Deciphering the Reliability Scheme of the Neurons One Ion Channel at a Time Invited Paper

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ABSTRACT
A revived interest on consecutive systems due to novel nano-architectures (where reliability estimates are of high interest) as well as particular nanoscale communications (where the reliability of transmission needs to be assessed) is to be expected. The reason is that certain nano-technologies, like, *e.g.*, molecular ones (but also nano-fluidic, nano-magnetic and even FinFETs) could be mapped onto consecutive systems. This paper will start by reviewing bounds for such systems and will then use these bounds to show how one could perform very accurate simulations of large consecutive systems (*e.g.*, arrays of ion channels used for axonal communications), arguing that bounds could be extremely useful in the exploratory phases for getting a quick and reasonably good understanding of the reliability of such nano-architectures (including molecular, nano-fluidic, nano-magnetic, and FinFETs).

Categories and Subject Descriptors
B.8.1 [**Reliability, Testing, and Fault-Tolerance**]
C.4 [**Performance of Systems**]: Fault tolerance; Reliability, availability, and serviceability.

General Terms
Algorithms, Design, Reliability, Theory.

Keywords
Reliability, consecutive-*k*-out-of-*n*:F system, algorithm, bound.

1. INTRODUCTION
A consecutive system, also known as a consecutive-*k*-out-of-*n*:F system, is a system having components placed one after another (*i.e.*, in a row), which fails if and only if consecutive components fail simultaneously. It was introduced as *r*-succesive-out-of-*n*:F system by Kontoleon in 1980 [1]. One year later Chiang & Niu [2] discussed several applications and also named them "consecutive-*k*-out-of-*n*:F systems." Many generalizations have been suggested later [3]–[8]. The interested reader should consult [9] as well as [10]–[13].

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With the advancements of nanotechnologies, it is of high interest to evaluate as precisely as possible the reliability of certain nanotechnologies (like, *e.g.*, molecular, nano-fluidic, nano-magnetic and FinFETs), as well as of their associated communication schemes. Some of these can be mapped onto consecutive-*k*-out-of-*n* systems. The exact calculation of the reliability of such systems is conceivable even for reasonably large and [14]–[16], but might take significant time and/or require large memory. This is one reason why approximations have been investigated by Fu [17], and many lower and upper bounds have been reported over the last thirty years.

This paper will briefly review lower and upper bounds in Sections 2 and 3. Section 4 will present a few preliminary reliability estimates and focus on arguing why such bounds can be extremely useful for a detailed analysis of the particular case of axonal communication based on arrays of ion channels. Conclusions and future directions of research are ending the paper.

2. LOWER BOUNDS
Besides and , the following notations will be used: is the probability that a component works correctly; is probability that a component fails (*i.e.*,); and is the reliability of a consecutive-*k*-out-of-*n*:F system with component failure probability .

The first lower bound was introduced by Chiang & Niu [2]:

. (1)

This lower was rediscovered later by Fu in 1985 (eqs. (5) and (20) in [18]) and 1986 [19], by Papastavridis & Koutras in 1993 (Theorem 1 in [20]), as well as by Muselli in 1997 (eq. (5) in [21]).

Salvia in 1982 (eq. (3) in [22]) proposed a different lower bound:

(2)

It took a decade till Barbour *et al.* [23], using a different approach, established:

(3)

This was improved later by Barbour *et al.* [24] as:

(4)

(5)

Table 1. Lower bounds for the reliability of consecutive-*k*-out-of-*n*:F systems.

Year	Author(s)	Lower Bound	Condition(s)	Color
1981	Chiang & Niu			Cyan
1982	Salvia			Black
1992	Barbour *et al.*			Blue
1995	Barbour *et al.*			Blue
2000	Muselli	( )	/	Green
2000	Muselli	[ ]		Yellow
2000	Muselli			Red
2000	Muselli		/	Red
2014	Dăuş & Beiu	[ ]		Red (dotted)

{ [ ]}

. (6)

Finally, Muselli ([25], [26]) presented several lower bounds as follows:

( ) (7)

which holds for / , and also [25]:

[ ] (8)

⌊ [ ]⌋ (9)

⌊ ⌋ (10)

which holds for

Corollary 3 in [25] detailed another lower bound:

(11)

⌈ [ ]⌉ (12)

while [26] revealed yet one more:

(13)

( ) (14)

which is valid for /

Recently, a fresh class of lower bounds has been introduced by Dăuş & Beiu [27], the first lower bound from the class being:

[ ] (15)

which holds for

All of these lower bounds can be seen in a compact form in Table 1.

3. UPPER BOUNDS

Upper bounds were in most, but not all, cases introduced together with lower bounds. Chiang & Niu [2] proposed in 1981 the following upper bound:

⌊ ⌋ (16)

rediscovered by Muselli [21] in 1997.

Salvia (eq. (3) in [22]) proved in 1982 that:

. (17)

Table 2. Upper bounds for the reliability of consecutive-*k*-out-of-*n*:F systems.

Year	Author(s)	Upper Bound	Condition(s)	Color
1981	Chiang & Niu	⌊ ⌋		Cyan
1982	Salvia			Black
1985	Fu	or		Yellow
1992	Barbour *et al.*			Blue
1995	Barbour *et al.*			Blue
2000	Muselli	( )	/	Red
2014	Dăuş & Beiu	[ ]		Red (dotted)

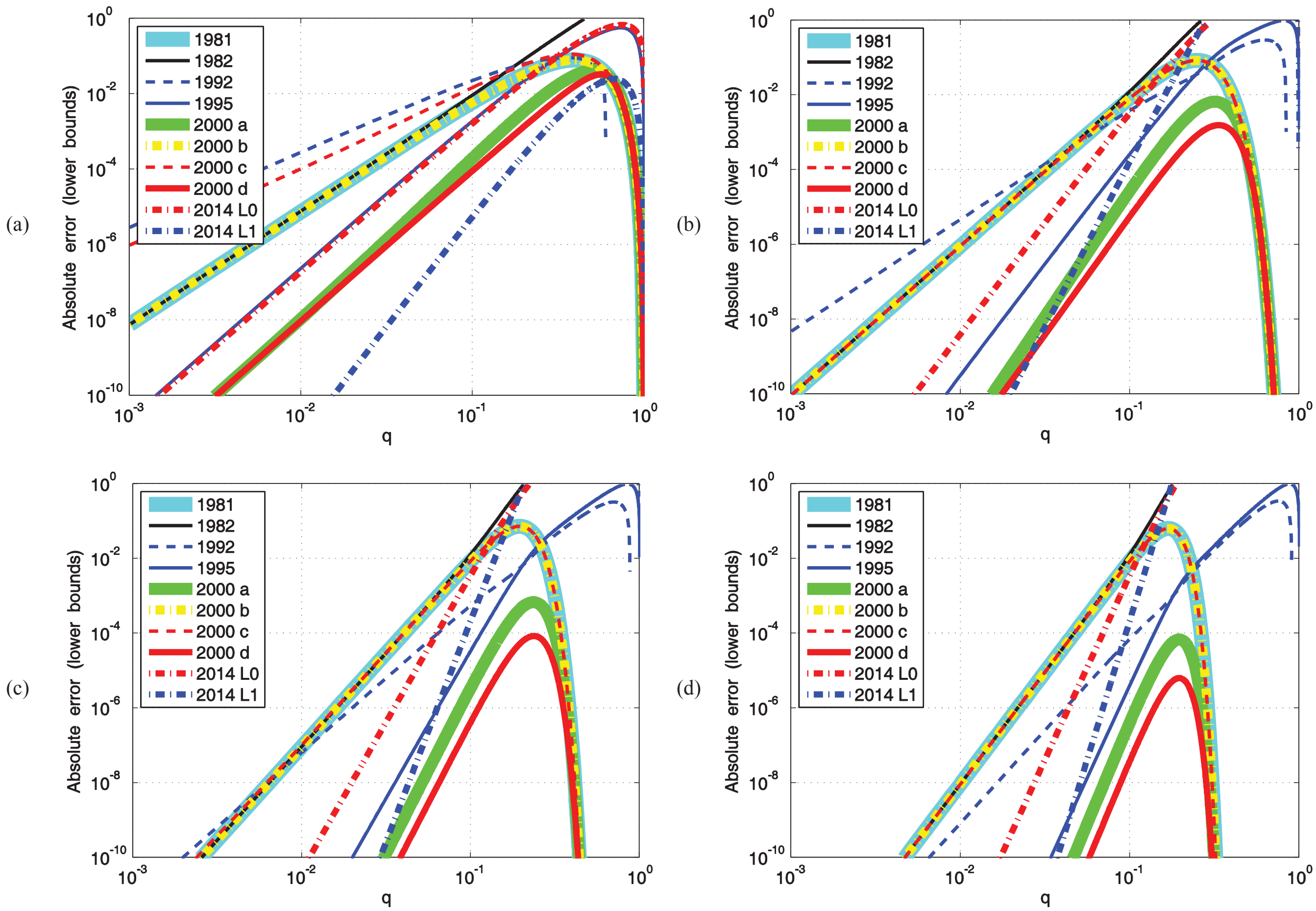

Figure 1. Absolute errors for consecutive: (a) 2-out-of-10; (b) 3-out-of-100; (c) 4-out-of-1000; and (d) 5-out-of-10000.

Fu in 1985 [18], and Papastavridis & Koutras in 1993 [20] showed that:

. (18)

Fu *et al.* [17] mentioned that this upper bound is in fact equivalent to one described by Gončarov in 1944 [28].

In 1992 Barbour *et al.* [23] established that:

(19)

and later improved on it [24]:

(20)

with and given by (5) and (6).

In 2000, Muselli shown that [25]:

( ) . (21)

Finally, Dăuş & Beiu [27] introduced a fresh class of upper bounds, the first upper bound from the class being:

[ ] (22)

which holds for .

All of these upper bounds can be seen in a compact form in Table 2.

4. CONSECUTIVE SYSTEMS REVISITED

All the equations presented above have been coded in Matlab. Additionally, two exact solutions of this well-known problem first stated by de Moivre [29] were also coded based on:

- the solution presented by de Moivre [29] and detailed by Uspensky in 1937 [30];
- the Markov chain method as developed by Fu in 1986 [31] and 1987 [32], as well as by Chao in 1989 [33].

The simulation results show the accuracy obtained by using the lower (see Fig. 1) and upper (see Fig. 2) bounds , *i.e.*, . The step used for all the simulations was , while we have used and . Both horizontal and vertical axes for all these plots are logarithmic (the base being 10).

From Figs. 1 and 2 it becomes apparent that, for all the lower and upper bounds B analyzed here, there is a particular value (depending on both k and n), starting from which the absolute errors | | depend linearly on (as all the plots look like straight lines for), therefore:

| | (23)

where . Since there exists such that , (23) can be rewritten as:

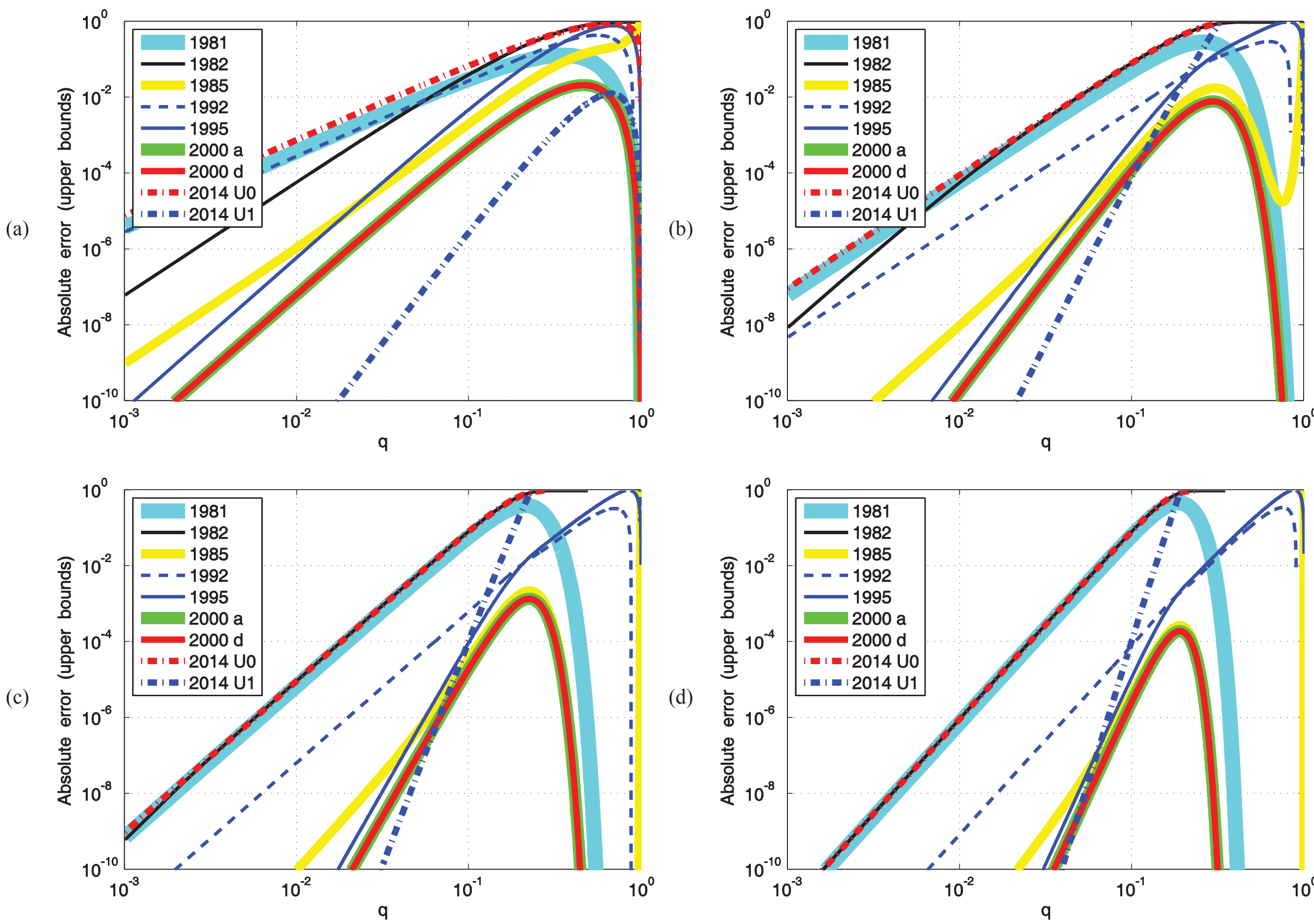

Figure 2. Absolute errors for consecutive: (a) 2-out-of-10; (b) 3-out-of-100; (c) 4-out-of-1000; and (d) 5-out-of-10000.

| | (24)

which gives:

. (25)

It follows that, for small enough (*i.e.*,), the absolute

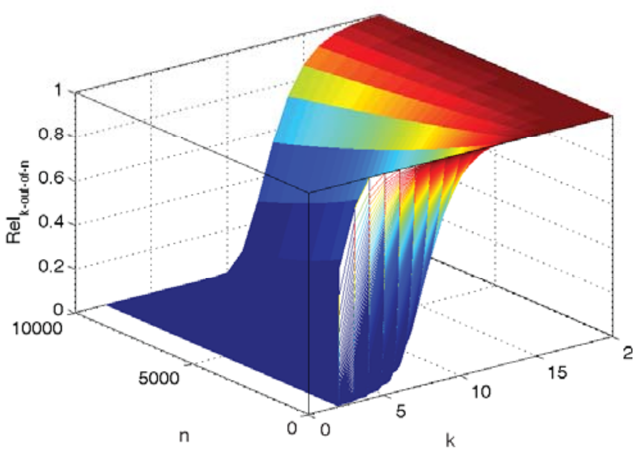

Figure 3. Reliability of consecutive-*k*-out-of-*n* for $n \le 10{,}000$ and $k \le 5\log n$.

approximation errors of all the lower and upper bounds presented here are off by an exponential . Obviously, the larger and the smaller the better the approximation.

The second set of simulation results presents estimates of the reliability of consecutive-*k*-out-of-*n* systems when , *i.e.*, , and can be seen in Fig. 3. These are in fact estimations of the reliability of axonal communication [35], as ion channels are considered to open and close randomly (hence). The fact that ion channels on the axons are distributed regularly was revealed recently [36]. This makes it that axonal communication corresponds to consecutive-*k*-out-of-*n* systems. That is why we have been trying to generate such 3D plots for other values of (ideally in steps of , as done for Figs. 1 and 2), and have been aiming for as large as 10^6 (not at all unreasonable when considering axonal communications).

Unfortunately, a 3D figure like the one presented in Fig. 3 requires repeated calculations of , ideally for each and every pair. That is why we have looked at very efficient algorithms for doing such calculations, like those presented by Hwang & Wright [14], Lin [15], and Cluzeau *et al.* [16]. They need , , and arithmetic operations (with), respectively. Estimations of the numbers of operations required by such algorithms for generating all the pairs needed for Fig. 3 are presented in Fig. 4. Obviously, using one of the bounds presented

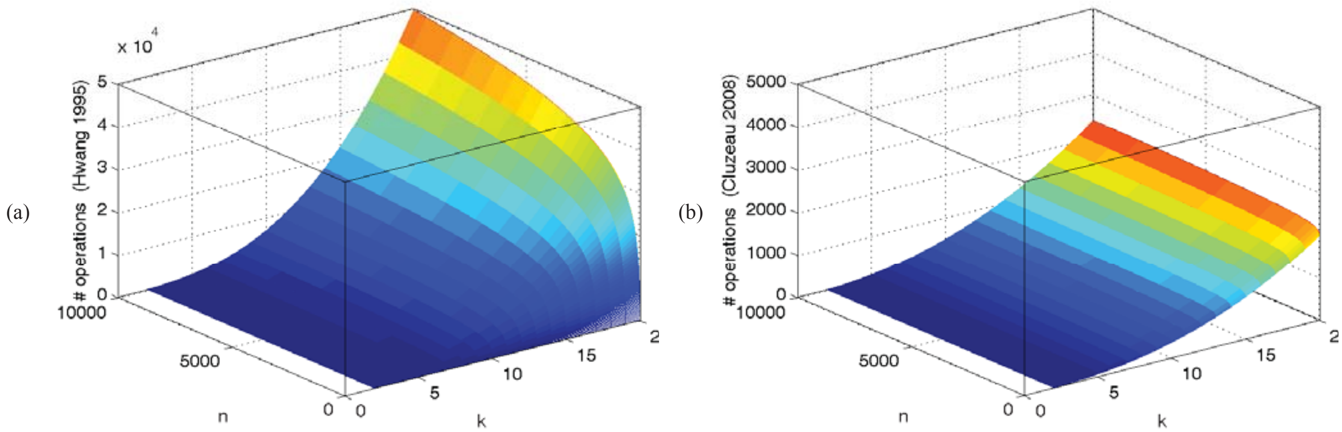

Figure 4. Number of operations versus *k* and *n* when using: (a) Hwan & Wright [14]; (b) Cluzeau *et al.* [15].

in Sections 2 or 3 would require only a fixed—and significantly lower—number of arithmetic operations per pair.

A comparison of three different solutions: (*i*) Hwang & Wright [14]; (*ii*) Lin [15]; (*iii*) Cluzeau *et al.* [16]; and (*iv*) using bounds [27], with respect to the total number of operations (proportional to the execution time) required for generating a 3D plot like the one shown in Fig. 3 (when is increased from 10 to 10,000 and is between 2 up to) is detailed in Fig. 5. It becomes clear that [16] is about one order of magnitude faster than [14], but quite similar to [15], while relying on bounds gets to be two orders of magnitude faster than [16] (and three orders of magnitude faster than [14], [15]) when .

5. CONCLUSIONS

Consecutive systems are a strong contender to the problem of reliable circuits using less reliable devices: simple, practical and within reach, but the increased connectivity is raising concerns which need to be addressed. It is also very interesting that this solution is used by the Brain at the lowest level, represented by arrays of ion channels relying on nano-fluidic communication.

Using bounds instead of exact calculations is very tempting at these early stages of our preliminary investigations on the

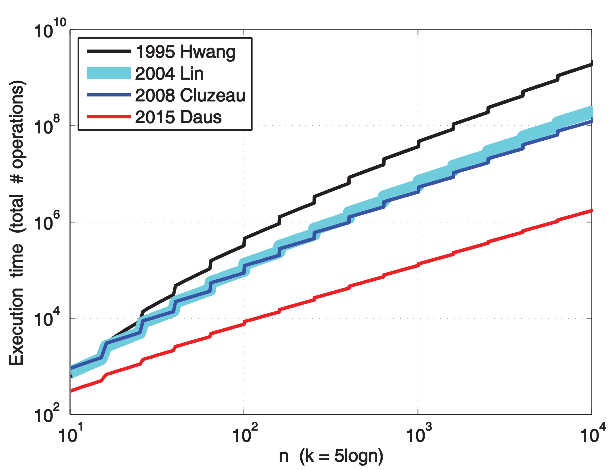

Figure 5. Total number of operations (related to execution time) for *n* up to 10,000 and *k* up to 5log*n*.

reliability of arrays of ion channels (with potential applications to molecular, nano-fluidic, nano-magnetic, and FinFETs) as:

- the known bounds are quite accurate (see Figs. 1 and 2);
- the execution time is two orders of magnitude lower than the best known exact algorithm [16].

Future directions of research will focus on:

- identifying the fastest (while accurate enough) bound approximation;
- generating plots for larger and varying ; currently in doing, but these take very long time even when using bounds (as an example, for , a 3D plot like the one presented in Fig. 3 took almost 3 days, while we would want to repeat such calculations many times with varying);
- trying to enhance the accuracy of estimating the reliability of axonal communication by using (*i*) combinations of circular and consecutive systems; (*ii*) 2-dimensional consecutive systems [7], [8], [37]; or (*iii*) connected-(r, s)-out-of-(m, n) lattice system [6], [38]–[40].

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