Development of link tension index for flocking agents

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ABSTRACT
Techniques are needed to control mobile agents as a flock. These agents observe each other within a neighboring distance. The agent and its observing relation can be considered as nodes and links of a graph. To control the flock, the graph needs to be connected. However, following the rules of flocking, a flock splits into smaller flocks; i.e., the graph becomes disconnected. We thus propose a link tension index that predicts which links in the graph will remain intact and which will break a short time later. We believe that the index can be applied to the control of a flock. The link tension index represents whether the maximum distance of the link between two agents is within a local observing area. In an actual flock, there is no guarantee that this local index always makes correct predictions because there are other agents besides the agents directly connected. Thus, this paper evaluates the validity of the index for a flock of many agents. We show that the index correctly predicts the links of the flock that will remain intact a short time later but poorly predicts the links that break because the index focuses on the energy of the relative motion between two agents.

Categories and Subject Descriptors
I.2.9 [**Robotics**]: Autonomous vehicle.

General Terms
Theory

Keywords
Boids model, dynamics, multiple-agent system, swarm robotics, flocking, stability

1. INTRODUCTION
Techniques need to be developed to control multiple mobile robots efficiently. In the near future, it will be possible to control multiple mobile robots following a disaster such as an earthquake or for the support of farming. However, it will be difficult to control a large number of robots efficiently. In the worst case, robots might cause a secondary disaster by, for example, colliding with each other.

To control many robots, this study considers that the robots should form and move as a single flock. Each robot should observe its surroundings individually and determine its direction of motion autonomously. The number of robots can thus be changed without changing the method of control and the information processing mechanism, and the method is robust because each robot's behavior is determined locally. However, a robot isolated from the flock is not able to move with the flock. When considering a robot and its observing relation as a node and link, all robots must form a single connected graph if mobile control of the flock is to be realized.

The control theory and connections of a flock of multiple agents have been studied. Reynolds proposed a local behavior rule called the Boids model in which agents are simulated as a flock in the natural world [1]. That work focused on the visual effects of computer graphics and did not address the stability of the flock.

In contrast, by developing the control of motion and keeping the distance between agents constant, Tanner et al. discussed stability using a model in which the observing relation does not rely on the distance between agents [2] and a model in which an agent can observe its vicinity [3]. Moreover, Tanner did the same for a network of information transmission and a network of observation that differ. Tanner's studies demonstrated that it is possible to form a flock when the agent's observing range is sufficiently wide and there is little difference in the velocity of each agent. He showed that it is possible to form a flock if an observation graph is connected throughout time [4].

In other work, Shimizu et al. applied a method of analyzing a many-particle system to control a flock, using only the agent's local information [5]. Chazelle applied asymptotically tight bounds to time to ensure flocks can combine [6]. Martin and coworkers determined sufficient conditions for the initial positions and velocities of agents that ensure that the agents will asymptotically achieve flocking. For this purpose, they defined the notion of robustness of graphs using a metric interaction rule [7]. Olfati-Saber constructed a mathematical framework to discuss the Boids model mathematically and thus the stability of the flock [8]. His study showed that a flock is not stable globally; i.e., it is impossible to form a single flock from arbitrary initial conditions. A global observation is essential for the formation of a stable flock. Zavlanos et al. developed an algorithm that changes the formation of a flock according to a graph structure estimated by local observation so as to enhance the capability of the form of the flock [9].

The above studies argued that it is not possible to form a single flock from arbitrary conditions completely by local control. However, we are interested in revealing each agent's motion in maintaining a single flock, or which part of the flock initiates a split of agents. In future work, we will apply our proposed method to control a massive flock locally (or sublocally from global observation).

In this paper, we propose a link tension index that expresses which links in the graph will remain intact and which will break. The index can be determined from a number of local observations. The index contributes to the control and maintenance of the flock because it indicates the part of the flock prone to separating. The controller can thus grasp where the flock requires external control.

The index is a local value that expresses whether the maximum distance between agents is within an observation range. In the

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case of a real flock, the local index is not guaranteed to be effective in predicting the cohesion of a flock because agents interact with each other. In this study, we test the validity of the index for a system of a large number of agents forming a flock. As a result, we show that the index can predict which links of the flock will remain intact a short time later. The prediction of disconnection using the index is difficult because it focuses on the energy of the relative motion between two agents.

2. PHYSICAL MODEL OF FLOCK FORMATION

The Boids model expresses the behavior of agents constituting a flock (Figure 1). The model comprises the following equations.

$$\boldsymbol{u_i} = \sum_{j\in N_i} k_p(\|\boldsymbol{p_j}-\boldsymbol{p_i}\| - d)\boldsymbol{n_{ij}} + k_v \sum_{j\in N_i}(\boldsymbol{v_j}-\boldsymbol{v_i}) \quad (1)$$

$$N_i = \{j \mid \|\boldsymbol{p_j}-\boldsymbol{p_i}\| \le r\} \quad (2)$$

$$\boldsymbol{n_i} = \frac{\boldsymbol{p_j}-\boldsymbol{p_i}}{\|\boldsymbol{p_j}-\boldsymbol{p_i}\|} \quad (3)$$

$$\boldsymbol{v_i^{t+\Delta t}} = \boldsymbol{v_i^t} + \boldsymbol{u_i}\Delta t \quad (4)$$

$$\boldsymbol{p_i^{t+\Delta t}} = \boldsymbol{p_i^t} + \boldsymbol{v_i}\Delta t \quad (5)$$

Each agent can observe an area with radius r. Equation (2) gives the set of agents N_i that is within the observing range r of agent i. The agent attempts to keep a stable distance d from other agents in its observing area and to match its velocity with those agents. Neighboring agents behave according to the stable distance between them; i.e., agents traveling side by side move toward each other if they are separated by more than the stable distance and move away from each other if they are within the stable distance. In the case that an agent is outside the observing range r, equation (1) is not applied; i.e., the agent splits from other agents. $\boldsymbol{u_i}$ denotes the force of the i-th agent, and k_p and k_v are spring and damper coefficients, respectively. The positions of the i-th and j-th agents are respectively denoted $\boldsymbol{p_i}$ and $\boldsymbol{p_j}$ and the velocities are $\boldsymbol{v_i}$ and $\boldsymbol{v_j}$. Equation (3) gives the unit vector $\boldsymbol{n_i}$ between agents i and j. Employing the Euler method, we obtain the position and velocity of agent i in the next time step according to equations (4) and (5).

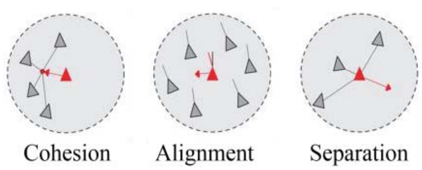


Figure 1. Flocking states.

3. PRINCIPLE AND FORMULIZATION OF THE LINK TENSION INDEX

The model used here is defined as a virtual spring and damper as shown in Figure 2. Therefore, the maximum distance of separation of two agents is in balance with the kinetic energy of their relative motion. We simplify the model by excluding the virtual damper. Initially, agent j's kinetic energy for relative motion of agents i and j is stored in the virtual spring. When the relative velocity is zero, the elongation of the spring is a maximum (Figure 3). The position at a later time can be found by applying the principle of conservation of mechanical energy described by equation (6).

$$\frac{1}{2}m_j(\boldsymbol{v_{ij}}\cdot\boldsymbol{n_{ij}})^2 + \frac{1}{2}k_p\left(\|\boldsymbol{p_j}-\boldsymbol{p_i}\| - d\right)^2 = \frac{1}{2}k_p\left(\|\boldsymbol{p_j}-\boldsymbol{p_i}\| - d + \Delta d\right)^2 \quad (6)$$

Solving the above equation, Δd is determined. Δd is the elongation of the spring from the natural length of the spring as given by equation (7).

$$\Delta d = \sqrt{\left(\|\boldsymbol{p_j}-\boldsymbol{p_i}\| - d\right)^2 + (\boldsymbol{v_{ij}}\cdot\boldsymbol{n_{ij}})^2} \quad (7)$$

When the sum of the natural length and the elongation $d + \Delta d$ is greater than r, agents separate. Thus, the link tension index is given by equation (8).

$$Index_i = d + \Delta d - r \quad (8)$$

If the above index is positive, the maximum distance will be beyond r and it can be said that the link will break. In contrast, if the index is negative, the agent will be within the observing area.

The index is completely local because it is based on the local positions and velocities of the i-th and j-th agents. The locality is maintained when we apply the index to the control of the flock. Moreover, millions of agents in a massive flock can be easily visualized by parallel computing because the computational complexity increases only linearly with the number of links.

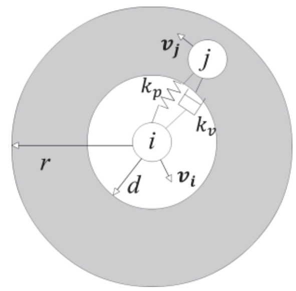


Figure 2. Flocking model comprising a virtual spring and damper.

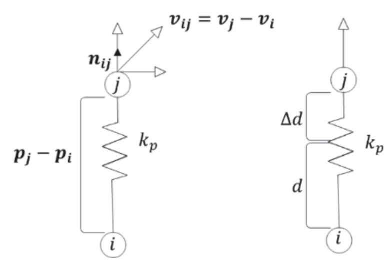


Figure 3. Spring model (right). Principle of prediction of the maximum distance (left).

4. VERIFICATION OF THE CONNECTION INDEX IN A NUMERICAL EXPERIMENT

4.1 Configuration of the experiment

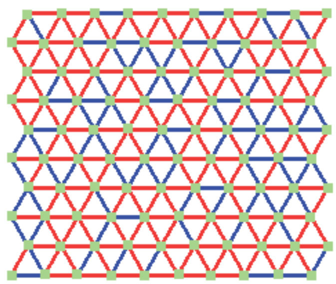
The connection index was defined using the spring model for two agents. However, many agents form a flock. We thus need to investigate the validity of using the index. In particular, the effectiveness of the index would be reduced by the effect of other agents around the link. However, it is impossible to simulate and investigate all cases. Focusing on two physical quantities, the position and velocity, we simulate and investigate the use of the index. Additionally, we divide the simulation time into two periods and compare the results obtained in each to check the accuracy of the prediction of the index.

In the experiment, there are 100 agents. An agent's observing range and initial velocity are determined by coefficients α and β and the stable distance d as expressed in equation (9). We use combinations of $\alpha = 1.3, 1.6, 1.9$ and $\beta = 1.0, 3.0, 5.0$. The stable distance is set as $d = 1$. Agents are visualized as green points; a red line indicates a link that will break while a blue line indicates a link that will not.

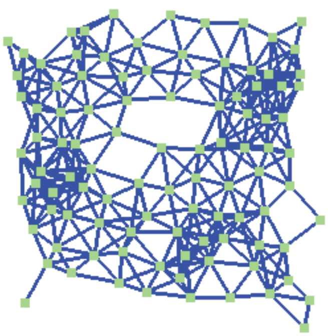
$$\begin{cases} r = \alpha d \\ |\boldsymbol{v}| = \beta d \end{cases} \tag{9}$$

4.2 Simulation focusing on velocity variations

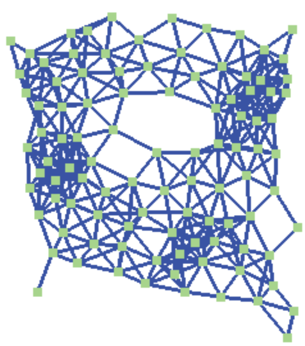
In the first simulation, agents are arranged in a pattern of regular triangles having an edge length that is the stable distance d. This arrangement allows the agents to connect with each other strongly. Agents are assigned a uniform random velocity having a norm less than β. At t = 0, the proposed index predicts that many links will break (Figure 4a). However, most links are intact at t = 1 (Figure 4b). At t = 2, the links remain intact as predicted at t = 1 (Figure 4c).



(a) The simulation at *t* = 0



(b) The simulation at *t* = 1



(c) The simulation at *t* = 2

Figure 4. Simulation of 100 agents focusing on velocity variations.
$\alpha = 1.6, \quad \beta = 3.0$

The results obtained for $\alpha = 1.6$ and $\beta = 3.0$ are given in Table 1. The columns express the predicted numbers of broken and intact links. The rows give the actual numbers in the experiment.

Table 1. Numbers of links that break and remain intact in the first simulation.

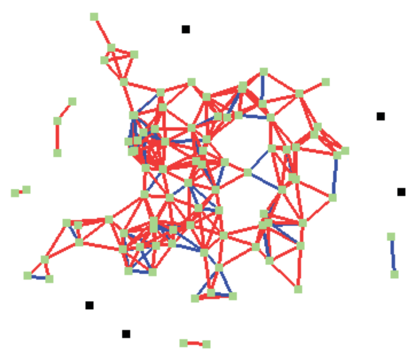
(a) $\alpha = 1.6, \beta = 3.0$. Simulation from t = 0 to t = 1.		
Actual/Predicted	**Broken**	**Intact**
Broken	2	0
Intact	210	50
(b) $\alpha = 1.6, \beta = 3.0$. Simulation from t = 1 to t = 2.		
Actual/Predicted	**Broken**	**Intact**
Broken	0	3
Intact	0	369

In Table 1 (a), the index calculated at t = 0 indicates that 212 links would break; yet only two links broke and 210 remained intact. In Table 1 (b), the intact links did not break in the next time step, because the agents reached a consensus in terms of their velocity. Results obtained for other combinations of α and β are almost the same.

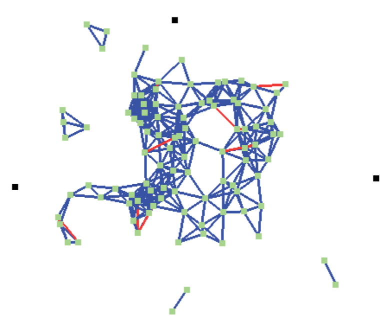
4.3 Simulation focusing on positional variations

In the second simulation, each agent is initially arranged as in the first simulation but shifted by a uniform random value less than γ in x and y directions. We use combinations of $\alpha = 1.3, 1.6, 1.9$ and $\gamma = 1, 3, 5$. Agents initially do not have uniform velocity.

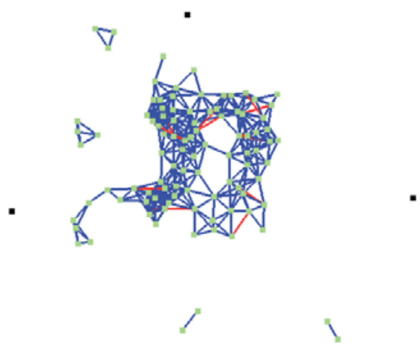
At t = 0, the index predicts that most links will break (Figure 5a). The links remain intact at t = 1, although some are predicted to break in the next time step (Figure 5b). At t = 2, however, the links remain intact (Figure 5c).



(a) The simulation at *t* = 0



(b) The simulation at *t* = 1



(c) The simulation at $t = 2$

Figure 5. Simulation of 100 agents focusing on positional variations.
$\alpha = 1.6, \quad \gamma = 3.0$

Table 2. Numbers of links that break and remain intact in the second simulation.

(a) $\alpha = 1.6, \gamma = 3.0$. Simulation from $t = 0$ to $t = 1$.

Actual/Predicted	Broken	Intact
Broken	10	0
Intact	217	40

(b) $\alpha = 1.6, \gamma = 3.0$. Simulation from $t = 1$ to $t = 2$.

Actual/Predicted	Broken	Intact
Broken	0	0
Intact	38	305

Table 2 (a) shows that the index calculated at $t = 0$ predicts that 227 links will break; however, only 10 links broke and 217 remained intact after 1 s. Table 2 (b) shows that the index calculated at $t = 1$ predicts some links will break yet there were no broken links at $t = 2$. The results obtained for other combinations of α and β are almost the same.

The results show that the proposed index can predict the links that remain intact; however, it is poor at predicting broken links. The poor prediction of broken links relates to the index being defined by the energy of the relative motion between two agents. When two agents are closer than the stable distance, the virtual spring between the two agents shrinks and stores energy. Red links are thus not links that will certainly break but links that are prone to breaking.

In the simulation focusing on positional variations, there were many red links because the agents were arranged randomly. Such an arrangement results in high latent energy. In the simulation focusing on velocity variations, the agents were arranged in a pattern of regular triangles having edge lengths equal to the stable distance, and the latent energy was high because of the difference in velocity at $t = 0$. However, the velocity started to converge soon after the simulation began. The difference in velocity thus reduced and there were no red links after $t = 1$.

5. CONCLUSION

Using a spring model, we developed a link tension index that can be used to predict and control the motion of a flock. The index was determined by the local position and velocity between two agents. To verify the effectiveness of the index, we conducted simulations under different conditions and investigated the number of links that remained intact and the number that broke. We conducted separate simulations focusing on variations in velocity and position. In the first, we arranged agents in a triangular lattice and assigned them random velocities. In the second, we shifted the positions of agents by a random distance and assigned a uniform velocity. These simulations of 100 agents forming a flock revealed that a negative index correctly predicts the part of the flock that remains intact. In contrast, a positive index reveals links prone to breaking. We thus consider the index to be a practical and valid tool for predicting the connection of many agents forming a flock and expressing the latent possibility of disconnection.

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