# Metric and Topological Neighborhoods in Flocking Models

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## ABSTRACT
Since the Boids model was proposed by Reynolds(1987), grouping behavior of animals and agents have been extensively studied and applied. Recent improvement in movie analysis enabled the discovery of scale-free correlation and phase transition in real flocks. We execute the comparison of two kinds of distances that define the behavior of the individuals in groups.

## Keywords
Collective behavior, Flocking model, Topological-distance Metric-distance

## 1. INTRODUCTION
In nature, various organisms form flocks and behave as if they are collectively a single creature. This phenomenon has attracted many people and lead them conduct observations and researches to study the flock since long ago. According to these studies, the most dominant theory is that collective behavior in nature is a bottom-up system based on simple interaction between individuals rather than a top-down system in which each individual follows a leader. Since Reynold who advocated a bottom-up theory proposed the Boids model, many researches has been conducted from various angles [1].

In the Boids model, individuals in a flock are modeled to interact with other individuals surrounding them. The distance that regulates this interaction is one of the most important factors in a flock model. The definition of the distance mainly has 2 aspects: *metric distance* defines individuals within a fixed range as the neighbors to interact with, whereas *topological distance* selects several individuals in order, starting from the closest.

After the discovery of the topological distance, researchers have proposed flock models that are somewhere between the two distance models and proposed alternatives, such as Metric-topological interaction (MTI) model which switch between 2 distances [4], and Limited Interaction (LI) model that employs stochastic and asynchronous interaction [5]. Although these models have already been compared to the existing flock models, the models with intermediate characteristics have not been compared to each other and further study is in need.


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Recent advances in observation and analysis enabled to test the descriptive values of flock models. We direct our attention to the data regarding the starling flock, published by Cavagna et al. and the phenomenon of *a flock within a flock* shown empirically. It describes the presence of a part of a flock with a particular behaviour distinct from the average behaviour of the rest of the flock, with its size proportional to the entire flock [2]. We perform comparison among models.

## 2. A FLOCK WITHIN A FLOCK AND MODELS
We will explain a flock within a flock, starting from Fig. 1. The left figure in Fig. 1 shows a distribution of the change in velocity calculated from a flock within a unit time [2]. Based on this data, velocity fluctuation of each individual is estimated. Velocity fluctuation is the difference between the average velocity of the entire flock and each individual's velocity; the result is shown in right. A group of individuals appears in this right figure, which has remarkably distinct velocity fluctuation than the rest, seemingly forming another flock, that is, a flock within a flock. Only a certain region of a flock specifically synchronizes and moves toward the same direction. A flock is

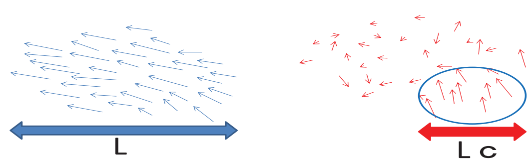



**Figure 1: Velocity configuration (left) and velocity fluctuation configuration (right). Both figures represent a same flock.**

indeed not uniform but contains certain structures within. In the data, a ratio of correlation length ($Lc$) to the entire length of the flock (maximum distance among individuals in the flock) ($L$:$Lc$) is given as a constant, regardless of flock size [2]. Such case in which the flock length correlates with the correlation length is known as a scale-free correlation, meaning that it maintains a constant proportion in the whole.

### 2.1 Boid Model
The Boids model features three simple rules, separation, alignment, and coherence. The individuals in the model intend to adjust the direction and velocity with other individuals; its alignment rule currently set bases for interactions in almost all the flock models [1]. The three rules are as follows:

- Separation: individuals avoid each other.
- Alignment: align with the other individuals in the neighborhood.

- Coherence: approach to other individuals in the neighborhood.

The algorithm of Boids model is shown below. The location of an individual is updated as follows.

$$\vec{x}_i(t+1) = \vec{x}_i(t) + \vec{v}_i(t+1) \qquad (1).$$

$\vec{x}_i(t)$ is a location of an individual at time $t$ and $\vec{v}_i(t+1)$ is velocity vector of and individual $i$ at time $t+1$. The individual $i$ has three radiuses, $R_{sep}$, $R_{ali}$, and $R_{coh}$, which determine the ranges for separation, alignment, and coherence, respectively. With the radiuses, the rules expressed as three vectors are defined as follows, and the velocity vector $\vec{v}_i(t+1)$ is the composition of the three.

$$\vec{v}_i^{sep} = \sum_{l \in \left(0 < |\vec{v}_i - \vec{v}_l| \le R_{sep}\right)} \frac{\vec{x}_i - \vec{x}_l}{|\vec{x}_i - \vec{x}_l|} \qquad (2)$$

$$\vec{v}_i^{ali} = \sum_{l \in \left(R_{sep} \le |\vec{v}_i - \vec{v}_l| \le R_{ali}\right)} \frac{\vec{v}_l}{|\vec{v}_l|} \qquad (3)$$

$$\vec{v}_i^{coh} = \sum_{l \in \left(R_{ali} \le |\vec{v}_i - \vec{v}_l| \le R_{coh}\right)} \frac{\vec{x}_l - \vec{x}_i}{|\vec{x}_l - \vec{x}_i|} \qquad (4)$$

$$\vec{v}_i(t+1) = |\vec{v}_i(t)| \frac{\vec{v}_i(t) + \vec{v}_i^{sep} + \vec{v}_i^{ali} + \vec{v}_i^{coh}}{\left|\vec{v}_i(t) + \vec{v}_i^{sep} + \vec{v}_i^{ali} + \vec{v}_i^{coh}\right|} \qquad (5)$$

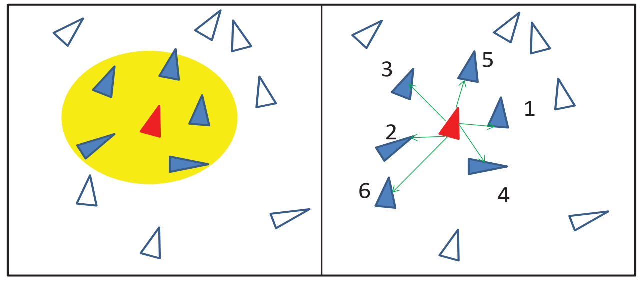
## 2.2 Metric Distance and Topological Distance

An individual in a flock model interacts with the other individuals in the neighbourhood and renews its velocity and location. Hence it is important to determine with whom one interacts. Selection of the individuals to interact with depends on the range or definition of the neighbourhood. The neighbourhood can be, for example, defined as the visual range of an individual. The definition of distance that determines the neighbourhood is classified into two kinds.

One is called *metric* distance. This classic definition for neighbourhood is adopted in the Boids model. It determines a constant range of neighbourhood around an individual and the individual interacts with all the other individuals within the range. Metric distance tends to form a solid flock with higher density. This is because each individual can enter into the neighbourhood of each other, which is known as "overlapped distance". However, with this kind of distance, when the wholeness of the flock is fluctuated by some means, the flock can be easily ramified.

The other is called *topological* distance, recently discovered by Ballerini; in the observation of starling flocks, they found that flocks' behaviour is better described with this distance [3]. Topological distance differs from metric distance in that it only interacts with several closest individuals in order defined as the rank in the metric distance. As it does not concern the metric distance, the interaction easily occurs even with a flock after ramification.

Figure 1 illustrates metric and topological distances. The red individual is in focus, and it will renew the velocity, referring to the blue individuals defined to be in its neighbourhood. In thee left figure, the red interacts with five individuals within the yellow-circle, defined by metric distance. The right figure illustrates topological distance, which selects six individuals in order, starting from the closest to the red.




(a)Metric distance (b)Topological distance

**Figure 2: metric distance and topologicaldistance**

## 2.3 Metric-Topological Interaction Model

MTI model proposed by Niizato switches between metric distance and topological distance according to the conditions described below [4]. When topological distance is used, if the difference between the direction of a certain individual and the average direction within the neighbourhood is less than threshold $a$, it switches to metric distance. On the contrary, when metric distance is used, and if the difference between the directions of two (randomly chosen) individuals within the distance is larger than threshold $b$, it returns to the topological distance.

$$\vec{v}_k^{t+1} = \vec{v}_k^{t} + \langle\vec{v}\rangle_{N-TOP}\Delta\theta \qquad (6).$$

$$if\left(\exists i \in N-TOP, \left|\theta_i^t - \langle\theta\rangle_{N-TOP}\right| < a\right) \qquad (7).$$

$$N-TOP \equiv \{l \in N \mid rank(l) \le 6\} \qquad (8).$$

Individual $k$ first uses topological distance. It selects the closest six individuals as the neighbors to interact with. $\langle\vec{v}\rangle_{N-TOP}$ indicates the average velocity vector of individuals within the neighbourhood. Equation (6) defines the next velocity according to the alignment rule. Formula (7) defines the topological distance and is used after interaction. $rank(l)$ represents the topological distance, the rank of closeness to individual $k$. When the difference between the direction of an individual $i$, located within the vicinity of and individual $k$ and selected randomly, and an average direction of individuals within the vicinity is less than threshold α, the distance used in the next time step will be switched to metric distance.

$$\vec{v}_k^{t+1} = \vec{v}_k^{t} + \langle\vec{v}\rangle_{N-MET}\Delta\theta \qquad (9).$$

$$if\left(\exists i,j \in N-MET, \left|\theta_i^t - \theta_j^t\right| > b\right) \qquad (10).$$

$$N-MET \equiv \{l \in N \mid 0 < |\vec{x}_k - \vec{x}_l| \le R\} \qquad (11).$$

When using the metric distance, the neighbourhood is defined as a range with the radius of $R$. $\langle\vec{v}\rangle_{N-MET}$ is an average individual velocity vector within the neighbourhood. After the interaction, equation (10) determines whether it needs to be switched from metric distance. If the difference between directions of two individuals selected randomly within the vicinity of the individual $k$ is larger than threshold $b$, the distance is switched to topological in the next time-step.

## 2.4 Limited Interaction Model

Around the same time with MTI, Bode proposed the Limited Interaction model (LI) to reproduce Ballenini's et al.'s data [3]. LI features stochastic and asymmetric interactions. The update algorithm of LI is as follows.

- Randomly select an individual $i$ for renewal
- Select an individual $j$ within the neighbourhood of an individual $i$
- Align with the individual $j$ and renew the individual $i$
- Repeat above described procedures for certain times

When an individual $i$ chooses another individual for interaction, the probability of selecting an individual $j$ depends on the distance between $i$ and $j$. The shorter the distance is, the higher the probability is defined. As the individual that is renewed the velocity is chosen randomly every time, some individuals may be renewed multiple times within a certain time step whereas others may not be renewed at all.

## 2.5 Metric-Topological Composition model

In this study, we propose the metric-topological composition (MTC) model, a new flock model with intermediate characteristics between metric and topological distances. This model is used for confirming the effect of metric and topological distances on flock behavior. A characteristic of this MTC model is that it conducts dual interactions based on both metric distance and topological distances, and the two velocity changes are averaged with a weight.

$$\vec{v}_k^{t+1} = \vec{v}_k^{t} + (c\langle\vec{v}\rangle_{N-MET} + (1-c)\langle\vec{v}\rangle_{N-TOP})\Delta\Theta \qquad (12).$$

First it calculates the average of the two velocity vectors from the metric and topological distances, and the average is weighted with $c$ and $(1 - c)$. The parameter $c$, which is proportional to the intensity of effect of the metric distance in the MTC model. When $c$ is closer to 1, the flock will be closer to the behavior with metric distance, and in contrast, when it's closer to 0, the flock approaches the behavior with topological distance. $c = 0.5$ indicates equal effects from both distances.

# 3. SIMULATION SETTING

The concept of scale-free correlation in a flock was introduced by Cavagna et al. [4]. $N$ is the total number of individuals in the flock. $V_i$ is the velocity of individual $i$. $u_i$ is the difference of velocity of $i$ from the average of the whole flock.

$$\mathrm{u}_i = V_i - \frac{1}{N}\sum_{K=1}^{N} V_k \qquad (13).$$

Correlation function $C_{(r)}$ is defined as follows:

$$C_{(r)} = \frac{\sum_{ij} u_i \cdot u_j \delta(r - r_{ij})}{\sum_{ij} \delta(r - r_{ij})} \qquad (14).$$

The distance between each agent is defined as $r_{ij}$. When $r - r_{ij} = 0$, $\delta(r - r_{ij}) = 1$, and otherwise $\delta(r - r_{ij}) = 0$. The data of the real flocks by Cavagna et al. showed that the correlation function decreases in proportion to the increase of $r$. A larger distance leads to a smaller, or negative value. This indicates that when individuals are close to each other, their velocities correlate strongly and when the distance between individuals is large, they are less likely to correlate well. $\xi$ such that $C_{(r=\xi)} = 0$ is called the correlation length. Cavagna et al. showed that this correlation length is proportional to the flock size. The flock size $L$ can be determined by calculating the maximum distance between any two birds that belong to the flock. The relationship between correlation length and a flock size satisfies $\xi = cL$. Here, $c$ indicates the correlation slope. In the data of starling flocks was constant, $\xi = 0.353 \pm 0.022$. Here we compare and examine the $\xi$ values for each model.

The two-dimensional space is $2000\times2000$, with the periodic boundary condition like a spherical surface. Individual velocity $v$ was set as 4.0. The initial locations and directions are random. Three models, Boids model, MTI model, and LI model, were tested for comparison. Thresholds $a$ and $b$ used in MTI model were set as a = 0.05 and $b = 0.10$, as in [6]. To adjust renewal frequency of LI with other models, the renewal count per step time was set as $N$ in LI. In this paper, the number of individuals are $N = (30,\ 40,\ 60,\ 80, 100)$ and performed simulation for 100 times each. The results of 100 simulations of each $N$ and 500 simulations per model are shown.

# 4. SIMULATION RESULTS

We show the results of MTI, Boids (metric), Boids (topological), LI, and MTC (with $c = 0.5$) in each Fig. 3, 4, 5, 6, and 7, respectively. Y-axis is correlation length and X-axis is a flock size. The correlation length and a flock size both increase as it goes to the right upper of a graph.

Figure 3 indicates that a larger flock size results in a longer correlation length in MTI. That is, a distance between individuals that interact to each other increases in proportion to the correlation length, clearly demonstrating scale-free correlation in MTI. Scale-free correlation indicates that the flock has a kind of integrated behavior like a single body. MTI model achieves scale-free correlation by treating the vicinity ambiguously. In other word, metric distance is one of the topological distance models which have no constant individuals to select, and topological distance is one of the metric distance models which have no constant vicinity range. ndeed, some results show that fish can discern small number and distinguish flocks according to their ratios such as 1:2 and 2:3. Correlating and non-correlating regions seems to appear as a result of looking at the surroundings with 2 ways [7][8].

In Figure 6, a larger flock size does not lead to a larger correlation length, so not proportionally to the flock size. This is likely because an individual for renewal is chosen randomly, therefore fail to renew all the individuals that belong to the flock and maintain the flock size. If there is no correlation region as a flock structure, scale-free correlation never appears. For this reason, at least a part of a flock needs to be renewed in a kind of similar manner. When LI renews all the individuals, probability is involved, which is likely to explain the lack of scale-free correlation in LI. Stochastic selection of partners for alignment also fails to stabilize the correlation length.

A formation of Boids' flock was tested in Figure 4 and 5. However, no scale-free correlation was observed either with metric distance or topological distance.

We have shown that neither of metric only nor topological only distance does not lead to scale-free correlation. It is only the MTI model that exhibits a significant scale-free correlation. MTI has a specific switching mechanism, and it is not very easy to see how the two kinds of distances are combined or interacting. This is why we introduce the MTC model. MTC linearly combines the velocity updates with metric and topological distances, with the parameter of $c$ from 0 to 1 that defines the weight of metric and topological distances. Fig. 7 shows the result with $c = 0.5$. It shows no scale-free correlation. We varied the parameter $c$ to $\{0.0, 0.25, 0.5, 0.75, 1.0\}$. As shown in Fig. 8,

## 5. CONCLUSION

In this study, we compared MTI model with Boids model, LI model, and MTC model with the index of the slope of scale-free correlation. We showed that not Boids and LI but only MTI showed the correlation. The MTC model that we introduced in this study showed that the idiosyncratic switching algorithm of MTI can be the key for exhibiting the scale-free correlation observed in bird flocks.

## 6. ACKNOWLEDGMENTS

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## 7. REFERENCES


[1] Reynolds, C. W. Flocks, herds, and schools: a distributed behavioral model. *SIGGRAPH Comput. Graph.* 21, 4, 25–34, 1987.

[2] Cavagna, A. et al. Scale-free correlation in the bird flocks. *PNAS*, 107, 11865–11870, 2010.

[3] Ballerini, M. et al. nteraction ruling animal collective behavior depends on topological rather than metric distance: evidence from a field study, *PNAS*, 105, 1232–1237, 2008.

[4] Niizato T., Gunji, Y.P. Metric–topological interaction model of collective behavior. *Ecological Modelling,* 222, 17, 3041–3049, 2011.

[5] Bode, W. F. N. et al. Limited interactions in flocks: relating model simulation to empirical data, *J. R. Soc. Interface* (Published Online), 2010.

[6] Niizato, T. *Theory of collective behavior observed on a basis of type-token interaction*, Dissertation, Graduate School of Science, Kobe University, 2012.

[7] Agrillo, C., Dadda, G., Serena, G., and Bisazza, A. Do fish count? spontaneous discrimination of quantity in female mosquitofish. *Animal Cognition*, 11, 495–503, 2008.

[8] Agrillo, C., Dadda, G., Serena, G., and Bisazza, A. Use of number by fish. *PLoS On,e* 4 (3), e478, 2009.


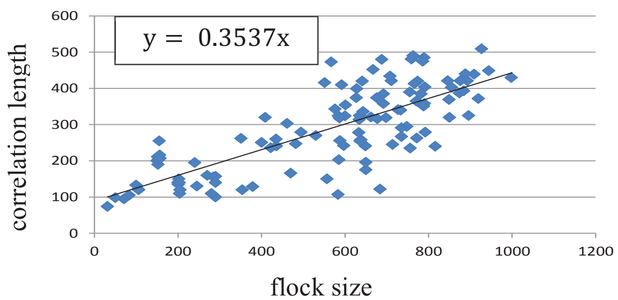


Figure 3: MTI model

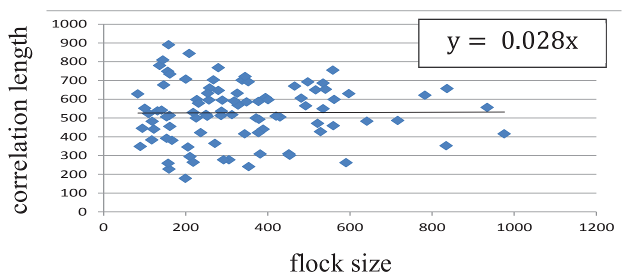


Figure 6: LI model

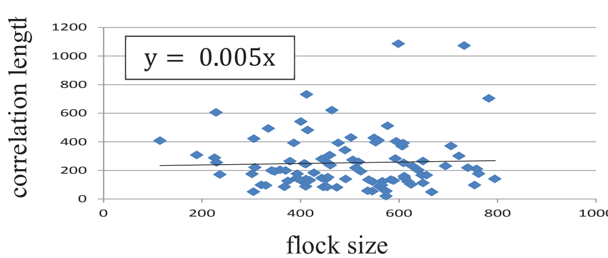


Figure 4: Boids (metric) model

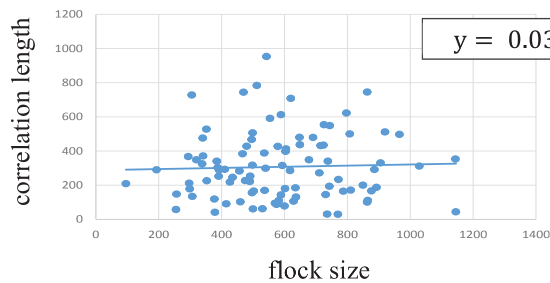


Figure 7: MTC ($c = 0.5$) model

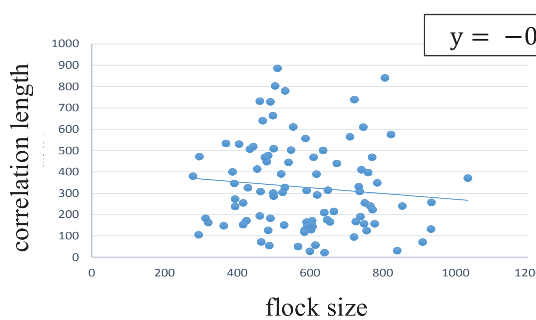


Figure 5: Boids (topological) model

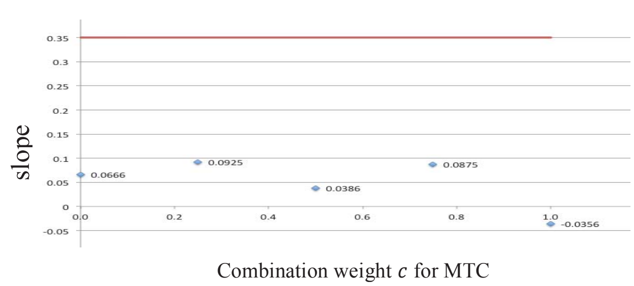


Figure 8: Slopes for MTC with various $c$ values