

Stationary solutions of discrete and continuous Petri nets with priorities*

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ABSTRACT

We study a continuous dynamics for a class of Petri nets involving priorities. We initially studied this class in [1] in the discrete setting, motivated by an application to the performance analysis of an emergency call center. In the discrete setting, limit time-periodic behaviors can occur. They may lead to asymptotic throughputs different from the affine stationary solutions of the dynamics, a pathology which motivates our study of a continuous version of the dynamics.

In a Petri net equipped with a continuous dynamics, the circulation of tokens is fluid, so that the dynamics can be represented by a system of ordinary differential equations or differential inclusions. Here, we address the situation in which the routing is specified by priority or preselection rules which are independent of the processing rates. To do so, it is convenient to attach times to places, instead of attaching firing rates to transitions, as in the standard continuous model, see [3].

The continuous dynamics of our class of Petri nets is piecewise linear, discontinuous. The nonlinearity arises from minimization operations corresponding to synchronization and priority configurations. We show that the continuous dynamics can be expressed in terms of *policies*. A policy is a map associating with every transition one of its upstream places. In this way, the dynamics of the Petri net can be written as an infimum of the dynamics of subnets induced by the different policies. The policies reaching the infimum indicate the places which are bottleneck in the Petri net. On any time interval in which a fixed policy reaches the infimum, the dynamics reduces to a linear dynamics.

Our main theorem provides a characterization of the stationary solutions in terms of the policies of the net. This allows us to set up a correspondence between the (ultimately affine) stationary solutions of the discrete dynamics that

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were described in [1] and the stationary solutions of the continuous dynamics. In particular, the stationary throughputs are proven to be the same for well-chosen initial markings.

An important problem is to relate the stationary flow to the initial marking. Difficulties arise from the nonmonotonicity of the dynamics (due to the priority routing rule). We present a partial result relating the continuous stationary solutions to the initial marking of the Petri net, under the assumption that the trajectory is associated with a constant policy. This result requires the semi-simplicity of the eigenvalue 0 of a matrix depending on the policy.

We finally provide some numerical simulations of the continuous dynamics. Our case of study is a model of emergency call center with two hierarchical levels for handling calls, taken from [1]. On this Petri net, numerical experimentations are led using the SpaceEx verification tool [2], which provides certified bounds on the trajectory of hybrid automata. Results illustrate the convergence of the trajectory towards the stationary solution, so that the pathological behavior observed in the discrete setting vanishes. We leave it for further work to see under which generality the convergence to the stationary solution can be established.

CCS Concepts

•Networks → *Network performance modeling*; •Theory of computation → *Timed and hybrid models*;

Keywords

Continuous Petri nets; Timed discrete event systems

1. REFERENCES

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