

# Headway Optimisation in Competitive Public Transport Systems

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**Abstract**—The paper presents a new control strategy of management of transport companies operating in competitive transport environment. It is aimed to optimise the headway of transport companies to provide the balance between costs and benefits of operation under competition. The model of transport system build using AnyLogic comprises agent-based and discrete-event techniques. The model combined two transport companies was investigated under condition of the competition between them. It was demonstrated that the control strategy can ensure the balance of interests of transport companies trying to find compromise between cost of operation and quality of service.

**Keywords**—transport systems; optimisation of control system; competition; headways optimisation

## I. INTRODUCTION

Primary purpose of a public transport system (TS) is to provide higher quality of services (QoS) ensuring efficient economic performance. Information and control systems are often used in administration and management of public transport to archive required quality and efficiency of the operation. In terms of mathematical description public transport is traditionally considered as queuing system where requests are formed by flows of passengers and processing units are transport carriers. Depending on the intensity of the flow a dispatcher control usually compute required number of transport carriers, their capacity and headway. The most important objective of such control system is headway adherence of transport traffic.

This approach is definitely applicable for management systems where there is no need to take into account economic parameters, for example, planning local in-plant transportation. However, in addition to passengers' satisfaction, the transport system must ensure profit of transportation. The headway adherence under reduction of passengers flow could bring the cost of transportation up and make it unprofitable. The profit of transportation becomes extremely crucial for a normal operation of a public transport company (TC) working in a competitive environment. Therefore the control methods must be flexible and adaptive to provide both higher QoS and

profitable operation of competitive TCs under variable passenger flow [1].

In order to provide reliable operation of public transport under required QoS most of methods suggest optimisation of timetable. For example, Xuan et al. [2] propose the method of dynamic scheduling based on bus holding strategy to provide stable operation of TS. The timetable optimisation reported in [3] is based on dynamic programming to ensure a balance of interests between TC and passengers.

On the other hand, Bartholdi and Emswiler [4] proposed an adaptive control method aimed to prevent bus bunching or timetable disturbance in one route of public transport completely excluding the concept of timetable scheduling or any previously defined headways. The system can freely correct the headways depending on current transport situation. The proposed algorithm balances the headways between all transport vehicles where the optimal headway is defined as the least common value based on the current vehicles capacity and operating restrictions. In case of a transport carrier breakage the control system can define the new positions for the remaining carriers to achieve a new least common value of headway which will be longer than previous. It is obvious that this method seeks to improve QoS for passengers by decreasing of dispersion of waiting time. The algorithm is based on knowledge of arrival time of each vehicle and strongly depends on passenger load of a transport network. The method based on alignment of headway is adaptive and can partly solves the problem of dynamic reaction on variable passenger flow in a transport network, but it can be used only in short routes with a low traffic.

The majority of passengers in megalopolises have to use more than one route. Thus, there is a problem of coordination of arrival and departure of buses to provide route transfer. Generally it can be solved using optimisation where criterion of efficiency is the delay of passengers in service. Teng et al [5] suggested the method of complex optimisation of the schedule for several transportation routes serving the same transport network and operating under centralised management.

It is extremely difficult to solve the problem of coordination of transport vehicles of different capacity to achieve an appropriate level of service. Typically, efficiency of transportation of the main passengers flow is considered as the criterion of control. The solution proposed in [5] optimises the bus headways to coordinate running schedule in passenger transport networks. In fact the optimisation forms a "corridor" for passengers following the most loaded route in the network.

The requirement to coordinate the city transport with the public transport travelling outside the urban area makes the headway optimisation much difficult. Another problem of scheduling appears because of the traffic congestion in transport corridors where public transport carriers often arrive at station unevenly leading to instable QoS. Sun et al [6] studied the optimisation of headways for the bus rapid transit (BRT) and introduced the scheduling combination as a characteristic of such transport system. It has been also suggested to divide the scheduling into normal scheduling, zone scheduling, and express scheduling in accordance to vehicle operation form and number of stops.

Scheduling of city transportation is the most common task. In contrast to city transport planning, the headways for BRT is not the key parameter to control. The arrival time at the end terminal of route is the main parameter for BRT to be controlled while the intermediate stops, where bus does not stop, can be ignored. Sun et al [6] are suggested to consider the BRT control as a task of multi-objective optimisation and use the method of convolution to decrease computing complexity both the model and the implemented algorithm.

High density public transport networks often maintained in urban areas can be reconfigured in order to optimise the headway and improve QoS. Zidi et al [7] proposed the routes and transport network topology control in real-time using ant colony optimisation algorithm.

However all works presented above aimed to optimise the headways and, therefore, QoS and do not consider operating cost of TCs under the defined headways. This paper presents the way to optimise the headway of TCs to bring balance between costs and benefits of operation for TC working in competitive transportation environment.

## II. ANALYSIS OF PARAMETERS OF TRANSPORT SYSTEMS

The task of a TC is transportation of passengers between route points. If TS considered as a queuing system, the passengers served by number of transport operators create waiting queues. Thus, the parameters of TS can be determined using statistical characteristics of passengers flow. In the simple case, headway of TS is based on the capacity and intensity of the passengers flow.

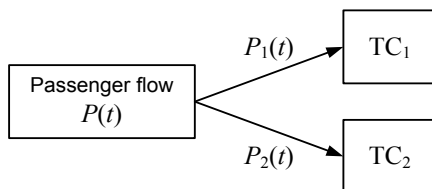


Figure 1. Competition of transport companies.

Let intensity of a flow of passengers be defined by function  $P(t)$ . Then number of the passengers appeared at time  $\Delta t = t_2 - t_1$  waiting for service, can be described as:

$$Q = \int_{t_1}^{t_2} P(t) dt \quad (1)$$

The first approach is based on domination of TC interest. Assuming that the capacity ( $C$ ) for all vehicles is the same then headway ( $h$ ) can be found by the following equations:

$$C = \int_0^h P(t) dt \quad (2)$$

Under uniform distribution of passengers flow  $P(t) = \lambda$ ,

$$h_1 = \frac{C}{\lambda} \quad (3)$$

Expressions (2) and (3) assume full load of vehicles. However the headway depends on profitability of transportation. Passenger load ( $L$ ) of vehicles, at which the transportation is profitable, significantly affects on the headway. The effective capacity of vehicle ( $D$ ) can be found as:

$$D = C \times L \text{ where } 0 < L \leq 1 \quad (4)$$

$$D = \int_0^h P(t) dt \quad (5)$$

$$h_2 = \frac{D}{\lambda} \quad (6)$$

According to (4)  $h_1 \geq h_2$ .

The second approach in calculation of the TS parameters is focused on consumers' service and based on average waiting time ( $T_w$ ) that passengers spent in queue waiting for transport. For a constant passengers flow  $P(t) = \lambda$  the headway can be found as:

$$h = 2 \times T_w \quad (7)$$

where waiting time is  $T_w = h/2$ .

The remaining parameter  $C$  is determined using (3)

$$C = \frac{2 \times T_w \times \lambda}{L} \quad (8)$$

Interests of the subjects in transportation process are contradictory. TCs are interested in the maximum loading of buses (3), while passengers are interested in minimising of waiting time (8). Therefore the transportation scheduling can be solved using multi-objective optimisation. In is supposed that the coefficient  $L$  enables to balance interests.

Equations (1) to (8) are used for calculation of the TS parameters operating under no competition; there is only one carrier on a route. Actually there is always competition between several companies offering various conditions of transportations. Passengers can impose to carriers variety of requirements such as service level, speed, cost of tickets, regular trips and many others. However the static planning can not provide reaction on dynamic changes of passenger flows and condition of transportation.

### III. MODELLING OF TRANSPORTATION UNDER COMPETITION CONDITIONS

Consider competition between two TCs shown in Fig. 1 where one TC is a state (TC<sub>1</sub>) whereas another is a private (TC<sub>2</sub>). The passenger flow  $P(t)$  is divided between the competing companies:

$$P(t) = P_1(t) + P_2(t) \quad (9)$$

The aim of the competition is to increase the profit which actually depends on the amount of passengers carried by a TC. That is why a company success is evaluated by the dynamic of  $P_i/P$  ratio. If the ratio is increased the strategy is winning, otherwise the strategy is losing.

### IV. CRITERION FUNCTION

The model was designed to investigate optimising strategy of TC behaviours in competing environment providing numerical assessment of controlled parameters. The most common criteria of QoS for queuing system is  $T_w$ , delay time of passengers in queue. However, in queuing systems this parameter is often called delay in service. It can be used as one of criteria for assessment of TS control system quality. Average value of delay of  $M$  passengers:

$$T_w = \frac{1}{M} \sum_{i=1}^M T_{delay}(i) \quad (10)$$

where  $T_{delay}(i) = T_{serv}(i) - T_{inp}(i)$  delay in service of  $i$  passenger,  $T_{serv}(i)$  is boarding time of  $i$  passenger,  $T_{inp}(i)$  arrival time to a stop of  $i$  passenger.

Profitability of TC depends on many factors such as tickets costs, bus loading and applied control algorithm. The criterion defining efficiency of control system of TC is coefficient of bus loading ( $B_{load}$ ). Analysis of average loading of  $N$  vehicles on time interval  $[t_1, t_2]$  is:

$$B_{load} = \frac{\sum_{i=1}^N \left( \int_{t_1}^{t_2} B_{load(i)}(t) dt \right)}{N \times (t_2 - t_1)} \quad (11)$$

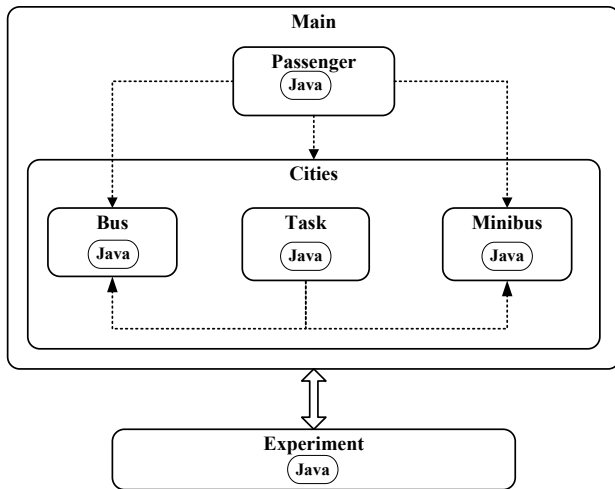


Figure 2. Structure of model of transport system.

The major problem of TC control system of is to find a compromise between:

$$T_w \rightarrow \min \quad B_{load} \rightarrow \max \quad (12)$$

The model of competing TCs should provide the value control of criteria functions (11) and (12) under changing of control strategy parameters. Therefore the main parameters of investigated system should be highlighted and the functional link between them and criterion expressions must be established.

Assume that the following factors impact on preferences of passengers:

- Queue length – coefficient  $\alpha_1$ ;
- Ticket prices ratio – coefficient  $\alpha_2$ ;
- Delay time – coefficient  $\alpha_3$ .

If values of all coefficients are within a range from 0 to 1, then the integrated coefficient  $K$  defines preferences of the passenger in the competitive market:

$$K = \alpha_1 \times \alpha_2 \times \alpha_3 \quad (13)$$

### V. HYBRID MODEL OF PUBLIC TRANSPORTATION SYSTEM

The model was developed using object-oriented approach often used to investigate complex systems. This approach allows developers to structure the model in AnyLogic to simplify and accelerate the steps of the model development.

The model structure and the relations between classes are presented in Fig 2.

According to the chosen paradigm, the model divided into the following classes:

- 1) Main class;
- 2) Passenger class;
- 3) Bus class;
- 4) Minibus class;
- 5) Cities class;
- 6) Task (Class of routes);
- 7) Experiment (class that used for simulation run).

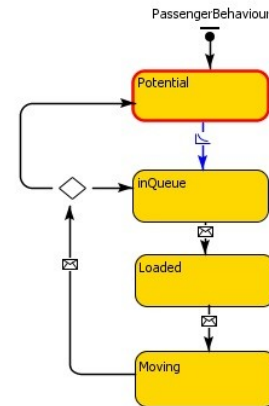


Figure 3. Statechart of passenger class.

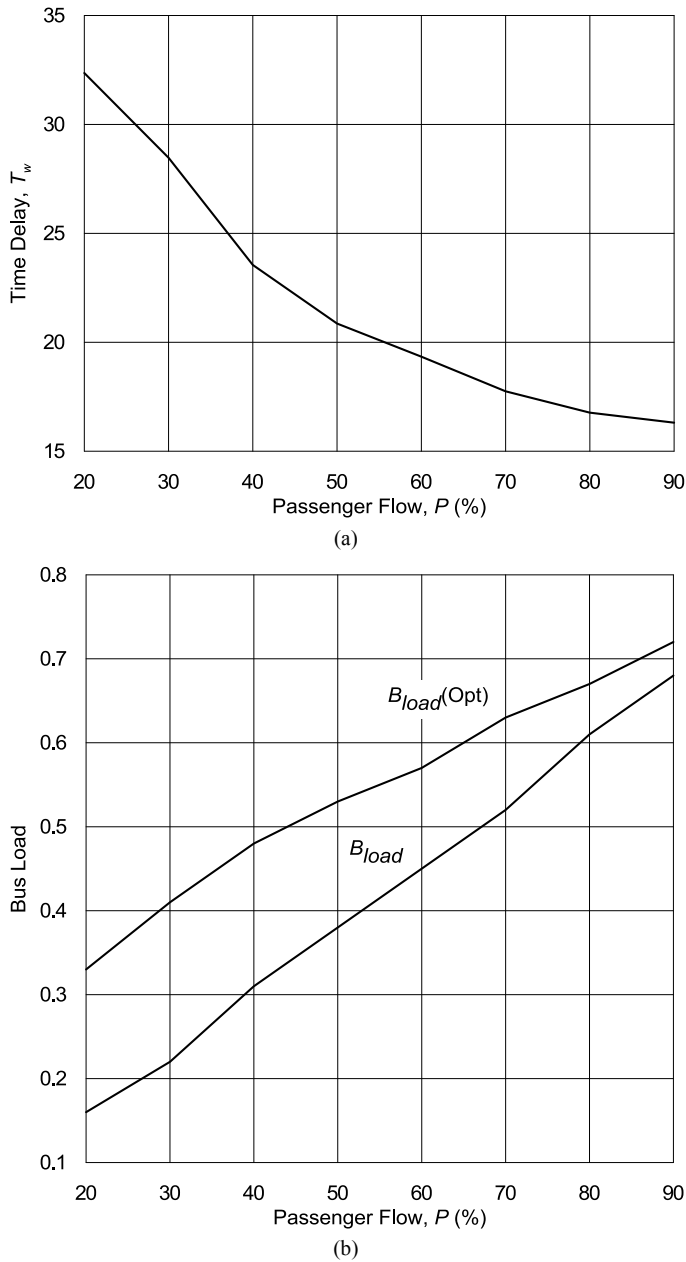


Figure 4. (a) Time delay under optimal parameters of control; (b) Increase of the bus loading under optimal control.

An object displaying events and defining behaviour of the model is called the active object. In object-oriented models the active object has dynamic properties.

According to the model, AnyLogic considers passengers as agent-based objects. The most effective way to simulate agent behaviour in the AnyLogic environment is statechart. The statechart can be used as a tool to assign behaviour of agents during discrete event modelling. The statechart of passenger class is presented in Fig. 3.

Due to the properties of AnyLogic platform the model of TS is presented as a hybrid combining different modelling and simulation technologies. While the agent-based approach is used for modelling of passenger behaviour the discrete-event

TABLE I. RESULTS OF THE OPTIMISATION

$P(\%)$	100*	90	80	70	60	50	40	30	20
$T_w$	15.14*	16.31	16.77	17.75	19.34	20.86	23.55	28.47	32.36
$B_{load}$	0.76*	0.72	0.67	0.63	0.57	0.53	0.48	0.41	0.33
$h_{opt}$	30*	32	33	35	39	42	47	55	65

method is applied to simulate the performance of transport carriers.

## VI. OPTIMISATION OF TRANSPORTATION SYSTEM PARAMETERS

The optimisation of TS parameters was conducted in order to verify reactions of control system on decrease of passengers flow.

It has been found that the increase in profitability of a transport carrier decreases QoS. However the compromise value of the headway  $h_{opt}$  obtained as a result of the optimisation provides the balance of interests of the subjects in the transportation process. Results of the optimisation in respect of passenger flow are given in Table I.

Results shown in Fig. 4a demonstrate that the dynamic scheduling of TS based on the headway adjustment in respect of intensity passengers flow can adapt the control system operation under wide range of bus loading. Fig. 4b shows the increase of bus loading due to implementation of optimal parameters. It can be seen that the optimal bus loading is increased in the range from 5 to 18% for various values of the passengers flow.

## VII. HEADWAY OPTIMISATION IN COMPETITIVE ENVIRONMENT

In order to reflect the competition between TCs the model utilises additional parameters such as competition level  $Comp_{level}$ , passengers' preferences  $K$ , capacity  $C$  and headways of transport corridors for  $TC_1$  and  $TC_2$ . The new parameter called competition level is introduced as following:

$$Comp_{level} = \frac{C_{TC_1} \times h_{TC_2}}{C_{TC_1} \times h_{TC_1} + C_{TC_2} \times h_{TC_2}} \quad (14)$$

Quality of services is defined as following:

$$QoS = \frac{T_w}{T_{w_{model}}} \quad (15)$$

where  $T_{w(model)}$  is the waiting time obtained from the model analysis.

Theoretically, the competition reduces the intensity of passenger flow for one of the companies. However the control reaction on that input could be quite various because of the system freedom. The multi-parameter structure of the system and operation under control algorithm utilising multi-objective optimisation make the search of optimum value is considerably complicate.

Figs 5–7 illustrate ability of the proposed model to find the optimum headway under change of the competition level in

TABLE II. RESULTS OF THE OPTIMISATION

$C_{TC2}$	$B_{loadTC1}$	QoS	Comp <sub>level</sub>	$h_{TC1}$
2	0.82	0.83	0.19	38
3	0.76	0.77	0.30	50
4	0.77	0.73	0.39	55

## IX. CONCLUSION

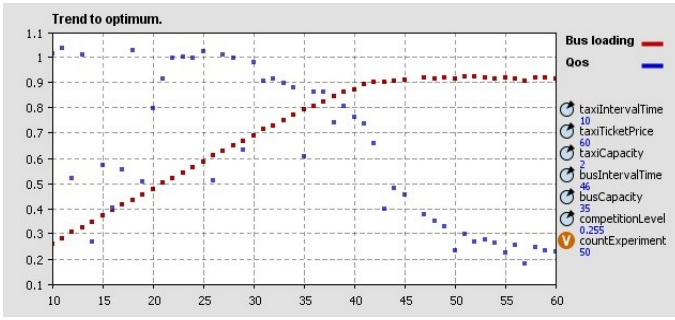
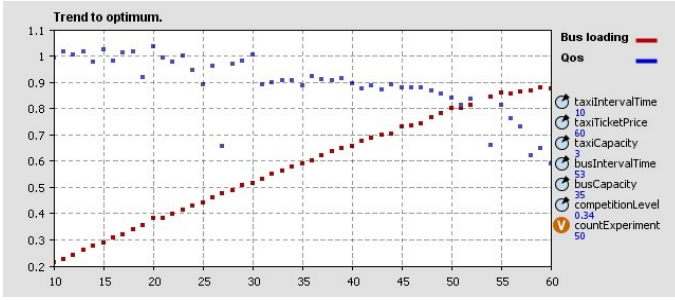
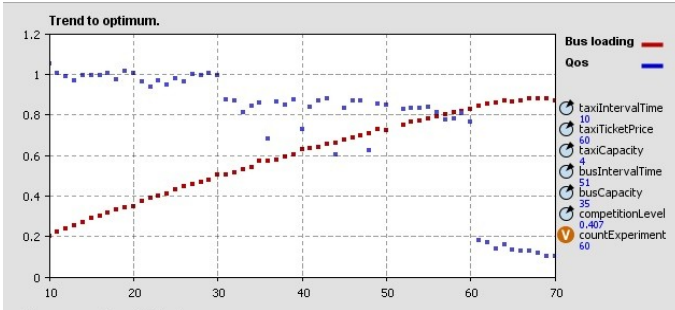
It has been discussed that modern transport networks become complex systems which management based on optimisation techniques. Most of TS control systems use optimisation of the headway to achieve effective operation of TC in order to keep QoS at a high level. However this approach does not take into account the cost of operation. The algorithms suggested above aimed to optimise the headway to provide balance between costs and QoS for TCs working in competitive transportation environment.

The model of TS reflecting performance of TCs under complete condition has been analysed and built using AnyLogic platform. The hybrid structure of the model has been achieved due to the features of AnyLogic where agent-based technique was used to model passenger flow and discrete-event approach was applied to simulate transport carriers behaviour.

The model of TS comprised two TCs was investigated under condition of the competition between the companies. It has been shown that the control strategy based on multi-objective optimisation can ensure the balance of uncomplimentary interests of TCs trying to find compromise between  $T_w \rightarrow \min$ ,  $B_{load} \rightarrow \max$  and, therefore, between costs and QoS.

## REFERENCES

- [1] Y. Li, W. Xu, and A. He, "Expected value model for optimizing the multiple bus headways," *Applied Mathematics and Computation*, vol. 219, no. 11, pp. 5849-5861, Feb. 2013.
- [2] Y. Xuan, J. Argote, and C.F. Daganzo, "Dynamic bus holding strategies for schedule reliability: Optimal linear control and performance analysis," *Transportation Research Part B: Methodological*, vol. 45, no. 10, pp. 1831-1845, Dec. 2011.
- [3] W. Zhu, and R. Hu, "Research on dynamic timetables of bus scheduling based on dynamic programming," in *Proceedings of IEEE 33rd Chinese Control Conference*, Nanjing, China, 2014, pp. 8930-8934.
- [4] J.J. Bartholdi, and D.D. Eisenstein, "A self-coordinating bus route to resist bus bunching," *Transport Research Part B: Methodological*, vol. 46, no. 4, pp. 481-491, May 2012.
- [5] J. Teng, X. Li, X. Yang, and M. Zhao, "Coordinated optimization of bus headways for passenger corridors," in *Proceedings of IEEE International Joint Conference on Computational Science and Optimization*, Sanya, Hainan, 2009, vol. 2, pp. 981-985.
- [6] C. Sun, W. Zhou, and Y. Wang, "Scheduling combination and headway optimization of bus rapid transit," *Journal of Transportation Systems Engineering and Information Technology*, vol. 8, no. 5, pp. 61-67, Oct. 2008.
- [7] S. Zidi, S. Maouche and S. Hammadi, "Real-time route planning of the public transportation system," in *Proceedings of IEEE Conference on Intelligent Transportation Systems*, Toronto, Canada, 2006, pp. 55-60.

Figure 5. Optimisation of  $h_{TC1}$  ( $C_{TC2} = 2$ ).Figure 6. Optimisation of  $h_{TC1}$  ( $C_{TC2} = 3$ ).Figure 7. Optimisation of  $h_{TC1}$  ( $C_{TC2} = 4$ ).

the TS. The optimisation was performed in interests of  $TC_1$  company using the following initial values of parameters: bus capacity  $C_{TC1} = 35$ ,  $h_{TC2} = 10$ , value of  $C_{TC2}$  is changed from 2 to 4, competition level is variable (not fixed).

Results of optimisation of  $h_{TC1}$  are presented in Table II. If  $TC_1$  increases the headway  $h_{TC1}$  to ensure profitability of transportations it does not affect on QoS because it is compensated by the increase of passenger flow for  $TC_2$ . Therefore QoS as a parameter based on both passenger flows is reduced insignificantly. Thus the system controls the objects by means of the only parameter – headway.

This example demonstrates the optimisation of the headway in complete transportation environment aimed to satisfy expectations of all objects of TS and find the balance between profitability of operation of a TC and quality of service at reasonable level.