparse levels of the signals. Utilizing different sparse domains of the SOI and the impulsive noise, the proposed algorithms recover the SOI and suppress the noise simultaneously. Simulations demonstrate the proposed algorithms are able to reconstruct the SOI and also to produce satisfactory performance in the noise suppression.

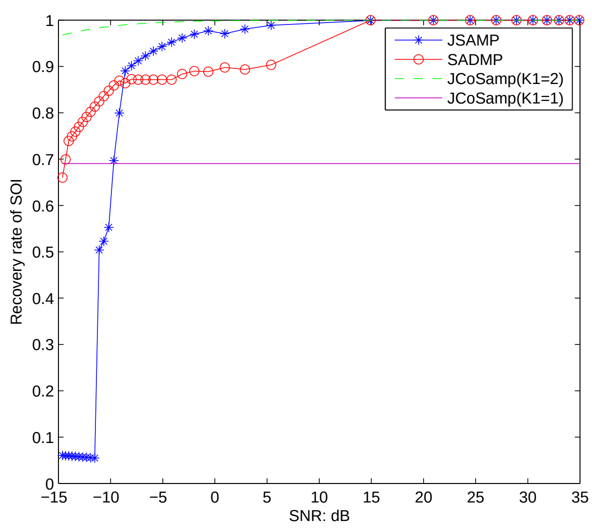


(a) The Reconstructed signal by JSAMP

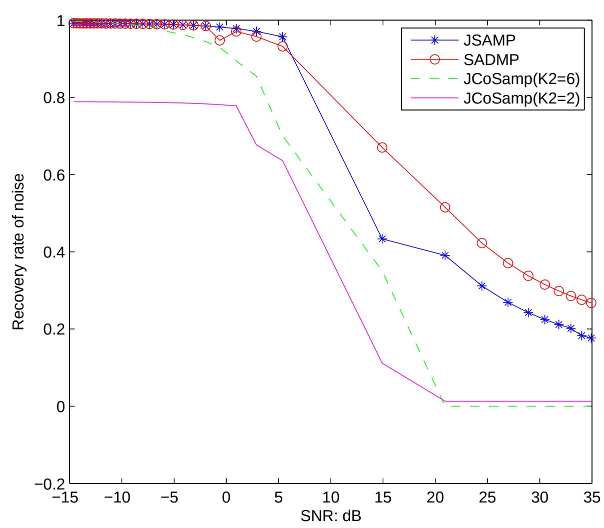


(b) The reconstructed signal by SADMP

Figure 3: The results obtained by different approaches



(a) SOI



(b) Noise

Figure 4: Recovery ratios obtained by different approaches

Table 1: Joint SAMP Algorithm.

Objective function: minimize $\|\mathbf{x}\|_1 + \tau\|\mathbf{e}\|_1$ subject to $\|\mathbf{y} - \mathbf{\Phi}\mathbf{x} - \mathbf{I}\mathbf{e}\|_2 < \epsilon$
Inputs: $\mathbf{y}$, $\mathbf{\Phi}$, $\mathbf{I}$
Outputs: Estimates of $\mathbf{x}$ and $\mathbf{e}$

Initialization: $\mathbf{r} = \mathbf{y}$, $t = 1$, $\boldsymbol{r_x} = \mathbf{0}$ {residue for $\boldsymbol{\sigma}$}, $\boldsymbol{r_e} = \mathbf{0}$ {residual for $\boldsymbol{\alpha}$}, $\boldsymbol{x}_0$ and $\boldsymbol{e}_0$
Repeat
t=t+1
Step 1: Estimate the $\boldsymbol{x}$ as $(\boldsymbol{x}^t, \boldsymbol{r}_{\boldsymbol{x}}^t) \leftarrow$ SAMP($\mathbf{y}$,$\mathbf{r}$,$\mathbf{\Phi}$,$\boldsymbol{T_1}$)
Step 2: Estimate the $\boldsymbol{e}$ as $(\boldsymbol{e}^t, \boldsymbol{r}_{\boldsymbol{e}}^t) \leftarrow$ SAMP($\mathbf{y}$,$\mathbf{r}$,$\mathbf{I}$,$\boldsymbol{T_2}$)
Step 3: Update the global residue as $\mathbf{r} = \mathbf{y} - \boldsymbol{r}_{\boldsymbol{x}}^t - \boldsymbol{r}_{\boldsymbol{e}}^t$
Until $t > T$ {maximum iteration} or norm($\mathbf{r}$) $< \delta$ {predefined threshold}

Table 2: Sparsity Adaptive Dual Matching Pursuit (SADMP).

Objective function: minimize $\|\mathbf{x}\|_1 + \tau\|\mathbf{e}\|_1$ subject to $\|\mathbf{y} - \mathbf{\Phi}\mathbf{x} - \mathbf{\Omega}\boldsymbol{e}\|_2 < \epsilon$
Inputs: $\mathbf{y}$, $\mathbf{\Phi}$, $\mathbf{\Omega} = \mathbf{I}$
Outputs: Estimates of $\mathbf{x}$ and $\mathbf{e}$

Initialization: $\hat{\mathrm{x}} = 0$ $\hat{\mathrm{e}} = 0$ {estimation initialization}
r = y Residual initialization
$L_x = 1$, $L_e = 1${Sparsity Initialization}
$F_x = \varnothing, F_e = \varnothing$ {Finalist initialization}
t=0 {Iteration index}
Repeat
t=t+1
Step 1: $S_x = Max(|\ \Phi' R\ |, L_x)$ **{Preliminary Test}**
$S_e = Max(|\ \Omega' R\ |, L_e)$
Step 2: $C_x = F_x \bigcup S_x$ **{Making candidate list}**
$C_e = F_e \bigcup S_e$
Step 3: $\hat{F}_x = Max(|\ \Phi'_{C_x} R\ |, L_x)$ **{Finalist test}**
$\hat{F}_e = Max(|\ \Phi'_{C_e} R\ |, L_e)$
Step 4: $\hat{x} = (\Phi'_{\hat{F}_x}\Phi_{\hat{F}_x})^{-1}\Phi_{\hat{F}_x} y$ **{Least square}**
$\hat{e} = (\Omega'_{\hat{F}_e}\Omega_{\hat{F}_e})^{-1}\Omega_{\hat{F}_e} y$
Step 5: $\boldsymbol{rnew_x} = y - \Phi_{F_x} \cdot \hat{x}$, $\boldsymbol{rnew_e} = y - \Omega_{F_e} \cdot \hat{e}$ **{Obtain new residual}**
If $rnew_x >= r_x$ **or** $rnew_e >= r_e$
if $rnew_x >= r_x$ then $L_x = L_x + 1$;
if $rnew_e >= r_e$ then $L_e = L_e + 1$;
t=t-1;**{iteration index remains}**
Else
$F_x = \hat{F}_x$ **{Update Finalist}**
$F_e = \hat{F}_e$
$r_x = rnew_x, r_e = rnew_e$
$R = y - rnew_x - rnew_y$ **{Update Total residual}**
Until $t > T$ {maximum iteration} or norm($\mathbf{R}$) $< \delta$ {predefined threshold}

ACKNOWLEDGMENT

This work was jointly supported by the National Natural Science Foundation of China under Grant 61501072, by Foundation and Advanced research projects of Chongqing Municipal Science and Technology Commission under Grant cstc2015jcyjA40040, by the Chinese Scholarship Council(CSC) under Grants 201607845024.

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