

Resource Allocation Algorithm for the Downlink of Multi-user OFDM System Based on Fairness

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ABSTRACT

This paper studies the resource allocation algorithm for the downlink of multi-user OFDM system based on fairness. The transmit capacity of the OFDM system is improved according to the requirements of the users' data rate. To reduce the complexity of the algorithm, sub-carriers and power are allocated separately. We allocate the sub-carriers following the principle of proportional fairness. First, assign one sub-carrier for each user, and then allocate the remaining sub-carriers for considering with the users' data rate and the predetermined ratio of users' rate. Power allocated to each user is determined by the number of sub-carriers the user has obtained. And then it is distributed to each sub-carrier in the same user according to the water-filling algorithm. Simulation results show that the algorithm improves the overall rate of the system while ensuring the performance of fairness better compared with the sub-optimal algorithm in [1].

Keywords

OFDM, fairness, data rate, resource allocation algorithm

1. INTRODUCTION

As the wireless communication technology has been used widely in this years, the requirement about its quality of service (QoS) is more and more strict. The

next-generation wireless communication system will develop in the direction of high data rate. Under the limited radio resources and the poor channel conditions, OFDM has become one of the core technology of the future wireless communication because of its significant advantages. OFDM technology is a high-speed transmission technology, suitable for mobile radio channels with multi-path propagation and Doppler shift. It is effective against with multi-path effects and frequency selective fading. Also, it can eliminate inter-symbol interference (ISI) and provide high spectrum efficiency. This technology makes the future multimedia services more flexible and perfect.

In order to make more effective use of limited radio resources, adaptive resource allocation problem of multi-user OFDM system has been studied popularly in recent years. It can be divided into two categories mainly. One is called the margin adaptive (MA) algorithm, as described in [2], [3]. It is in order to minimize the total transmit power with the limit of the minimum users' data rate. The other is to maximize the overall system throughput while meeting each user's minimum data rate and bit error rate (BER) requirement, as in [4]-[10]. It is known as the rate adaptive (RA) algorithm. This article is concerned about RA algorithm which is based on fairness. That is maximizing the total capacity under the requirement of the users' proportional data rate.

In [1], a joint sub-carrier and power algorithm was proposed using water-filling method. Once a sub-carrier is allocated, the total power of water-filling will be updated in accordance with the number of sub-carriers the corresponding user allocated. Then the power each sub-carrier allocated is obtained by water-filling algorithm, and the user's real-time data rate can be calculated out then. The remaining sub-carriers are distributed according to the users' real-time data rates and the predetermined values of users' rate proportion. The algorithm is able to get comparable results with [7], but its power distribution method is much more practical than the water-filling algorithm in [7].

A low-complexity sub-optimal algorithm was

proposed in [12], which distributed sub-carrier and power separately. In the algorithm, sub-carrier allocation is first performed by assuming an equal power distribution, and then each sub-carrier is assigned to the user with the best sub-carrier efficiency function. After sub-carrier allocation, they use the best and linear water-filling to inject power. It is shown to be superior to the traditional power allocation algorithm, while reducing the complexity from exponential to linear in the number of sub-carriers. Differently, a low-complexity worst sub-carrier avoiding (WSA) sub-carrier allocation scheme is proposed in [11], in order to avoid assigning users the sub-carriers experiencing severe fading. This method takes better use of power avoiding to be wasted on those sub-carriers with poor performance.

Another thought is proposed to reduce the signaling overhead in [13]. Same data rate is allocated to all the sub-carriers assigned to a user, while the rates to sub-carriers of different users may differ. It aims to maximize the total weighted data rate with the limited total power. When the channel changes rapidly, this algorithm shows much better performance than the water-filling algorithm.

The proposed algorithm pays more attention to fairness problem. The power distribution draws the method in [1]. Let users' power be proportional to the number of sub-carriers they obtained and power for each sub-carrier belonging to a user is calculated by water-filling algorithm. This method not only reduces the computational complexity, but also ensures the practicality. Sub-carrier allocation takes fairness as an important factor. In the first step, offer each user a sub-carrier based on the required ratio of users' data rate. In the second stage, distribute the remaining sub-carriers considering about the real-time users' data rate and the predetermined rate proportion.

The following article is organized as follows. Section II outlines the model of multi-user downlink OFDM system. In Section III, the new proposed algorithm is described. Section IV focuses on the comparison with other algorithms and the simulation results and analysis are given out. Finally, the conclusion about this article is shown in Section V.

2. SYSTEM MODEL OF MULTI-USER DOWNLINK OFDM SYSTEM

A multi-user downlink OFDM system is shown in figure 1. In the base station, it is assumed that all channel information is known through the feedback channels from all the users. Suppose there are K users and L sub-carriers in the system and all the users are gathered in the same cell. Now a resource allocation algorithm should be found in every L OFDM symbols, while L is determined by the real-time channel condition and the time duration of an OFDM symbol. In a fast time-varying environment L cannot be too large.

At the transmitter, the serial data from K users are fed into the sub-carrier and bit allocation block which allocates bits from different users to different

sub-carriers. Suppose that the bandwidth of each sub-carrier is much smaller than the coherence bandwidth of the channel. Using the channel information, the transmitter applies the resource allocation algorithm to assign different sub-carriers to different users and determines how many bits to be transmitted on each sub-carrier every OFDM symbol. Depending on the number of bits assigned to a sub-carrier, the adaptive modulator will use a corresponding modulation scheme, and the transmit power level will be adjusted according to the combined resource algorithm. The complex symbols at the output of the modulators are transformed into the time domain samples by inverse fast Fourier transform (IFFT). Cyclic extension of the time domain samples, called the guard interval, is then added to ensure orthogonality between the sub-carriers.

The signal passes through different frequency selective fading channels to different users' receiver. At the receiver, the guard interval is removed to eliminate the ISI, and the time samples of the k th user are transformed by the FFT block into modulated symbols. The bit allocation information is used to configure the demodulator while the sub-carrier allocation information is used to extract the demodulated bits from the sub-carriers assigned to the k th user. Then we can get user's data at the rate of R_k bit/OFDM symbol.

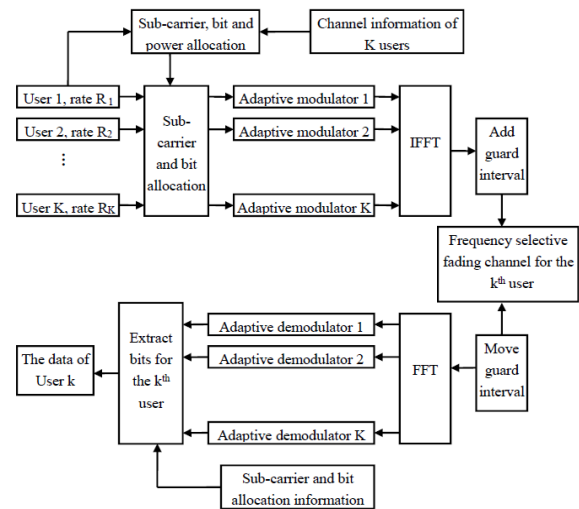


Figure. 1 Block diagram of multi-user OFDM system with sub-carrier, bit, and power allocation.

3. ADAPTIVE RESOURCE ALLOCATION FOR MULTI-USER OFDM SYSTEM

It was proved in [6] and [14] that a multi-user OFDM system can obtain the maximum capacity if each sub-carrier is assigned to the user with the best channel condition and the power it got is calculated by water-filling algorithm in [7]. This is called greedy algorithm. However, as the path loss of each user is all different from each other, the user with higher channel gain will occupy most of the resources, and those who suffer from bad channel gain will get a small part of the resource. Thus, it will not make the users' quality

of service meet the requirements. So we need to find an optimal algorithm to meet all the users' requirements. The objective function we studied in this paper is shown as follows:

$$\max_{\omega_{k,n}, p_{k,n}} R_T = \sum_{k=1}^K \sum_{n=1}^N \omega_{k,n} r_{k,n} = \sum_{k=1}^K \sum_{n=1}^N \omega_{k,n} \log_2 \left(1 + \frac{\gamma_{k,n}}{\Gamma} \right) \quad (1)$$

Subject to

$$\begin{aligned} C1: & \omega_{k,n} \in \{0,1\}, \forall k, n \\ C2: & \sum_{k=1}^K \omega_{k,n} = 1, \forall n \\ C3: & p_{k,n} \geq 0, \forall k, n \\ C4: & \sum_{k=1}^K \sum_{n=1}^N \omega_{k,n} p_{k,n} \leq P_{total}, \\ C5: & R_1 : R_2 : \dots : R_K = \alpha_1 : \alpha_2 : \dots : \alpha_K \end{aligned} \quad (2)$$

$\omega_{k,n}$ is the sub-carrier allocation index. It can only be either 1 or 0, indicating whether sub-carrier n is used by user k or not. It shows that each sub-carrier can only be used by one user. N_0 is the power spectral density of AWGN. Assume that the fading each user experienced is not related and $g_{k,n}$ is the channel gain for user k in sub-carrier n . Let the corresponding channel-to-noise ratio (CNR) be

$$h_{k,n} = \frac{g_{k,n}^2}{N_0 B / N} \quad (3)$$

SNR for the k th user in sub-carrier n can be described as $\gamma_{k,n} = p_{k,n} h_{k,n}$, where $p_{k,n}$ is the power for sub-carrier n of user k . When we use a square M-level QAM with Gray bit mapping, the transmission bit error rate (BER) is

$$BER_{MQAM}(\gamma_{k,n}) \approx 0.2 \exp\left(\frac{-1.6\gamma_{k,n}}{2^{r_{k,n}} - 1}\right) \quad (4)$$

Then we can get the number of bit in sub-carrier n for user k as:

$$r_{k,n} = \log_2 \left(1 + \frac{\gamma_{k,n}}{\Gamma} \right) = \log_2 \left(1 + \frac{p_{k,n} h_{k,n}}{\Gamma} \right) \quad (5)$$

Where Γ is the power loss which is the difference between the SNR needed to achieve a certain data transmission rate for a practical system and the theoretical limit called a constant SNR gap. It is determined by the target BER, the coding gain and the performance of system:

$$\Gamma = -\frac{\ln(5BER)}{1.6} \quad (7)$$

In (2), C4 shows there is a power constraint P_{total} on the total transmit power of the system. C5 means that each users' data rate should be proportional to ensure the users' fairness. where $\{\alpha_i\}_{i=1}^K$ is a set of

predetermined values that are used to ensure proportional fairness among users.

$$R_k = \sum_{n=1}^N \omega_{k,n} r_{k,n} = \sum_{n=1}^N \omega_{k,n} \log_2 \left(1 + \frac{\gamma_{k,n}}{\Gamma} \right) \quad (8)$$

Due to the complexity of resource allocation algorithm under the proportional fairness constraint is very high, we need to find a less complex and as close to the optimal solution algorithm. To reduce the complexity of the algorithm, assume that the power allocated to each user is proportional to the number of sub-carriers assigned to it. The specific steps of the algorithm are described as follows:

Initialization:

$$\Omega = \{1, 2, \dots, N\}, \quad \Omega_k = \emptyset, \quad \forall k, k \in \{1, 2, \dots, K\}$$

Step 1 Pre-assignment: sort the user in descending order according to the data rate proportion to let the user with a higher rate requirement choose a sub-carrier first. Assume that $\alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_{K-1} \geq \alpha_K$,

then we should distribute a sub-carrier to user from 1 to K in turn :

for $k = 1$ to K

$$n_k = \operatorname{argmax}_{n \in \Omega} h_{k,n}$$

$$\Omega_k = \{n_k\} \cup \Omega_k$$

$$\Omega = \Omega - \{n_k\}$$

$$\omega_{k,n} = 1$$

$$R_k = \log_2 \left(1 + \frac{(P_{total}/N) h_{k,n}}{\Gamma} \right)$$

Step 2 Allocate the remaining sub-carriers according:

$$a) \text{ When } \Omega \neq \emptyset, \text{ find } i = \operatorname{argmin}_{i \in \{1, 2, \dots, K\}} R_i / \alpha_i$$

$$b) j_i = \operatorname{argmax}_{n \in \Omega} h_{i,n}, \quad \Omega_i = \{j_i\} \cup \Omega_i, \quad \Omega = \Omega - \{j_i\},$$

$$\omega_{i,j_i} = 1, \quad p_i = p_i + P_{total}/N$$

$$c) R_i = \sum_{j \in \Omega_i} \omega_{i,j} \log_2 \left(1 + \frac{p_{i,j} h_{i,j}}{\Gamma} \right), \text{ where } p_{i,j} \text{ is}$$

determined by the water-filling method with the total

$$\text{water power } P_i: \quad p_{i,j} = \frac{1}{m_i} \left(p_i + \sum_{j \in \Omega_i} \frac{1}{h_{i,j}} \right) - \frac{1}{h_{i,j}}$$

m_i is the number of sub-carriers allocated to user i .

d) If $\Omega \neq \emptyset$, return to a) until all the sub-carriers are used.

An index F is introduced to evaluate the performance of fairness about the algorithm^[15]:

$$F = \frac{\left(\sum_{k=1}^K \frac{R_k}{\alpha_k} \right)^2}{K \sum_{k=1}^K \left(\frac{R_k}{\alpha_k} \right)^2} \quad (9)$$

F is a real number in the interval (0, 1] with the

maximum value of 1 in the case that the achieved rate exactly meet the requirement. At this time, the performance of fairness is the best and it gets worse as F being smaller.

4. SIMULATION RESULTS

Simulation results are shown for an $N=1024$ sub-carriers and $K=24$ multi-user OFDM system using Rayleigh fading channel with bandwidth $B=1\text{MHz}$. Set the total power P_{total} available at the transmitter be 1W . The noise power spectral density (PSD) is $N_0=1.1565 \times 10^{-8}$ and let $BER=10^{-5}$. The sub-optimal algorithm in [1] and the greedy algorithm are taken to be compared with the proposed algorithm.

Figure 2 shows the data rate each algorithm has got as the number of users being 2,4,6,...,24. It is obvious that the proposed algorithm achieves better result than the sub-optimal algorithm, which is closer to the greedy one.

Figure 3 is a histogram for the ratio of all 24 different users' rate and the total. We can see the result of the proposed and the sub-optimal algorithm is very close to the predetermined values. In contrast, the greedy algorithm gets very poor performance. So the fairness index of the first two algorithms is much better than the later as shown in Figure 4.

Figure 4 shows the fairness index for each algorithm in the case of different number of users respectively.

Figure 5 is an enlarged view of figure 4 to see the difference between the proposed and sub-optimal algorithm clearly. From these figures, we can see the proposed algorithm performs much better than the other two on fairness.

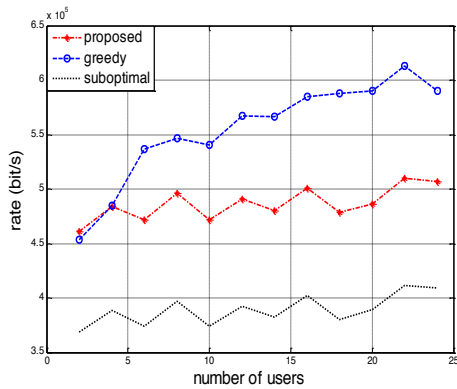


Figure. 2 Data rate of the three algorithm vs the number of users

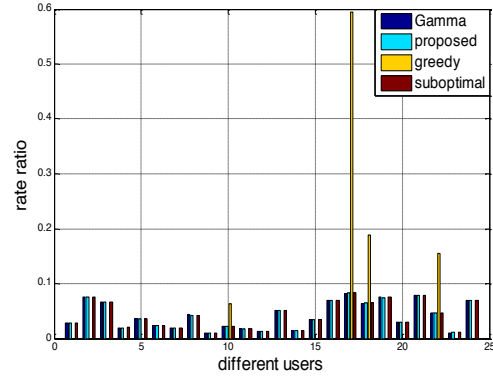


Figure. 3 Data rate proportion vs the different 24 users

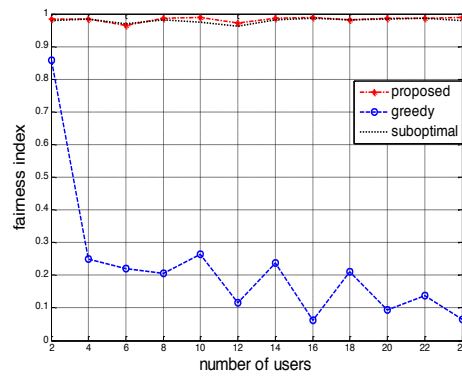


Figure. 4 Fairness index vs the number of users

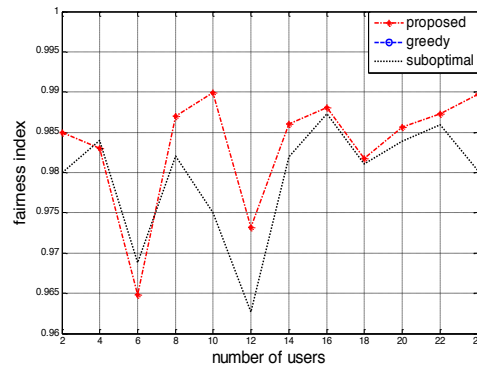


Figure. 5 An enlarged view of Figure 4

5. CONCLUSIONS

This paper proposes a kind of RA algorithm based on fairness of users' rate proportion. The power allocation of the algorithm combines equal and water-filling distribution. Compared to the traditional optimal power allocation algorithm in [6], it can greatly reduce the complexity and do almost negligible impact on system performance. Sub-carrier allocation is divided into two steps and always follows the main principle of fairness. Users are assigned sub-carriers with better channel gain as far as possible. It can not only ensure a better performance of proportional fairness, but also obtain a higher total

system data rate.

To further optimize the system performance, a lot of work should be done, such as the MIMO-OFDM technology. Its superior performance arouses people's attention, like the references [16] and [17]. But the technical complexity is increasing accordingly. So the proposed algorithm could be considered to be used in this technology next, due to its low complexity and practicability.

6. ACKNOWLEDGMENTS

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7. REFERENCES

- [1] Mohanram C, Bhashyam S. A sub-optimal joint subcarrier and power allocation algorithm for multiuser OFDM[J]. *IEEE Commun. Lett.* 2005, 9:685-687.
- [2] Wong C Y, Tsui C Y, Cheng R S, et al. A real-time sub-carrier allocation scheme for multiple access downlink OFDM transmission[J]. *Proc. IEEE VTC*, 1999,2: 1124-1128.
- [3] Wong C Y, Cheng R S, Lataief K B, et al. Multiuser OFDM with adaptive subcarrier, bit, and power allocation[J]. *Selected Areas in Communications*, *IEEE Journal on*, 1999, 17(10): 1747-1758.
- [4] Yin H, Liu H. An efficient multiuser loading algorithm for OFDM based broadband wireless systems[J]. *Proc. IEEE Globecom*, 2000,1:103-107.
- [5] Rhee W, Cioffi J M. Increase in capacity of multiuser OFDM system using dynamic subchannel allocation[J]. *Proceedings of IEEE Conference on Vehicular Technology*, Boston, 2000. 1085-1089.
- [6] Zukang S, Andrews J G, Evans B L. Optimal power allocation in multiuser OFDM systems[J]. *Proceedings of IEEE Conference on GLOBECOM*, San Francisco, 2003. 87-341.
- [7] Zukang S, Andrews J G, Evans B L. Adaptive resource allocation in multiuser OFDM systems with proportional rate constraints[J]. *IEEE Trans Wireless Commun*, 2005, 4: 2726-2737.
- [8] Wong I C, Shen Z, Evans B L, et al. A low complexity algorithm for proportional resource allocation in OFDMA systems[J]. *Signal Processing Systems*, 2004. *SIPS 2004. IEEE Workshop on*. IEEE, 2004. 1-6.
- [9] Zhang Y J, Letaief K B. Multiuser adaptive subcarrier-and-bit allocation with adaptive cell selection for OFDM systems[J]. *Wireless Communications*, *IEEE Transactions on*, 2004, 3(5): 1566-1575.
- [10] Sadr S, Anpalagan A, Raahemifar K. A novel subcarrier allocation algorithm for multiuser OFDM system with fairness: User's perspective[J]. *Vehicular Technology Conference*, 2007. *VTC-2007 Fall*. 2007 IEEE 66th. IEEE, 2007. 1772-1776.
- [11] Liu T, Yang C, Yang L L. A low-complexity subcarrier-power allocation scheme for frequency-division multiple-access systems[J]. *Wireless Communications*, *IEEE Transactions on*, 2010, 9(5): 1564-1570.
- [12] Sun D, Zheng B, Xu X. Multi-user cognitive OFDM with adaptive sub-carrier and power allocation[J]. *Future Computer and Communication (ICFCC)*, 2010 2nd International Conference on. IEEE, 2010, 1: 159-163.
- [13] Liu C, Schmeink A, Mathar R. Equal rate resource allocation for multiuser OFDM[J]. *EURASIP Journal on Wireless Communications and Networking*, 2012, 1:1-9.
- [14] Jang J, Lee K B. Transmit power adaptation for multiuser OFDM systems[J]. *IEEE J Sel Areas Commun*, 2003, 21:171-178.
- [15] Rasouli H, Anpalagan A. An Asymptotically Fair Subcarrier Allocation Algorithm in OFDM Systems[J]. *Vehicular Technology Conference*, 2009. IEEE, 2009, 69:26-29.
- [16] Ho W W L, Liang Y C. Efficient resource allocation for power minimization in mimo-ofdm downlink[C]. *Vehicular Technology Conference*, 2008. *VTC 2008-Fall*. IEEE 68th. IEEE, 2008. 1-5.
- [17] Wu F, Abu-Rgheff M A. Efficient subcarrier allocation in downlink multiuser MIMO-OFDM systems[C]. *Wireless Communication Systems (ISWCS)*, 2010 7th International Symposium on. IEEE, 2010. 51-55.