Polynomial Mean-Centric Crossover for Directed Mating in Evolutionary Constrained Multi-Objective Continuous Optimization

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ABSTRACT

This paper proposes a mean-centric crossover to improve the effectiveness of the directed mating utilizing useful infeasible solutions in evolutionary constrained multi-objective continuous optimization. The directed mating selects a feasible solution as the first parent and a solution dominating the first parent in the objective space from the population involving infeasible solutions as the second parent. Since infeasible solutions having better objective values than feasible ones have useful variables, it helps to improve the search performance. So far, the commonly used simulated binary crossover (SBX) have been employed to generate offspring from two parents selected by the directed mating. However, it is not clear that the commonly used SBX is appropriate also for parents selected by the directed mating. When the Pareto front exists on the boundary between the feasible and the infeasible regions in the variable space, a mean-centric crossover generating offspring around intermediate area of two parents would be more effective than SBX which is a parent-centric crossover generating offspring around two parents. This work proposes the polynomial mean-centric crossover (PMCX) and combines it with the directed mating. The experimental results show that the proposed PMCX achieves higher search performance than SBX on several test problems.

CCS Concepts

•Computing methodologies → Optimization algorithms; •Artificial intelligence → Search methodologies;

Keywords

constrained multi-objective optimization; evolutionary algorithms; utilization of infeasible solutions; crossover operator

1. INTRODUCTION

Real-world optimization problems often involve multiple conflicting objectives and become multi-objective optimization problems (MOPs). The goal of MOPs is to approximate the Pareto front, the optimal tradeoff among objectives, with a set of solutions. Evolutionary algorithms are suited to solve MOPs since they use a solution set called the population and obtain multiple solutions to approximate the Pareto front from the population in a single run. Therefore, multi-objective evolutionary algorithms (MOEAs) have been intensively studied [1, 2] and successfully applied to real-world optimization problems so far [3]. Also, real-world optimization problems often involve several constraint conditions and become constrained MOPs (CMOPs). For solving CMOPs, we need to optimize the multiple objectives of solutions while satisfying the multiple constraints. To handle generated infeasible solutions not satisfying constraints during the search, several approaches have been studied and proposed so far [4, 5, 6, 7, 8]. To solve CMOPs, since infeasible solutions may have useful variable information to enhance the search, it is important to utilize them in the search to improve the search performance. As a MOEA employing this concept, this work focuses on TNSDM (Two-Stage Non-dominated Sorting and Directed Mating) algorithm [9] using the solution ranking based on both constraint violation values and objective values and the mating to utilize infeasible solutions useful for the search.

General MOEAs just discard infeasible solutions from the population, and they are not able to become parents since they do not satisfy constraint conditions. On the other hand, the directed mating in TNSDM selects a feasible solution as the primary parent and a solution dominating the primary parent in the objective space from the entire population involving infeasible solutions as the secondary parent. The previous work showed that the parent selection in the directed mating improves the search performance by enhancing the convergence of solutions toward the Pareto front by utilizing useful variable information of infeasible solutions on several discrete and continuous CMOPs [9]. In our previous studies about the directed mating [9, 10, 11], we have focused on ways to select parents so far. Therefore, to generate offspring from the selected parents, we just employ commonly used crossover and mutation operators. Concretely, we have used the simulated binary crossover (SBX) [12] for continuous CMOPs. However, it is not clear that whether the commonly used SBX is useful also for parents selected by the directed mating or not. In other words, little is known about appropriate crossover operator for parents selected by the directed mating.

This work focuses on continuous CMOPs and studies about crossover operators appropriate for parents selected by the directed mating. Since the conventional SBX is a parent-centric crossover, offspring are distributed around two parents with high probability in the variable space. For CMOPs having the Pareto front on the boundary between the feasible and the infeasible regions in the variable space, a mean-centric crossover generating offspring around intermediate area of two parents in the variable space would enhance the search for the Pareto front. This work proposes the polynomial mean-centric crossover (PMCX) using two parents and the similar probability density function to SBX and the polynomial mutation [12], and verify its effectiveness of the directed matings on several continuous test CMOPs.

2. EVOLUTIONARY CONSTRAINED MULTI-OBJECTIVE OPTIMIZATION

2.1 Constrained Multi-objective Optimization Problems

Constrained MOPs (CMOPs) are concerned with finding solutions $\boldsymbol{x}$ minimizing m objective functions f_i $(i = 1, 2, \ldots, m)$ subject to satisfy k constraint functions g_j $(j = 1, 2, \ldots, k)$. CMOPs are defined as

$$\begin{cases} \text{Minimize} & f_i(\boldsymbol{x}) \quad (i = 1, 2, \ldots, m) \\ \text{subject to} & g_j(\boldsymbol{x}) \geq 0 \quad (j = 1, 2, \ldots, k) \end{cases} \tag{1}$$

Solutions satisfying all k constraints are said to be *feasible*, and solutions not satisfying all k constraints are said to be *infeasible*. The constraint violation vector $\boldsymbol{v}(\boldsymbol{x})$ is defined as

$$v_j(\boldsymbol{x}) = \begin{cases} |g_j(\boldsymbol{x})|, & \text{if } g_j(\boldsymbol{x}) < 0 \\ 0, & \text{otherwise} \end{cases} \quad (j = 1, 2, \ldots, k) \tag{2}$$

In MOPs, generally, there is not an ideal solution minimizing all m objective functions due to the trade-off among objectives. Therefore, the concept of *Pareto dominance* is introduced. *Pareto dominance* between $\boldsymbol{x}$ and $\boldsymbol{y}$ is defined as follows: If

$$\forall i : f_i(\boldsymbol{x}) \leq f_i(\boldsymbol{y}) \ \wedge \ \exists i : f_i(\boldsymbol{x}) < f_i(\boldsymbol{y}) \ (i = 1, 2, \ldots, m) \tag{3}$$

is satisfied, $\boldsymbol{x}$ dominates $\boldsymbol{y}$ on objective function values, which is denoted by $\boldsymbol{x} \succ_f \boldsymbol{y}$ in the following. Also, a feasible solution $\boldsymbol{x}$ not dominated by any other feasible solution is said to be a non-dominated solution. The set of non-dominated solutions in the solution space is called Pareto optimal solutions (POS), and the trade-off among objective functions represented by POS in the objective space is called Pareto front. The goal of solving MOPs with MOEAs is to approximate the Pareto front with non-dominated solutions obtained during the solution search.

2.2 Constraint-handling in MOEAs

When we solve CMOPs by using evolutionary algorithms, infeasible solutions are generated during the search. To handle infeasible solutions in evolutionary algorithms, the death penalty approach discarding all infeasible solutions from the population, the repair approach transforming infeasible solutions into feasible ones by using previous knowledge of each optimization problem, and the penalty approach modifying objective function values based on constraint violation values have been studied so far [4, 5, 6]. Also, there is the comprehensive solution ranking approach evolving infeasible solutions to feasible ones based on solutions ranking individually considering the constraint violation values and the objective values [4, 5, 6]. From the last approach, this work focuses on TNSDM algorithm using the two-stage non-dominated solution ranking and the directed mating utilizing the useful variable information of infeasible solutions [9].

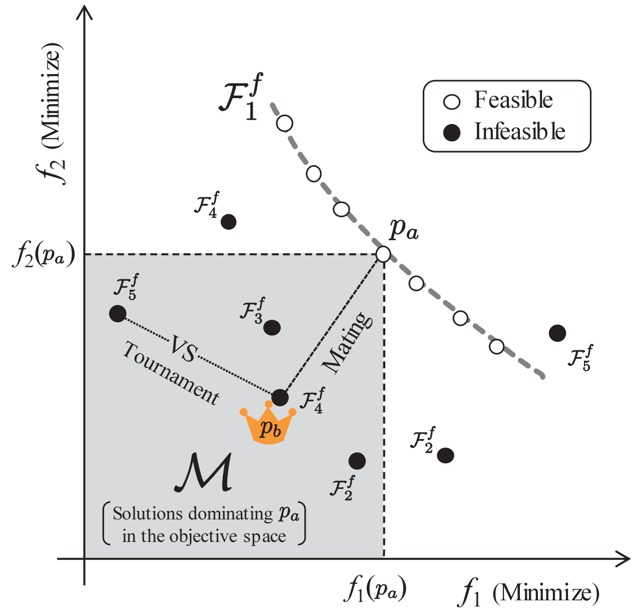


Figure 1: Conceptual figure of directed mating

3. TNSDM

TNSDM (Two-stage Non-dominated Sorting and Directed Mating) introduces Two-stage non-dominated sorting to rank solutions based on constraint violation values and objective function values and Directed mating to enhance convergence toward the Pareto front. TNSDM is designed based on the framework of NSGA-II [7]. The entire population $\mathcal{R}$ consists of the parent population $\mathcal{P}$ and the offspring population $\mathcal{Q}$, i.e. $\mathcal{R} = \mathcal{P} \cup \mathcal{Q}$.

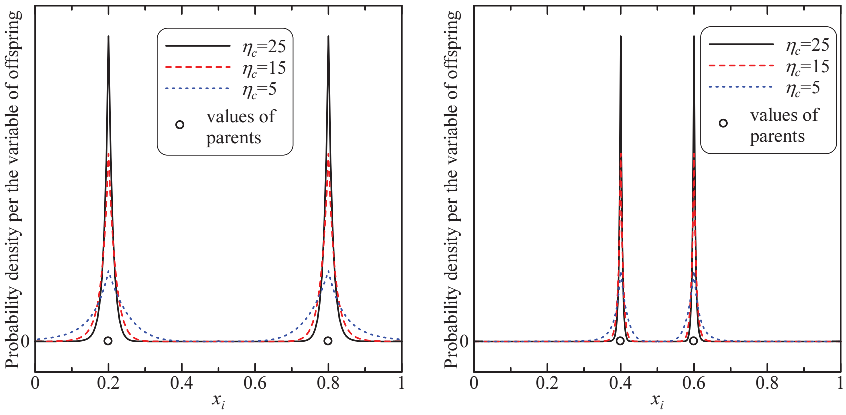
3.1 Two-Stage Non-dominated Sorting

Firstly, TNSDM classifies $\mathcal{R}$ into several fronts $\mathcal{F}_1^f, \mathcal{F}_2^f, \ldots$ by using the two-stage non-dominated sorting based on constraint violation values v_j $(j = 1, 2, \ldots, k)$ and objective function values f_i $(i = 1, 2, \ldots, m)$. As the result, upper front $\mathcal{F}_i^f$ with small index i includes solutions having lower constraint violation values and higher objective function values.

Next, the half of $\mathcal{R}$ is selected as a parent population $\mathcal{P}$ from upper fronts while simultaneously considering the crowding distance (CD) [7].

3.2 Directed Mating

A conceptual figure of directed mating is shown in **Fig. 1**. In this figure, solutions in the entire population $\mathcal{R}$ are distributed in the objective space and classified into $\mathcal{F}_1^f, \mathcal{F}_2^f, \ldots, \mathcal{F}_5^f$. Solutions belonging to $\mathcal{F}_1^f$ are selected as the parent population $\mathcal{P}$ since they are upper half solutions in term of the front rank. To generate one offspring, first, a primary parent $\boldsymbol{p_a}$ is selected from the parent population $\mathcal{P}$ by tournament selection. In the tournament, two solutions are randomly chosen from $\mathcal{P}$, and the solution belonging to the upper front (with a lower front index number) becomes parent $\boldsymbol{p_a}$. If both of them belong to the same front, the solution having a larger crowding distance (CD) [7] becomes parent $\boldsymbol{p_a}$. Next, we pick a set of candidate solutions $\mathcal{M}$ $(= \{\boldsymbol{x} \in \mathcal{R} \mid \boldsymbol{x} \succ_f \boldsymbol{p_a}\})$ dominating $\boldsymbol{p_a}$ in the objective space from the entire population $\mathcal{R}$ including infeasible solutions. If the primary parent $\boldsymbol{p_a}$ is feasible and the



(a) Distant parents

(b) Close parents

Figure 2: Probability density of variable values of offspring generated by SBX

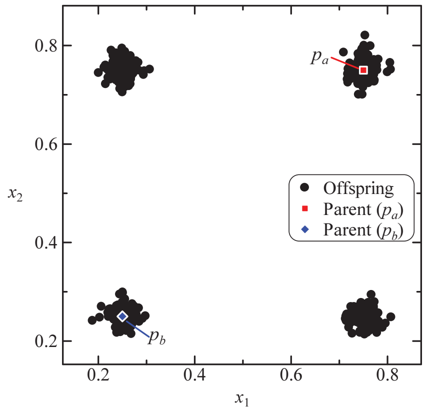
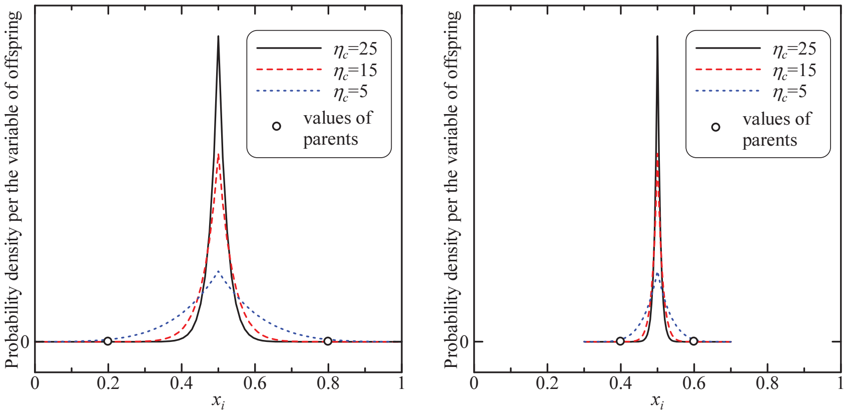


Figure 3: Offspring distribution by SBX in the two dimensional variable space



(a) Distant parents

(b) Close parents

Figure 4: Probability density of variable values of offspring generated by PMCX

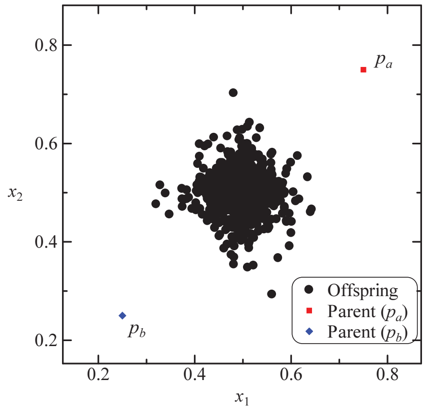


Figure 5: Offspring distribution by PMCX in the two dimensional variable space

number of solutions in $\mathcal{M}$ is more than or equal to two ($|\mathcal{M}| \geq 2$), the directed mating is performed. Otherwise, both parent selected from the parent population $\mathcal{P}$ and it is called conventional mating in this paper.

In **Fig. 1**, four solutions dominating the primary parent p_a are candidate solutions $\mathcal{M}$. Solutions belonging to $\mathcal{F}_5^f$ and $\mathcal{F}_4^f$ respectively is randomly chosen from $\mathcal{M}$ and then compare their front index numbers. As the result, the solution belonging to $\mathcal{F}_4^f$ is selected as a secondary parent p_b. Secondary parents p_b selected by directed mating are mainly infeasible solutions. However, since it dominates the primary parents p_a in the objective space, there is a possibility that $\boldsymbol{p_b}$ has valuable genetic information to enhance the convergence of primary $\boldsymbol{p_a}$ toward Pareto front.

3.3 Simulated Binary Crossover (SBX) [12]

In our previous studies of TNSDM, we have employed the simulated binary crossover (SBX) commonly used in MOEA community to generate offspring from parents. SBX varies each variable values of two parents and then copies each variable value from primary parent or secondary parent with the equal probability to their offspring.

For each variable i, first we generate a random number $u_i \in [0, 1]$ and calculate β_i $(i = 1, 2, \ldots, n)$ as

$$\beta_i = \begin{cases} (2u_i)^{\frac{1}{\eta_c+1}} & \text{if } u_i \leq 0.5, \\ (\frac{1}{2(1-u_i)})^{\frac{1}{\eta_c+1}} & \text{otherwise,} \end{cases} \tag{4}$$

where, η_c is the user-defined distribution parameter. Next, for each variable i, we respectively vary variable values $x_i^{p_a}$ and $x_i^{p_b}$ of two parents p_a and p_b with β_i as

$$\begin{aligned} x_i'^{p_a} &= 0.5\{(1+\beta_i)x_i^{p_a} + (1-\beta_i)x_i^{p_b}\}, \\ x_i'^{p_b} &= 0.5\{(1-\beta_i)x_i^{p_a} + (1+\beta_i)x_i^{p_b}\}. \end{aligned} \tag{5}$$

Finally, we exchange each variable of $\boldsymbol{x}'^{p_a}$ and $\boldsymbol{x}'^{p_b}$ with a probability of 0.5, and the exchanged variable vector become offspring.

In SBX, the amount of variation in each variable value is stochastically determined with the distance between two parents in the variable space and the distribution parameter η_c. **Fig. 2** shows examples of probabilistic density of offspring's variable value. **Fig. 2 (a)** shows the case of distant parents, and **Fig. 2 (b)** shows the case of close parents. Also, both figures involve three probabilistic densities with different distribution parameters $\eta_c = \{5, 15, 25\}$. From the example, we can see that offspring tends to be distributed close to parents when two parents are close and a large distribution parameter η_c is used.

Furthermore, to observe the effects of variable exchange mechanism in SBX, we conduct a simple experiment to show the distribution of offspring generated by SBX in a two-dimensional variable space. In the experiment, two parents $\boldsymbol{x}^{p_a} = (0.75, 0.75)$ and $\boldsymbol{x}^{p_b} = (0.25, 0.25)$ are used and repeatedly generates 1,000 offspring by SBX with a distribution parameter $\eta_c = 15$. **Fig. 3** shows the result of the simple experiment. From the result, we can see that offspring generated by SBX are distributed around parents or area combining the parents' variable values.

4. FOCUS ISSUE: APPROPRIATE CROSSOVER FOR DIRECTED MATING

In our previous work about the directed mating, we have focused on only selection methods of useful infeasible solutions [9, 10, 11]. After the selection of a pair of infeasible and feasible parents by the directed mating for solving continuous CMOPs, we have used SBX to generate offspring. Although SBX is a commonly used crossover operator for crossing feasible parents especially in general MOEAs, it might be not appropriate for feasible and infeasible parents selected by the directed mating.

In CMOPs having the Pareto front lying on a boundary between infeasible region and feasible one, it can be expected that the Pareto optimal solutions exist between an infeasible solution and a feasible one selected by the directed mating. However, since SBX is a parent centric crossover, offspring tend to be distributed around each parent with high probability, and we cannot expect to obtain offspring intermediate area of two parents by SBX. Especially for parents selected by the directed mating, a mean-centric crossover generating offspring in intermediate area of two parents in the variable space would be effective to search the Pareto optimal solutions. As mean-centric crossovers, the unimodal normally distributed crossover (UNDX) [14] and the simplex crossover (SPX) [15] seem to fit our purpose in this work, however they use more than three parents to generate one offspring. Since the directed mating select two solutions as parents, we cannot easily combine them with the directed mating. Therefore, we will study a novel mean-centric crossover operator using two parents and generating offspring in intermediate area of parents with high probability to improve the effectiveness of the directed mating. In other words, we study a crossover operator to promote the further utilization of the useful genetic information of infeasible solutions.

5. PROPOSED METHOD: POLYNOMIAL MEAN-CENTRIC CROSSOVER

5.1 Overview

To improve the effectiveness of the directed mating and the search performance on continuous CMOPs, In this work, we propose a crossover operator appropriate for parents selected by the directed mating. The proposed crossover is a mean-centric crossover and generates offspring in intermediate area of two parents in the variable space with high probability. The probabilistic density function is designed based on the polynomial mutation [16] and SBX. We call the proposed crossover operator the polynomial mean-centric crossover (PMCX).

5.2 Method

For each variable i, first we generate a random number $u_i \in [0, 1]$ and calculate δ_i $(i = 1, 2, \ldots, n)$ as

$$\delta_i = \begin{cases} (2u_i)^{\frac{1}{\eta_c+1}} - 1 & \text{if } u_i > 0.5, \\ 1 - \{2(1-u_i)\}^{\frac{1}{\eta_c+1}} & \text{otherwise,} \end{cases} \quad (6)$$

where, η_c is the user-defined distribution parameter. Next, for each variable i, we calculate variable value x_i^o $(i = 1, 2, \ldots, n)$ of offspring $\boldsymbol{o}$ as

$$x_i^o = \frac{x_i^{p_a} + x_i^{p_b}}{2} + \delta_i(|x_i^{p_a} - x_i^{p_b}|), \quad (7)$$

where, $x_i^{p_a}$ and $x_i^{p_b}$ are i-th variable values of two parents p_a and p_b, respectively.

Fig. 4 shows examples of probabilistic density of offspring's variable value in cases with different distance between two parents and η_c. **Fig. 4 (a)** shows the case of distant parents, and **Fig. 4 (b)** shows the case of close parents. Also, both figures involve three probabilistic densities with different distribution parameters $\eta_c = \{5, 15, 25\}$. From the example, we can see that offspring is distributed close to the mean of variable values of two parents. Also, we can see the probabilistic density is changed by the distance between two parents ${x_i}^{p_a}$ and ${x_i}^{p_b}$. Offspring distribution close to the mean of two parents is emphasized when the distance between two parents is short and a large distribution parameter η_c is used. Thus, although the probabilistic density of the proposed PMCX is derived from the polynomial mutation [16], note that the probabilistic density of the proposed PMCX is varied by the distance between two parents in the variable space even if the same distribution parameter η_c is used while the probabilistic density of the polynomial mutation is always constant.

5.3 Expected Effects

In the same manner as **Fig. 3**, we conduct a simple experiment to show the distribution of offspring generated by PMCX in a two-dimensional variable space. In the experiment, we use two parents $\boldsymbol{x}^{p_a} = (0.75, 0.75)$ and $\boldsymbol{x}^{p_b} = (0.25, 0.25)$ and repeatedly generate 1,000 offspring by PMCX with a distribution parameter $\eta_c = 15$. **Fig. 5** shows generated offspring and the parents in the variable space. From the result, we can see that offspring generated by PMCX are distributed around intermediate area between the two parents. In CMOPs having the Pareto front lying on the boundary between infeasible region and feasible one in the variable space, it is expected that the directed mating with the proposed PMCX enhances search the Pareto front effectively since the Pareto front would exist between two feasible and infeasible parents.

6. EXPERIMENTAL SETUP

6.1 Algorithms

To verify effects of the proposed PMCX used with the directed mating, this work compares search performances of two algorithms. The one is TNSDM algorithm with SBX, and the other is TNSDM algorithm with the proposed PMCX.

Note that the proposed PMCX is performed only for parents selected by the directed mating in this work. In TNSDM, if candidate solutions of secondary parent dominating each primary parent is less than two, the conventional mating selecting both parents from the parent population $\mathcal{P}$ is performed. In this case, SBX is performed to generate offspring in both algorithms.

6.2 Benchmark Problems

In this work, we use continuous function optimization problems, TNK [17], OSY [18] and mCDTLZ [11] as benchmark problems.

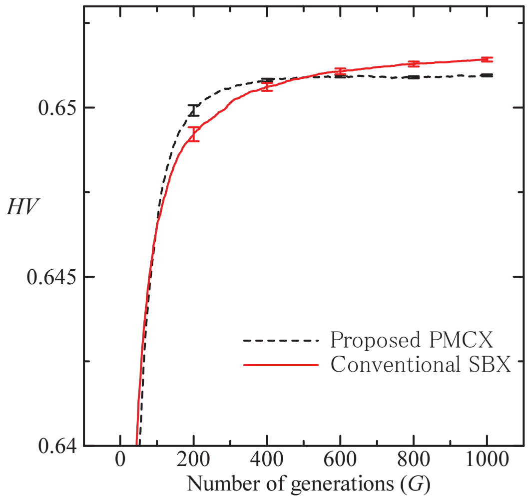


Figure 6: Transitions of HV on TNK

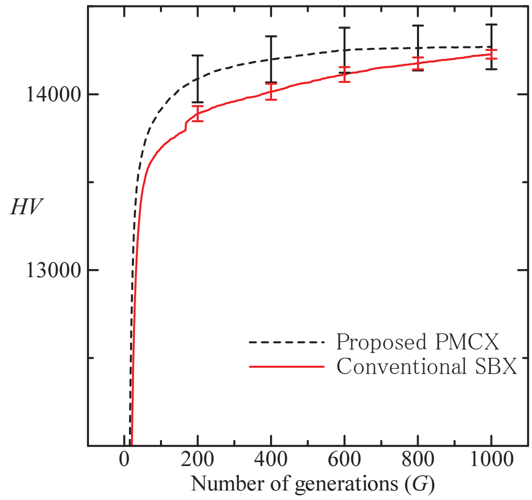


Figure 7: Transitions of HV on OSY

The Pareto fronts of these problems lying on the boundary between the feasible region and the infeasible one. mCDTLZ has scalability to vary correlation complexity between the objective space and the variable one by problem parameters.

TNK

TNK [17] is a minimization problem involving two objectives and two constraints. The Pareto front is discontinuous. TNK is defined as follows.

$$\begin{cases} \text{Minimize } f_1(\boldsymbol{x}) = x_1 \\ \qquad\qquad f_2(\boldsymbol{x}) = x_2 \\ \text{Subject to } g_1(\boldsymbol{x}) = x_1^2 + x_2^2 - 1 \\ \qquad\qquad -0.1\cos(16\arctan\frac{x_1}{x_2}) \geq 0 \\ \qquad\qquad g_2(\boldsymbol{x}) = (x_1 - 0.5)^2 + (x_2 - 0.5)^2 \leq 0.5. \end{cases} \tag{8}$$

A solution (variable vector) $\boldsymbol{x}$ consists of two variables (x_1, x_2), and both variables are real parameters in the range $[0, \pi]$.

OSY

OSY [18] is a minimization problem involving two objectives and six constraints. OSY is defined as follows.

$$\begin{cases} \text{Minimize } f_1(\boldsymbol{x}) = -[25(x_1-2)^2 + (x_2-2)^2 \\ \qquad\qquad +(x_3-1)^2 + (x_4-4)^2 + (x_5-1)^2] \\ \qquad\qquad f_2(\boldsymbol{x}) = {x_1}^2 + {x_2}^2 + {x_3}^2 + {x_4}^2 + {x_5}^2 + {x_6}^2 \\ \text{Subject to } g_1(\boldsymbol{x}) = x_1 + x_2 - 2 \geq 0 \\ \qquad\qquad g_2(\boldsymbol{x}) = 6 - x_1 - x_2 \geq 0 \\ \qquad\qquad g_3(\boldsymbol{x}) = 2 - x_2 + x_1 \geq 0 \\ \qquad\qquad g_4(\boldsymbol{x}) = 2 - x_1 - 3x_2 \geq 0 \\ \qquad\qquad g_5(\boldsymbol{x}) = 4 - (x_3-3)^2 - x_4 \geq 0 \\ \qquad\qquad g_6(\boldsymbol{x}) = (x_5-3)^2 + x_6 - 4 \geq 0 \end{cases} \tag{9}$$

A solution (variable vector) $\boldsymbol{x}$ consists of six variables $(x_1, x_2, \ldots, x_6)$, and all the variables are real parameters. The value ranges of variable values are $0 \leq x_1, x_2, x_6 \leq 10$, $1 \leq x_3, x_5 \leq 5$ and $0 \leq x_4 \leq 6$.

mCDTLZ

mCDTLZ (modified constrained DTLZ) [11] is a minimization problems involving m objectives and m constraints. mCDTLZ is defined as follows.

$$\begin{cases} \text{Minimize } f_i(\boldsymbol{x}) = \frac{1}{\lfloor \frac{n}{m} \rfloor} \sum_{l=\lfloor (i-1)\frac{n}{m} \rfloor}^{\lfloor i\frac{n}{m} \rfloor} x_l^{\alpha} \\ \text{Subject to } g_i(\boldsymbol{x}) = f_i(\boldsymbol{x})^2 + 4\sum_{l=1, l\neq i}^{m} f_l(\boldsymbol{x})^2 - 1 \geq 0 \\ \qquad\qquad (i = 1, 2, \ldots, m) \end{cases} \tag{10}$$

A solution (variable vector) $\boldsymbol{x}$ consists of n variables $(x_1, x_2, \ldots, x_n)$, and all the variables are real parameters in the range $[0, 1]$. Each bound $g_i = 0$ $(i = 1, 2, \ldots, m)$ becomes a part of Pareto front. In this problem, the numbers of objectives, constraints and variables are user-defined parameters. However, the number of constraints is equivalent to the number of objectives. Note that mCDTLZ defined in the literature [11] is a case with $\alpha = 0.5$. The exponent is parameterized as the problem parameter α in this work. Solution distribution bias toward the central area of the objective space is emphasized by decreasing α. Also, the number of variables n and the problem parameter α influence the correlation between the objective and the variable space.

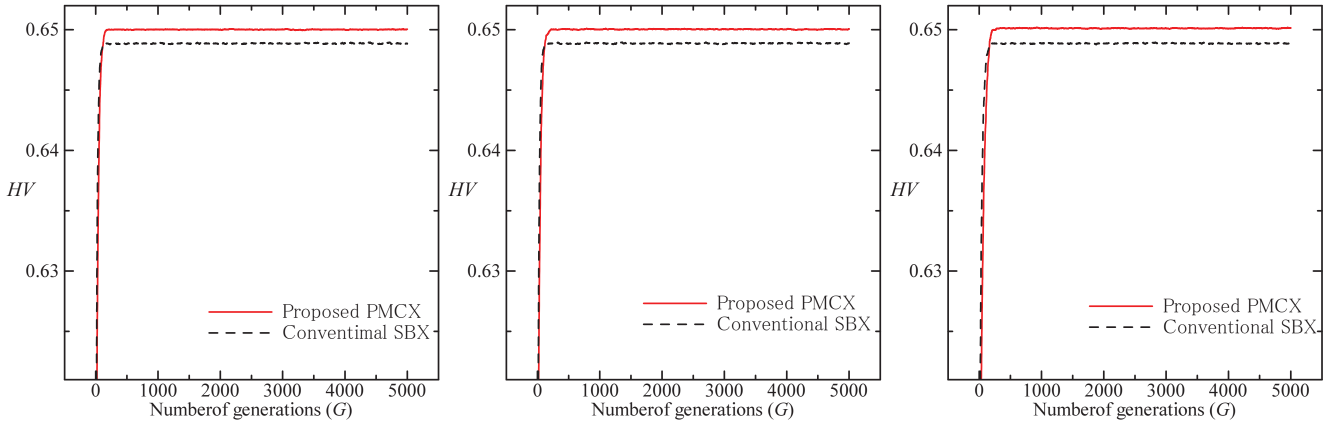
6.3 Parameters

This work uses mCDTLZ problems with $m = 2$ objectives (constraints), $n = \{2, 4, 6, 8, 10, 12\}$ variables and the problem parameters $\alpha = \{1.00, 0.75, 0.50\}$.

As genetic parameters, for both the proposed PMCX and SBX, the crossover ratio is set to $P_c = 0.8$, and the distribution parameter is set to $\eta_c = 15$. After crossed variables, we also use the polynomial mutation with the mutation rate $P_m = 0.2$ and the distribution parameter $\eta_m = 20$. As the termination criterion of algorithms, the total number of generations is set to $T = \{1000, 1000, 5000\}$ for TNK, OSY and mCDTLZ, respectively. The population size is set to $|\mathcal{R}| = 200$ $(|\mathcal{P}| = |\mathcal{Q}| = 100)$. Experimental results are shown by the average (mean) values of 100 runs.

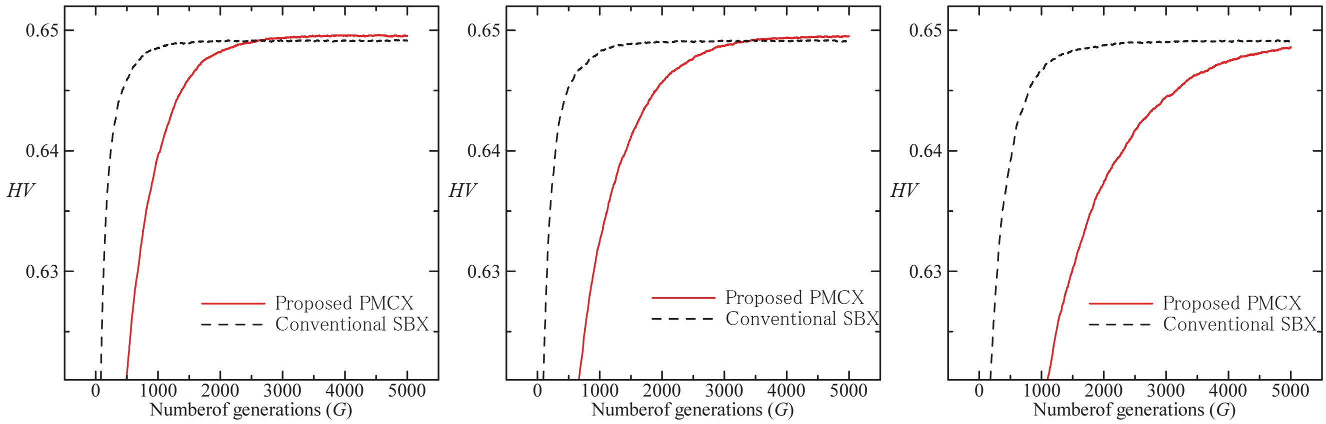
6.4 Performance Metric

To evaluate the obtained non-dominated set of solutions, we use Hypervolume (HV) [19]. HV measures m-dimensional volume



(a) $\alpha = 1.00$ (b) $\alpha = 0.75$ (c) $\alpha = 0.50$

Figure 8: Transitions of HV on mCDTLZ problems with $n = 2$ variables



(a) $\alpha = 1.00$ (b) $\alpha = 0.75$ (c) $\alpha = 0.50$

Figure 9: Transitions of HV on mCDTLZ problems with $n = 8$ variables

covered by obtained non-dominated set and a reference point $\boldsymbol{r}$ in the objective space. Higher HV values denote better search performance in term of both the convergence and the diversity of obtained solutions toward the Pareto front. We use $\boldsymbol{r} = (1.2, 1.2)$ for TNK, $\boldsymbol{r} = (-30, 80)$ for OSY and $\boldsymbol{r} = (1.1, 1.1)$ for mCDTLZ.

7. RESULTS AND DISCUSSION

7.1 Results on TNK

First, we observe the results on TNK problem. **Fig. 6** shows transitions of HV obtained by TNSDM with proposed PMCX and TNSDM with SBX, and error bars in every 200 generations indicate the 95% confidence intervals. From the results, we can see that TNSDM with the proposed PMCX shows lower HV than one with SBX from about 100 to 500 generations. However, TNSDM with the proposed PMCX achieves higher HV than TNSDM with SBX after about 500 generations. In TNK problem, since the objective space and the variable space are matched, offspring generation in intermediate area of two parents selected by the directed mating is effective to enhance the search the Pareto optimal solutions, and it contributes to the search performance improvement.

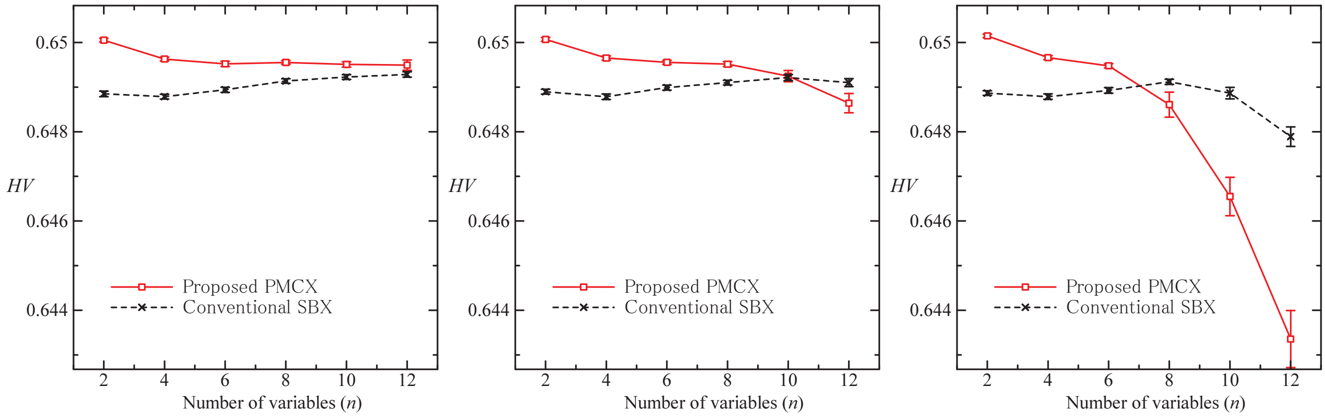
7.2 Results on OSY

Next, **Fig. 7** shows the transitions of HV obtained by TNSDM with proposed PMCX and TNSDM with SBX, and error bars in every 200 generations indicate the 95% confidence interval. From the reusults, we can see that TNSDM with proposed PMCX is inferior to TNDM with SBX in the view point of mean HV value, though the difference is not statistically significant at the final generation. Thus, the effectiveness of the proposed PMCX cannot be observed on OSY problem.

7.3 Results on mCDTLZ

Finally, we observed results on mCDTLZ problems. **Fig. 8** and **9** show the transitions of HV on mCDTLZ problems with $n = \{2, 8\}$ variables and the problem parameters $\alpha = \{1.00, 0.75, 0.50\}$.

From the results on problems with $n = 2$ variables shown in **Fig. 8**, we can see that both HV values obtained by the conventional SBX and the proposed PMCX are converged in early generations, and the proposed PMCX achieves higher HV than the conventional SBX. Also, we can see that the transitions of HV is not effected by decreasing the problem parameter α since these problems has $n = 2$ variables, the objective space and the variable space are matched, and these problems are relatively easy to op-



(a) $\alpha = 1.00$ (b) $\alpha = 0.75$ (c) $\alpha = 0.50$

Figure 10: Results of HV at the final generation on mCDTLZ problems with differnt number of variables n and the parameters α

timize. Next, from the results on problems with $n = 8$ variables shown in **Fig. 9**, we can see that the convergence of HV is slower than the case with $n = 2$ variables shown in **Fig. 8**. The tendency becomes significant especially on the proposed PMCX, and the decrease of α also has influence for it. However, the proposed PMCX achieves higher HV than the conventional SBX at the final generation on problems with $\alpha = \{1.00, 0.75\}$ and obtains solutions to finely approximate the Pareto front.

Next, **Fig. 10** shows the results of HV at the final generation on problems with $n = \{2, 4, 6, 8, 10, 12\}$ variables and the problem parameters $\alpha = \{1.00, 0.75, 0.50\}$. The error bars indicate 95% confidence intervals.

Fig. 10 (a) shows the results on problems with $\alpha = 1.00$. In the case of the problem with $n = 2$ variables, the objective space and the variable space are matched such as TNK problem. The relation between the objective space and the variable space becomes complicated as the number of variables n is increased. From these results, we can see that the proposed PMCX achieves the higher HV than the conventional SBX on problems with $\alpha = 1.00$ and any number of variables n. Also, we can see the tendency that HV achieved by the proposed PMCX is decreased with increasing the number of variables n. On the other hand, we can see that HV achieved by the conventional SBX is increased with increasing the number of variables n. The search difficulty is generally increased with increasing the number of variables n, however, the conventional SBX improves the search performance by increasing the number of offspring distribution areas in the variable space with the variable exchange mechanism employed in SBX as shown in **Fig. 3**. **Fig. 10 (b)** and **(c)** show the results on problems with the problem parameters $\alpha = \{0.75, 0.50\}$, respectively. In mCDTLZ problems, solution distribution bias for the central area of the objective space is strengthen by decreasing the problem parameter α. In problems with small number of variables, we can see that the proposed PMCX achieves higher HV than the conventional SBX. However, HV obtained by the proposed PMCX is deteriorated as decreasing α. As the results, HV of the proposed PMCX is lower than the conventional SBX on several problems with small α. This is because the proposed mean-centric crossover faces difficulty to obtain spread solutions to approximate a wide range of the Pareto front when the solution distribution bias for the central area of the objective space is strengthen as decreasing α.

These results reveal that the convergence of HV obtained by the proposed PMCX is slower than the conventional SBX but the proposed PMCX achieves higher HV than the conventional SBX at the final generation especially on problems having a strong correlation between the objective space and the variable space.

8. CONCLUSIONS

To improve the effectiveness of the directed mating and improve the search performance of TNSDM on continuous CMOPs, in this work, we proposed the crossover method appropriate for parents selected by the directed mating. The proposed polynomial mean-centric crossover (PMCX) generates offspring distributed in the intermediate area between two parents in the variable space with high probability. The mean-centric crossover is able to effectively search the Pareto optimal solutions locates between the feasible and the infeasible regions compared with the conventional parent centric SBX. Experimental results using TNK, OSY and mCDTLZ problems showed that the proposed PMCX achieved higher search performance than conventional SBX on problems having strong correlation between the objective space and the variable space.

Since the proposed PMCX changes variable values of two parents drastically compared with the conventional SBX, useful variable information of two parents selected by the directed mating might be destroyed. To overcome this problem and improve the search performance especially in early stage of optimization, as a future work, we will develop the crossover operator involving both parent mean-centric and parent-centric offspring generations.

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