

Modeling and shaping the lifetime of target detection sensor networks

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ABSTRACT

In this work we model the lifetime of a clustered sensor network deployed for target detection. We assume sensors are randomly dropped on a bidimensional field in order to detect target traversals occurring stochastically. Once a target enters the sensing area of a sensor, the sensor transmits such information to a cluster head, in charge of receiving and retransmitting the messages received from the sensors deployed on the field. The contribution of this work is threefold. We first identify the sensing nodes whose behavior is key to model the duration of sensing operations. We then proceed, providing a heuristic estimation of the traffic received by the cluster head to quantify its energy requirements, resorting to specific lifetime definitions. We finally evaluate the relationship between our probabilistic and heuristic models and the time until when the network fulfills its purpose, i.e., it remains capable of detecting and reporting the traversal of any target to a sink, as obtained by simulation.

Keywords

Surveillance sensor network; lifetime definition; stochastic analysis.

1. INTRODUCTION

It is difficult to delimit the fields of application of sensor networks, as they have reached any type of context and site (e.g., body, underwater, vehicular, Internet of Things, etc.) [1,5,16,17]. Among the many challenges these networks pose, finding ways of characterizing and increasing their lifetime amounts to one of the main ones [2]: as nodes deplete their energy and shutdown, the sensing power a network degrades, until no longer capable of fulfilling its scope.

Lifetime analysis is challenging, as no unique and widely accepted definition exists which well adapts to all possible application scenarios. In a context where the monitored variable is the surface temperature of a field, for example, an

acceptable metric may amount to the lifetime of the longest lasting sensor, when all deployed sensors are expected to provide sample values which are very close to each other, no matter where positioned. In a single barrier coverage situation, instead, the time of death of the first sensor may coincide with the lifetime of the network, as beyond such time the network may no longer be capable of guaranteeing the detection of a target crossing an area of interest. In this work we adopt a perspective which is close to this second example, as we study the key variables which influence most the efficacy and duration of a surveillance system composed by a number of sensors randomly dropped to monitor the traversal of a given bidimensional area.

In particular, the surveillance system amounts to a clustered sensor network whose scope is to detect and report specific type of events: the traversals of targets across an area of interest (e.g., herds of animals crossing a canyon, soldiers moving through a minecamp, etc.). As a target enters, moves across the area and reaches its exit, it enters the sensing region of different sensors. All those sensors that detect the presence of the target transmit a message to a cluster head, in charge of receiving and retransmitting the messages received by all sensors to a sink. In such scenario, our aim is to individuate and model the nodes whose lifetime matches the lifetime of the surveillance system, as determined by the following definition [9]: *the lifetime of a surveillance system is assumed to be represented by the lifetime(s) of its most critical traversal path(s), the path(s) that will first be available to a target for an undetected traversal.*

To proceed with such analysis, we assume a sufficient number of nodes are initially deployed to guarantee the detection of a target at any inner point within the area of interest. In addition, we assume the availability of prior: (a) target arrival and mobility models and, (b) an energy consumption model per target detection and message transmission/reception. We then probabilistically model and analyze two classes of key sensing nodes: (a) the node(s) which exhibit the highest probability of dying first, and, (b) a specific subset (which subset, will become clear in Section 4) of the node(s), located on the path(s) which traverse the node(s) determined at the previous point, with the highest probability of dying last. Resorting to such models, we provide an approximation of the probabilistic distribution function of a surveillance system lifetime as provided by the aforementioned definition. Finally, such results are also utilized to heuristically estimate the amount of energy required by the cluster head, in order to guarantee the forwarding of any messages received during the active phase of the surveillance

system.

The remainder of this paper is structured as follows. Section 2 provides an overview of the state-of-the-art, from a point of view of the stochastic analysis of sensor network lifetime. In Section 3 we delineate the system scenario of interest, whereas in Section 4 we provide our models. An evaluation of our model is provided in Section 5, whereas Section 6 concludes this work.

2. RELATED WORK

Studies evaluating the lifetime model of wireless sensor networks may be found in a plethora of scientific contributions [6, 14, 15, 19, 21, 23]. However, only a few have analyzed the node and network lifetimes in a pure probabilistic way [4, 7, 18, 20]. In the following, we provide an overview of three of such works.

[18] presents a lifetime analysis of a randomly deployed clustered network, where a cluster head collects data from the nodes within its cluster, performing data aggregation and compression, finally relaying such data to a sink. The main characteristics of this model are: (a) an approximation for the complementary cumulative distribution function of cluster lifetime is found for event-driven networks, (b) two sensing modes are considered, one where an event is only reported by the closest node and a second by all nodes, (c) the stochastic nature of the network lifetime follows from the randomness of the sensors deployment, the amount of energy required to transmit a data packet at different distances, the events' appearance time and location, and, (d) the sensor network lifetime is assumed active as long as β out of N sensors are still alive [19].

Also [20] proposes a stochastic analysis of the energy consumption in a random network environment providing: (a) a cross-layer Markov process based framework to characterize the energy consumption distribution during a given period of time, (b) where energy consumption is shown converge to a Normal distribution, if the time period is large enough, and, (c) with the consequent computation of node lifetime and network lifetime distributions. In addition, authors assume that event arrivals are independent and identically distributed at all nodes and in correspondence of an arrival an equal amount of data is transmitted.

The contribution presented in [7] assumes an event may not always be detected by a sensor network, modeling the probability of detecting an event adopting an exponential negative decay law dependent on the distance from the interested node [11]. In addition, a node is represented as a network entity which can transmit a maximum number of packets, not considering the fact that energy resources are spent also while in idle mode.

This main differences between our work and [7, 18, 20] are: (a) two different lifetime definition are considered, which well adapt to the specific scenario of interest, (b) events are not modeled as instantaneous and punctiform, an event is here also characterized by the path followed by a target when traversing the monitored area, and, (c) when a target traverses the monitored area, only those sensors whose sensing range cover the path are involved in the detection and reporting of the event.

3. SYSTEM MODEL

We consider a bidimensional field A of arbitrary shape,

where N sensors are randomly dropped to monitor the traversals of targets which may appear at random times. The N dropped nodes are supposed to be sufficient to cover every single point in A and to report the detection of a target to a fixed cluster head. The cluster head is also equipped with a limited amount of energy, which may be more than the one provided to single sensors. The following summarizes the system assumptions made to formulate our analysis.

A sensor node, in general, consumes energy for sensing, receiving, transmitting, data processing, sleeping and also during the idle mode when no data sensing, processing or exchange happens. Such model is here simplified, considering the following modes of operation as the ones necessary to determine the lifetime of a node for the scenario of interest: sensing, transmitting and idle. The idle state does not correspond to the sleep one, as a sensing node is capable of detecting an event during idle mode. We also assume that the energy consumed while active, monitoring for the occurrence of events, is much lower than the energy spent during sensing and transmitting phases. A similar model is also adopted for the cluster head, with the difference that the cluster head is solely devoted to receive and retransmit messages to a sink node. Hence, during reception and transmissions, we assume the cluster head consumes an amount of energy which is much higher than the amount of energy spent during idle mode, when scanning the channel for the reception of any transmission. Now, instead of energy, we will directly deal with the corresponding lifetime quantities. Instead of considering the total energy of a given node i , we will refer to its maximum lifetime, $l_{max,i}$, here defined as the lifetime of node i while permanently staying in idle mode. For what concerns detection and transmission phases (or receival and retransmission at the cluster head), we assume i consumes a constant amount of lifetime equal to $l_{eh,i}$. As in previous works, events are assumed arriving to the monitored area according to a Poisson process. The Poisson distribution has been widely used in the literature to model events whose time/place of occurrence are random and independent from each other [3]. Also, in vehicle interarrival in highway scenarios, the Poisson distribution has been found an interesting model [13]. Unlike previous approaches, events do not popup at given points in space, but are detected by sensors as targets move along random trajectories within the area under observation. The shape of the monitored area may be random, depending on the application needs and on how the sensors spread when dropped. Our assumption is that it may always be possible to individuate an entry and an exit to the area. Different approaches may, then, be utilized to represent mobility within the area. A successful approach, adopted in literature, amounts to n -th order Markov Chains, $MC(n)$, where the probability of reaching a given state depends upon the n previously visited ones [12].

As anticipated, we consider a scenario where a sensor network organizes itself into clusters. With the adoption of a clustering scheme, nodes self-organize themselves into hierarchical layers. We assume three layers: (a) a bottom layer composed of sensing nodes in charge of detecting targets entering their sensing region and transmitting such information to the cluster head, (b) an intermediate one, made of a cluster head which receives and retransmits messages and, (c) a top one where a sink node collects all the information received by the cluster head. Assuming such type of orga-

nization, a time division multiple access (TDMA) scheme may be employed at the MAC layer, where the cluster head takes care of coordinating the time scheduling among the nodes. TDMA well fits our scenario of interest, as it is a common choice, at the MAC layer, for clustered WSNs [22]. Finally, such assumption also ensures the avoidance of collisions, allowing the modeling of intra-cluster transmissions as ideal.

4. LIFETIME MODEL

As anticipated, we consider two classes of sensing nodes to compute the lifetime of the surveillance system. The first class is very simple to identify: these are the node(s) that die first (exhaust their energy first) [8]. The second class, instead, is individuated as the subset of the node(s), which lying along the paths that contain the node(s) which are expected to die first, will die last [10]. Which subset we are referring to will become clear by the end of this Section.

Before proceeding, however, let us remind that in this paper we assume a generic sensing node i is equipped with an initial lifetime $l_{max,i}$, the time node i would live in case it was always idle. A sensing node i which, hence, utilizes part of its energy resources to detect targets and transmit messages, lives for an interval of time which is lower than $l_{max,i}$. Lifetime l_i may hence be written as: $l_i = l_{max,i} - l_{EH,i}$, where $l_{EH,i}$ amounts to the lifetime reserved and available for sensing and message transmission activities. We also assume all sensing nodes in the network are equipped with the same amount of energy at the beginning and that the amount of energy spent in correspondence of a sensing/message transmission activity is constant and equal for all nodes.

Now, the lifetime of a sensing node depends on how often it is involved in sensing/message transmission operations, i.e., on its probability of activation. The probability of activation, in turn, depends on the position of the sensing node within A and on the mobility model of targets. This means that depending on its position, but also on its sensing reach, a sensing node may be more often, or less, activated.

The individuation of the node that will die first, among all sensing nodes, can be performed individuating the position(s) with the highest activation probability, i.e., the highest probability of detecting a target: in fact, the sensors located in such position(s) will most probably exhaust their energy before any other.

The position(s) of the node(s) that will die first can also be put to good use to determine the sensor network lifetime according to the second approach of interest. In fact, with the individuation of the node(s) that will, with the highest chance, exhaust their energy first, it is also possible to individuate the path(s) that will, most probably, exhaust the energy of the monitoring sensors first, and their lifetime, as follows. Say, for example, only one position with the maximum activation probability value may be found in a specific area A . Denote such point as (x_M, y_M) , with a corresponding activation probability equal to $p(x_M, y_M)$. Next, among all possible paths that traverse (x_M, y_M) , individuate the path(s) characterized by the highest probability to be used to traverse the critical area (for simplicity, from now on we assume only one of such paths exist and indicate it as T). Such condition implies T is the path with the highest probability of exhausting its energy resources first, becoming viable for a target seeking an undetected traversal. Please

note that the lifetime of T , and hence of the sensor network, may be analyzed modeling the behavior of the node exhibiting the lowest probability of activation: the lifetime of such node(s) amounts to the second lifetime metric considered in this work. Clearly, resorting to such probability metrics concerning the critical path one can establish several other heuristics which may result useful to compute stochastic bounds of the variable of interest.

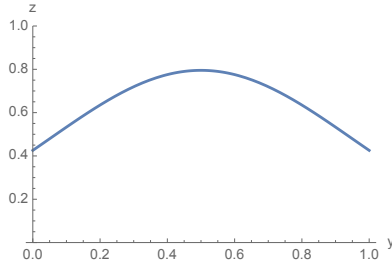
To visualize how the location of the two nodes of interest may be individuated, consider Figure 1. In particular, Subfigure 1b represents the activation probability values of a 1×1 square, where targets can enter from any point on the $y = 0$ segment, traverse the area walking along a straight line, exiting from any point on the $y = 1$ segment. According to Subfigure 1b, the node that is expected to die first, i.e., the one with the highest probability of activation, falls exactly at the center of the area. Now, the position of the second node of interest can be found as follows: of all paths which start at some point on the $y = 0$ segment and end on the $y = 1$ one, take the one exhibiting the maximum value of all the minimum activation probabilities. Following such rule, we find such path as the one crossing the area along the $x = 0.5$ segment (the activation probabilities along the $x = 0.5$ segment are plotted in Subfigure 1a). Finally, referring again to Subfigure 1b, we conclude saying that the first lifetime of interest may be computed modeling the behavior of a node located in $(0.5, 0.5)$ (the location with highest activation probability), whereas second lifetime metric can be obtained analyzing the node(s) located in $(0.5, 0)$ and $(0.5, 1)$. In fact, when the sensor(s) located in $(0.5, 0)$ and $(0.5, 1)$ exhaust their energy, it is probabilistically plausible that the path connecting $(0.5, 0)$ to $(0.5, 1)$ will result free of active sensing devices. This is the most critical path in such scenario.

Before concluding, please note that the cluster head amounts to an important bottleneck for the flow of information from the field of sensing nodes to the sink. Because of this, we will show how it is possible to put to good use the lifetime definitions provided in this Section to compute the amount of energy required by the cluster head. In the following we show how it is possible to obtain: (a) a stochastic model for sensing nodes lifetime according to the two aforementioned definitions, and, (b) a heuristic function which may be used to define the energy requirements of a cluster head. To do this, we first show how it may be possible to compute the activation probability of a node within a cluster and the move on to model the probability of activating a given number of nodes in correspondence of a traversal.

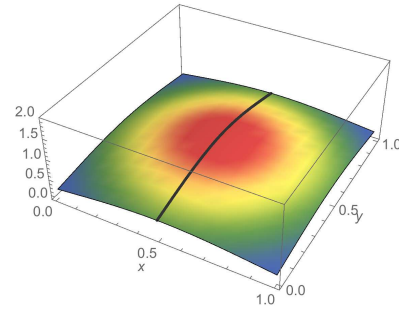
4.1 Node activation probability

From now on, for the sake of simplicity, we consider a situation where A is a square, of size X^2 , with N sensor nodes located inside. N is big enough to guarantee that each point (x, y) in A is covered by at least one sensor, assuming each sensor can detect, according to a boolean model, a target moving within a radius r .

We want to compute the probability a generic node i , located in (x_i, y_i) , is activated every time a target traverses A . Such probability may be computed searching, first of all, for the probability that i is activated when a single target traverses A . When a target traverses A , all the sensors that are within a distance of r units are activated, hence if i lies in a stripe determined by such quantity, i is activated. To



(a) Trajectory, among those crossing (x_{max}, y_{max}) , which exhibits the maximum among the minimum probability of activation.



(b) Probability of activation field along with the trajectory that traverses (x_{max}, y_{max}) and exhibits the maximum among the minimum activation probabilities.

Figure 1: Activation probability example.

find such probability hence, it is necessary to compute the probability that a random segment, describing the traversal of a target, falls within distance r from i .

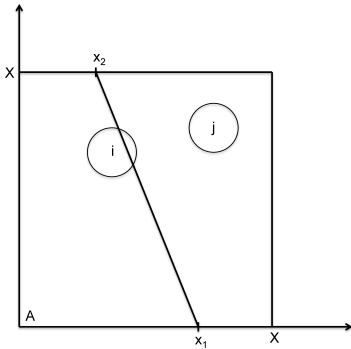


Figure 2: Computing the probability of activation of sensor i .

Consider now the situation depicted in Figure 2. Assume a target may only enter from the bottom edge of the square and leave from the opposite one. The entrance and exit points are random, though. Such locations are chosen uniformly in $[0, X]$. Hence, according to the reference system adopted in Figure 2, the entrance point is given by $(x_1, 0)$ and the exit point by (x_2, X) , where x_1 and x_2 are i.i.d. uniform random variables $U[0, X]$. The event that i is activated amounts to the event that a possible trajectory T intersects the sensing area of i . Such event is here indicated as $T \cap i$ and its probability is given by:

$$P(T \cap i) = \int_T P(T \cap i|T) \times P(T), \quad (1)$$

where $P(T \cap i|T)$ amounts to the probability of intersection, for a given trajectory T , and $P(T)$ to the probability of a given trajectory T . Let us start considering the problem of determining $P(T)$. Its value can be easily found considering the fact that $P(T) = P(x_1 \wedge x_2)$. Reminding that x_1 and x_2 are i.i.d. uniform random variables $U[0, X]$, we have that $P(T) = 1/X^2$. For what concerns $P(T \cap i|T)$, we have that:

$$P(T \cap i|T) = \begin{cases} 1 & \text{if the sensing area of } i \text{ and } T \text{ intersect,} \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

Now, it is possible to determine whether T and the sensing area of i intersect, for example, computing the distance between i 's coordinates, (x_i, y_i) and trajectory T . Hence, may be written as $P(T \cap i|T) = \mathbb{1}_{\{d((x_i, y_i), T) \leq r\}}$. Equation 1 may hence be rewritten as:

$$P(T \cap i) = \frac{1}{X^2} \times \int_0^X \int_0^X \mathbb{1}_{\{d((x_i, y_i), T) \leq r\}} dx_1 dx_2. \quad (3)$$

4.2 Number of detections per traversal

Computing the cluster head lifetime entails finding the number of events it will be handling. This quantity, in turn, depends on the number of targets traversing A and on the number of sensor nodes that are activated in correspondence of each single traversal. We here start computing the probability a given number of sensing nodes are activated in correspondence of a single traversal. Such function may be computed observing that, in correspondence of a traversal, all the nodes that fall within distance r will be activated. In essence, it is possible to approximate the area where all activated sensing nodes fall as a rectangle of dimensions $2 \times r$ by x , where $X \leq x \leq \sqrt{2} \times X$. With such assumption, we determine the probability a given number of sensing nodes falls in a rectangle of dimensions $2 \times r \times x$, given x . Such probability can be obtained resorting to a Binomial distribution, as our initial assumption is that nodes are dropped in a uniform way in A :

$$P(N' = n|x) = \binom{N}{n} \times \left(\frac{2 \times r \times x}{X^2} \right)^n \times \left(1 - \frac{2 \times r \times x}{X^2} \right)^{N-n}. \quad (4)$$

Now, Equation 4 may be put to good use the compute the probability of activating resorting to the law of total probability:

$$P(N' = n) = \int_X^{\sqrt{2}X} P(N' = n|x) \times P(x) dx, \quad (5)$$

with $P(x)$ the probability a target traverses A walking along a path of length x . It is possible to show geometrically that $P(x) = 2 \times x \times (X - \sqrt{x^2 - X^2})/X^2$, from which we obtain:

$$\begin{aligned}
P(N' = n) &= \int_X^{\sqrt{2}X} \binom{N}{n} \times \left(\frac{2 \times r \times x}{X^2} \right)^n \times \\
&\times \left(1 - \frac{2 \times r \times x}{X^2} \right)^{N-n} \times \frac{2 \times x \times (X - \sqrt{x^2 - X^2})}{X^2} dx,
\end{aligned} \tag{6}$$

Equation that is here put to good use to compute a heuristic function which may return a significant value of the cluster head lifetime and consequent energy requirements.

4.3 Event lifetime consumption model

In this Subsection we proceed first providing a model which may be put to good use to compute the probability distribution function of the lifetime of any node in the cluster and then move on to provide the heuristic function which is utilized to estimate the energy requirements of the cluster head. Clearly, the proposed consumption model may be subject to modifications depending on the particular application scenario. The same may be said for the heuristic proposed to compute the energy required by the cluster head.

4.3.1 Sensing node lifetime

Assuming targets arrive in A according to a Poisson model with intensity γ , each single sensor i keeps seeing Poisson traffic with intensity $\gamma_i = p_i \times \gamma$, where p_i is obtained as in Equation 3.

Assuming a constant amount of lifetime $l_{eh,i}$ is lost at node i every time a target crosses its range, the problem amounts to a simple budgetary problem: an equal amount of energy is subtracted from the initial lifetime $l_{EH,i} = l_{max,i} - l_i$ available per each message detection and transmission procedure.

The probability of spending less than $l_{EH,i}$ units of lifetime assuming Z events are detected and handled is:

$$P(Z \times l_{eh,i} < l_{EH,i} | Z) = \begin{cases} 1 & \text{if } Z < l_{EH,i}/l_{eh,i}, \\ 0 & \text{if } Z \geq l_{EH,i}/l_{eh,i}. \end{cases} \tag{7}$$

Using an alternative notation, $P(Z \times l_{eh,i} < l_{EH,i} | Z)$ amounts to the unit step function $u(l_{EH,i}/l_{eh,i} - Z)$. Taking now into account all possible arrivals, resorting to the total probability theorem, we can write:

$$\begin{aligned}
F(l_{EH,i}, l_i) &= \sum_{Z=0}^{\infty} F(l_{EH,i}, l_i | Z) \times e^{-\gamma_i l_i} \frac{(\gamma_i l_i)^Z}{Z!} = \\
&= e^{-\gamma_i l_i} \times \sum_{Z=0}^{\lfloor \frac{l_{EH,i}}{l_{eh,i}} \rfloor} \frac{(\gamma_i l_i)^Z}{Z!}.
\end{aligned} \tag{8}$$

Equation 8, computed for $F(l_{max,i} - l_i, l_i)$, returns the probability of spending less than the amount of available lifetime for event handling, when the lifetime of node i is l_i . In other words, it provides us with the probability of i lasting at least l_i units of time.

4.3.2 Cluster head lifetime heuristic function

The lifetime $l_{EH,CH}$ spent by the cluster head for event handling may be obtained as the sum of all detections made by the cluster multiplied by the amount of energy spent

per each relay operation. If Z traversals occur, the amount of battery lifetime required at the cluster head to manage all can be written as $l_{EH,CH} = l_{eh,CH} \times \sum_{i=1}^Z n_i$, where n_i amounts to the number of nodes activated in correspondence of the i -th traversal and $l_{eh,CH}$ the constant amount of lifetime spent at the cluster head in correspondence of an event handling.

This said, the heuristic function that is proposed in this work to assess the amount of battery lifetime $\tilde{l}_{EH,CH}$ required by the cluster head for event handling is:

$$\tilde{l}_{EH,CH} = Z^{90\%} \times n^{90\%} \times l_{eh,CH}, \tag{9}$$

where $Z^{90\%}$ amounts to the 90-percentile of random variable representing the number of arrivals in A during the given mission time and $n^{90\%}$ the 90-percentile of the number of activations per traversal. In Section 5 we verify how the value of $l_{max,CH} - \tilde{l}_{EH,CH}$ relates to the cluster head lifetime value l_{CH} obtained by simulation.

5. EVALUATION

We here validate our models resorting to simulation results, produced with a custom simulator developed in Mathematica. We consider a square area A with $X = 6$ units. The situation is the one presented in Subsection 4.1: intruders have an equal probability of entering at any point of only one edge of square A and of exiting from any point of on the edge opposite to the entrance (no targets can enter from the remaining two edges). Parameter values are $N = 100$, $l_{max,i} = 100$, $l_{eh,i} = 5$, $\gamma = 1$ and $r = 0.5$ for all sensing nodes i .

In Figure 3 we plot all alive and dead nodes observed in 100 simulations within A , considering intermediate time steps before 80 units of time. As expected, the nodes that first stop operating are those closer to the center of the square. At time $t = 60$ the detection system is no longer reliable (Figure 3c).

5.1 Sensing nodes lifetime

At this point we proceed computing the lifetime probability of two virtual nodes, one located in $(3, 0)$ and one in $(3, 3)$. In fact, in A , only one activation probability maximum exists, located at the center of the square ($p(3, 3) = 0.33$). Given the uniform condition on the entrance and exit edges, infinite trajectories T may be found, all crossing $(3, 3)$. All of them exhibit the minimum activation probability on the entrance and exit edges, which is the same for all. We hence consider, as second point of interest, $(3, 0)$. Equation 8 is plotted for $l_{max,i} = 100$, $l_{eh,i} = 5$, $\gamma = 1$ (hence $\gamma_{(3,0)} = 0.17$ and $\gamma_{(3,3)} = 0.33$), as a function of l_i in Figures 4a and 4b for positions $(3, 0)$ and $(3, 3)$, respectively. As l_i approaches the value of $l_{max,i}$ the probability of being capable of handling incoming events goes to zero. Please note both Figures 4a and 4b represent complementary cumulative distribution functions, as a function of l_i , as $l_{EH,i} = l_{max,i} - l_i$.

5.2 Cluster head energy requirements

To quantify the energy requirements of the cluster head, we first analyze the number of detections that occur in correspondence of one traversal resorting to Equation 6 and to our simulator. Figures 5a, 5b and 5c, in particular, show the empirical probability of being detected by a given number of

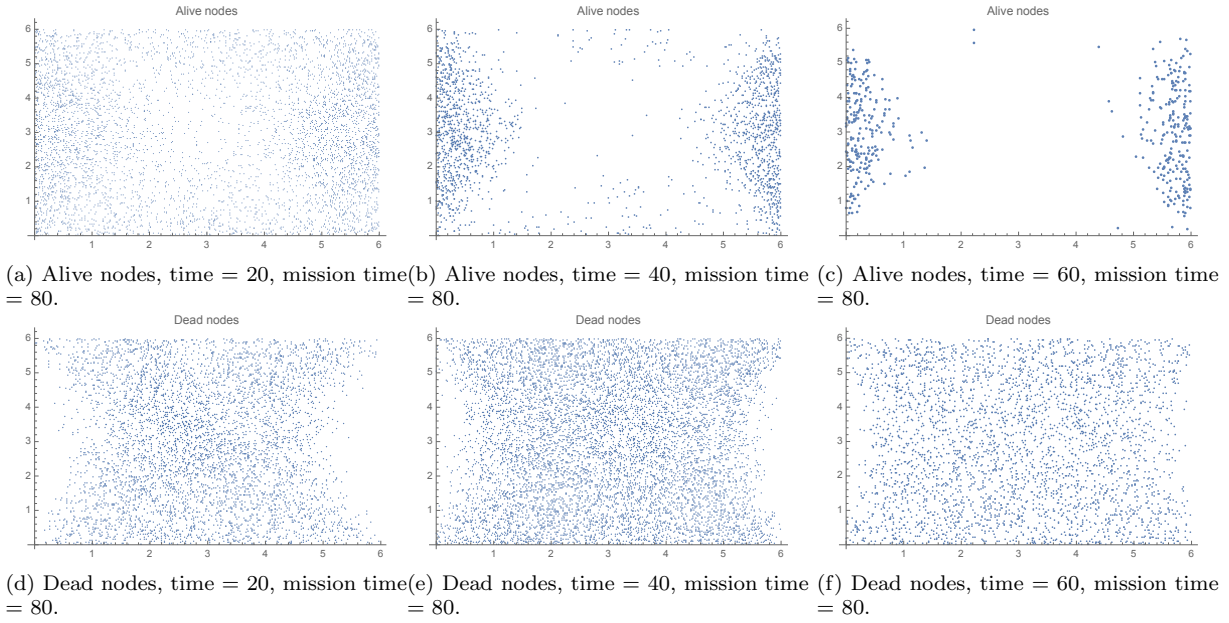


Figure 3: Spatial distribution of alive and dead nodes as a function time, before the end of mission time.

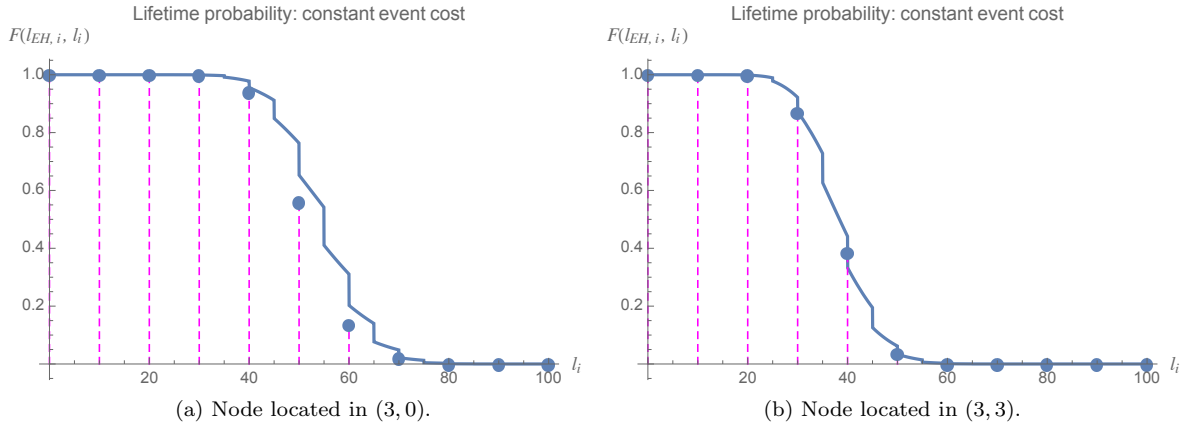


Figure 4: Computing the lifetime of a sensor network: model (continuous line) vs simulation (dots).

nodes in correspondence of a traversal: as the mission time increases, more nodes deplete their battery resources and the probability of not being detected by any sensing node increases. In Figure 5d we, instead, show that our model perfectly matches simulation results, in the case where sensing nodes are equipped with an infinite amount of energy.

We now assume battery lifetime the cluster head consumes in correspondence of an event is set to $l_{eh,CH} = 10$. The 10-percentile of lifetime of nodes located in $(3, 0)$ and $(3, 3)$ is instead obtained utilizing Equation 8: $l_{(3,0)}^{10\%} = 45$, whereas $l_{(3,3)}^{10\%} = 30$. Such values amount to two possible hypothetical mission times which we will assume from now on. Resorting to our heuristic function reported in Equation 9, for mission times 30 and 45, $\hat{l}_{EH,CH}$ results equal to 8,418 and 12,233, respectively. We now aim at understanding how well such heuristic function works, utilizing fraction of such values as initial battery lifetime in 1,000 simulation runs for each row reported Tables 1 and 2, where $l_{CH}^{10\%}$ amount to the lifetime of the cluster head obtained as the 10-percentile of the

1,000 simulation results. Tables 1 and 2 report $l_{CH}^{10\%}$ values as higher fractions of the computed battery lifetime are employed. For mission time 30, 80% of the battery lifetime given by our heuristic is sufficient to probabilistically guarantee that in the 90% of cases the cluster head will also last at least 30 units of time. Something similar happens also when the mission time is equal to 45, thus confirming the fact that the heuristic rule provided in Equation 8 tends to return amounts of battery lifetime that are larger than those that are really necessary.

5.3 System lifetime: models vs. simulation

Our analysis, to this point, tells us that with an initial maximum battery lifetime $l_{max,i} = 100$ we may obtain a lifetime of 30 units of time, when taking as a reference node $(3, 3)$ and equipping the cluster head with an initial battery lifetime greater than $l_{max,CH} = 6,764$. If, instead, we take as a reference the sensing node located in $(3, 0)$, such lifetime grows to 45 units of time, when equipping the

CH battery lifetime used in simulation (% of $\tilde{l}_{max,CH}$)	10-percentile of CH lifetime $l_{CH}^{10\%}$ obtained running 1,000 simulations
5,081 (60%)	22
5,923 (70%)	26
6,764 (80%)	30

Table 1: Cluster head 10-percentile lifetime obtained from 1,000 simulations for mission time = 30 as a function of different fractions of the initial battery lifetime value ($\tilde{l}_{max,CH} = l_{CH} + \tilde{l}_{EH,CH} = 30 + 8,418 = 8,448$).

cluster head of a battery lifetime that is at least equal to $l_{max,CH} = 8,608$. The two different reference nodes return remarkably different lifetime values and cluster head battery lifetime requirements, thus of paramount importance is to understand which, of the considered nodes, may be considered a reliable indicator of the sensor system lifetime. To this aim, we designed a test that builds upon the results plotted in Figure 3. In essence, in our simulations, after an event is handled, we check whether a potential undetected path, i.e., an open path, has become available. However, to limit its computation time, we limit our search to all the feasible paths containing (3,3). When fixing the mission time to 30 units of time, we find that no open path becomes available. When the mission time amounts to 45, instead, we obtain a 10-percentile of 41.5 units of time (i.e., in the 90% of cases an open path will become available after 41.5) and a minimum value of 28.6. The statistical values of interest are plotted in Figure 6 for different mission times. Note that for mission times 45, 50 and 60 the maximum values amount to 45, 50, and 60 respectively, as simulations are interrupted at such times. Beyond mission time 60, instead, an undetected path opens up always before the completion of mission time. In this Figure, we may also observe that the median and 10-percentile values (49.2 and 37.7, respectively) grow up to 50, falling beyond this value. As the mission time increases, less lifetime resources are available for target detection and message transmission. For what, instead, concerns our original question, which sensing node appears as the best sensing system lifetime indicator, we observe that no open path becomes available when the mission time equals 30, whereas in the 90% of cases a path opens after 41.5 units of time when the mission time amounts to 45. To be thorough, we find that in the 82% of cases an open path opens at or after 45 units of time. In essence, the lifetime value obtained observing node (3,3) appears too low and conservative, whereas the lifetime value obtained observing (3,0) is slightly optimistic.

6. CONCLUSION

In this work we modeled the lifetime of a clustered sensor network deployed for target detection. Compared to other approaches, we here considered the influence that the target mobility model has on the duration of surveillance operations. The validity of the adopted approach has been proven confronting our models with our simulation results.

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CH battery lifetime used in simulation (% of $\tilde{l}_{max,CH}$)	10-percentile of CH lifetime $l_{CH}^{10\%}$ obtained running 1,000 simulations
6,161 (50%)	28
7,385 (60%)	37
8,608 (70%)	45

Table 2: Cluster head 10-percentile lifetime obtained from 1,000 simulations for mission time = 45, as a function of different fractions of the initial battery lifetime ($\tilde{l}_{max,CH} = l_{CH} + \tilde{l}_{EH,CH} = 45 + 12,233 = 12,278$).

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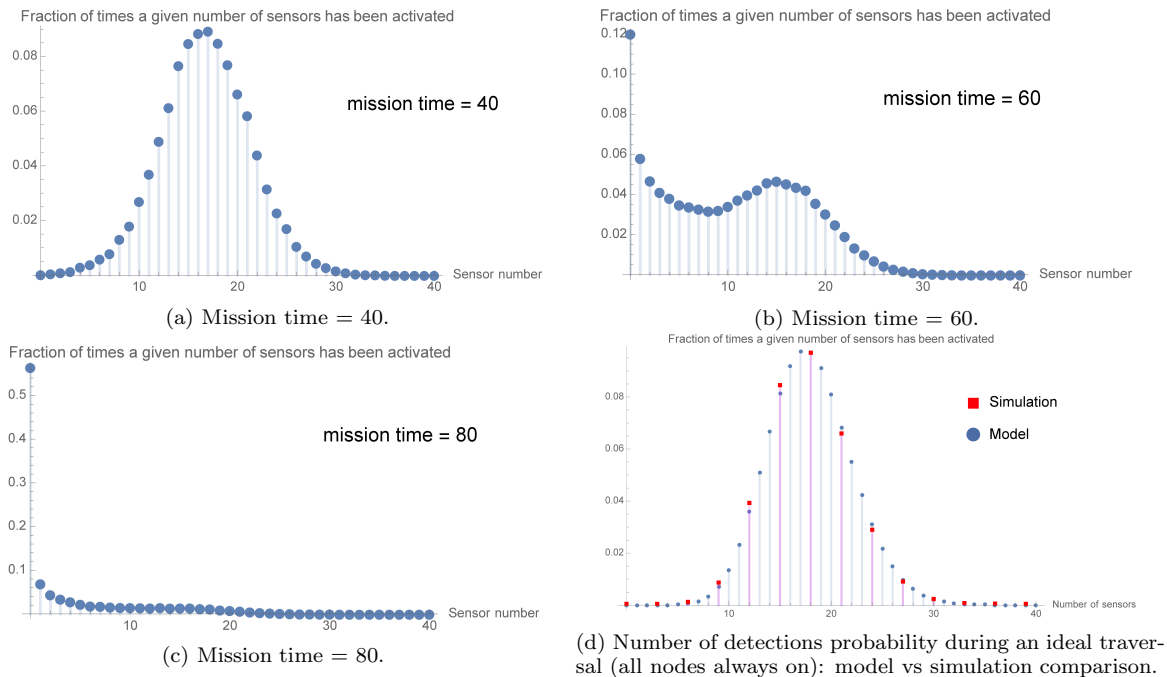


Figure 5: Number of detections per traversal probability when sensor nodes stop working as they deplete their batteries (Figures 5a, 5b and 5c). Ideal situation, when nodes never die Figure 5d.

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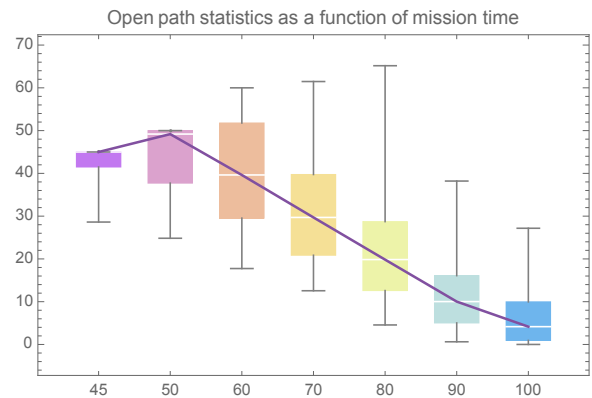


Figure 6: Maximum, 90-percentile, median, 10-percentile and minimum value obtained for the open path variable from 1,000 simulations in correspondence of different mission time values.