P. polycephalum Can Compute Shortest Paths*

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P. polycephalum is an interesting example of an organism believed to perform natural optimization. Despite lacking a brain or a nervous system, the slime mold seems well equipped to solve various complex problems. Famously, it has repeatedly demonstrated its ability to produce the shortest path between two food sources in a maze.

To model this behavior one can view *P. polycephalum* as a connected network of flow-carrying tubes with time-dependent capacities [3]. The time evolution of the capacities is determined by the following intuitive assumption: Tubes that carry a large amount of flow accommodate by increasing their capacity over time, whereas tubes that carry a small amount of flow will gradually decrease their capacity.

More formally, let $G = (V, E)$ be a graph describing the pipe network with a node-set V denoting the junction points where different tubes connect and an edge-set E denoting the set of tubes. For each edge $e \in E$ let ℓ_e denote its constant length and x_e its time-dependent capacity. The *evolution equation* formalizes the above notion of how edge capacities change over time:

$$\dot{x}_e = |q_e(r_e)| - x_e \tag{1}$$

Where q_e denotes the flow on edge e as a function of its (ohmic) resistance $r_e = \frac{\ell_e}{x_e}$.

Our work investigates the *evolution equation* in the context of the well-known Shortest-Path problem. Given a graph G and two distinct nodes, the source $s \in V$ and the sink $t \in V$, we prove that the given system (1) converges to the shortest $s - t$ path in the following sense [2]: The capacities of the edges on the shortest $s - t$ path converge to 1 while the capacities of all other edges tend to 0. This result holds true for any graph and any initial conditions for the edges under very mild assumptions.

Furthermore, the *evolution equation* can be discretized, yielding an algorithm guaranteed to compute an ϵ-approximation of the shortest $s - t$ path in polynomial time [1]. Although this algorithm is not competitive in practice, its operation is remarkably similar to the behavior of real *P. polycephalum* as observed in the original maze experiments. Just like the real slime mold, the algorithm keeps closing in on the shortest path by gradually retreating from everywhere else.

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