

Partial Transmit Sequence (PTS) based PAPR reduction for OFDM using improved harmony search evolutionary algorithm

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ABSTRACT

Partial Transmit Sequence (PTS) is one of the promising techniques, that provides good Peak-to-Average Power Ratio (PAPR) reduction for Orthogonal Frequency Division (OFDM) signal. The traditional PTS (T-PTS) has high complexity, as it involves extensive random search over all the combinations of allowed phase vectors. Moreover, the search complexity increases exponentially with the increase in number of phase vectors. In this paper, a suboptimal meta-heuristic algorithm for phase optimization based on Improved Harmony Search is proposed. Harmony Search mimics the improvisation process of music players. The proposed improved harmony search based PTS (IHS-PTS) provides improved performance compared to harmony search algorithm. Simulation results show significant reduction in PAPR using simple network structure compared to conventional harmony search algorithm and firefly algorithm.

Categories and Subject Descriptors

C.2.1 [Network Architecture & Design]: Wireless Communication; C.4 [Performance of Systems]: Metrics—*performance attributes*

General Terms

Measurement, Performance, Theory, Verification

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Keywords

Improved Harmony Search (IHS), Orthogonal Frequency Division Multiplexing (OFDM), Traditional Partial Transmit Sequence (T-PTS), Peak to Average Power Ratio (PAPR)

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) system is employed in variety of wireless broadband communication systems like Digital Audio Broadcasting (DAB), Terrestrial Digital Video Broadcasting (DVB-T), wireless LAN, WiMAX, smart grid system etc. OFDM systems inspite of their desirable qualities have major drawback on large envelope variations resulting in high Peak-to-Average Power Ratio (PAPR). This significantly reduces power efficiency of the system. Hence, efficient methods for PAPR reduction is essential in high speed wireless communication systems. In addition to this, reduction in PAPR is instrumental in removal of non-linear effect and power efficiency of power amplifier [12].

Several PAPR reduction schemes have been proposed, among which Partial Transmit Sequence (PTS) has been the most promising one due to its good PAPR reduction capability without restriction on the number of subcarriers [13]. In PTS, the data block is divided into disjoint sets called sub-blocks and the subblocks are combined followed by multiplication of a phase vector. In PTS, design of the optimal transmit phase selection of the optimum phase vector from a set of known solutions is most challenging.

In Traditional PTS (T-PTS), the exhaustive search space for optimal phase factor rises exponentially with the number of sub-blocks [14]. Rise in the search space leads to increased computational complexity. Numerous sub optimal PTS method have been developed [5]. The iterative flipping algorithm for PTS has the computational complexity linearly proportional to number of sub blocks [3]. Evolutionary based techniques have attracted attentions of researchers in recent time for solving variety of optimization problems. Some of these includes Tabu Search, Particle Swarm Optimization, Genetic Algorithm, Ant Colony algorithm, Differential Evolution etc. Recently, optimization algorithms have been applied to PAPR reduction of OFDM. These includes Particle Swarm Optimization [6], Genetic Algorithm [9], Artificial Bee Colony algorithm [15], Harmony Search [8] etc. An improved harmony search algorithm based on

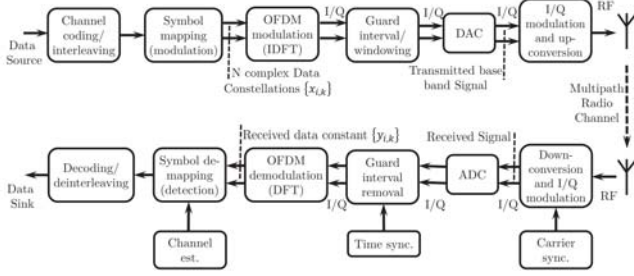


Figure 1: The block diagram of OFDM system

harmony search has been proposed recently [10]. This paper provides performance analysis for PAPR reduction in OFDM using this improved harmony search algorithm as compared to FF-PTS and HS-PTS.

The rest of this paper is organized as follows. In Section 2, the OFDM signal is analyzed in terms of PAPR and Complementary Cumulative Distribution Function (CCDF). Section 3 discusses Traditional PTS (T-PTS) technique in detail. Then, in Section 4, evolutionary optimization algorithms like harmony search (HS) and its variant IHS is explained and proposed IHS-PTS algorithm is described. Performance of proposed IHS-PTS is compared and evaluated based on various simulation result in Section 5. Finally, the paper is concluded in Section 6.

2. OFDM AND PEAK TO AVERAGE POWER RATIO (PAPR)

OFDM is a multicarrier modulation technique, where a block of N data symbols $X_n, n = 0, 1, \dots, N - 1$ form one OFDM signal to be transmitted using one sub-carrier for each symbol. Figure 1 shows the block diagram of a OFDM system. The transmit bit stream is divided into several orthogonal sub-carriers, each modulated at low rate [12]. Each OFDM signal modulates a different sub-carrier from the set $f_n, n = 0, 1, \dots, N - 1$. The N sub-carriers are orthogonal i.e. $f_n = n\Delta f$, where $\Delta f = \frac{1}{NT}$ and T is the symbol period. The complex baseband representation of a multicarrier signal consisting of N sub-carriers is given by

$$x_n = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi n t f_n} \quad (1)$$

where, $0 \leq t \leq T$

As per the definition, PAPR of the transmitted signal is the ratio of peak power to the average power of the signal, i.e.

$$PAPR = \frac{\max |x_n|^2}{E \{|x_n|^2\}} \quad (2)$$

Usually, CCDF is used to show performance measure for PAPR reduction techniques. It calculates the probability that the PAPR of a data block exceeds a given threshold z . The CCDF of the PAPR of N symbols of a data block with Nyquist rate sampling is defined as

$$P_r(PAPR \geq PAPR_0) = 1 - P_r(PAPR \leq PAPR_0) = 1 - (1 - e^{-PAPR_0})^N \quad (3)$$

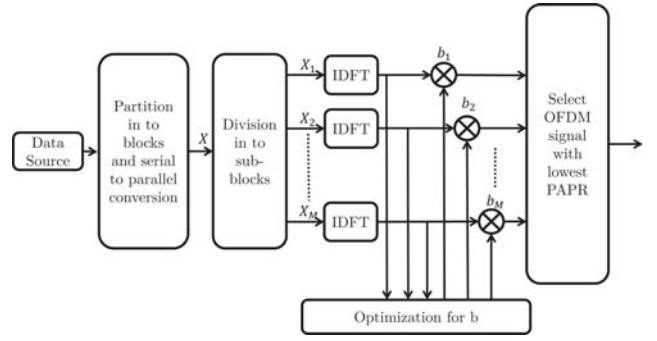


Figure 2: Block Diagram of Traditional Partial Transmit Sequence Technique

The CCDFs are usually compared in a graph such as Figure 3, which shows the CCDFs of the PAPR of an OFDM signal with 256 subcarriers for quaternary phase shift keying (QPSK) modulation.

3. TRADITIONAL PARTIAL TRANSMIT SEQUENCE TECHNIQUE

In Traditional Partial Transmit Sequence (T-PTS) technique as shown in Figure 2, the whole data Block X is partitioned into M disjoint sub-blocks as $X = [X_0, X_1, \dots, X_{M-1}]^T$. Then calculating individual sub-block IFFT's as

$$x_m = [x_{m,0}, x_{m,1}, \dots, x_{m,M-1}]^T \quad (4)$$

where $m = 1, 2, \dots, M$

Following this, all these disjoint sets are multiplied with random phase vectors b_1, b_2, \dots, b_M . This is the stage, where phase optimization techniques are employed to determine the optimal phase vector to provide lowest PAPR. After this, only the sets with optimal phase vectors having lowest PAPR is transmitted. T-PTS technique involves selection of OFDM signal with lesser phase factor set, results more complex in nature. This requires the transmitter to keep track the information of the sent data blocks[7].

Subblock partition for PTS OFDM is a method of division of subbands in to multiple disjoint subblocks. There are three kinds of sub-block partitioning scheme: adjacent, interleaved and pseudo-random partitioning [11]. The input signal (candidate signal) is the sum of the products of phase rotation vector and corresponding sub-block. Thus, the candidate signal is given by

$$x^c = \sum_{m=1}^M b_m^c x_m = [x_0^c, x_1^c, \dots, x_{N-1}^c]^T \quad (5)$$

where $c = 1, 2, \dots, C$

The main issue with PTS technique is its complexity. Since the PTS technique requires all the combinations of allowed phase factors for an optimum exhaustive search [14], leading to exponential rise in subblocks. Subblock partitioning also affects the PAPR reduction performance in PTS. The PTS technique works with an arbitrary number of subcarriers and all modulation scheme.

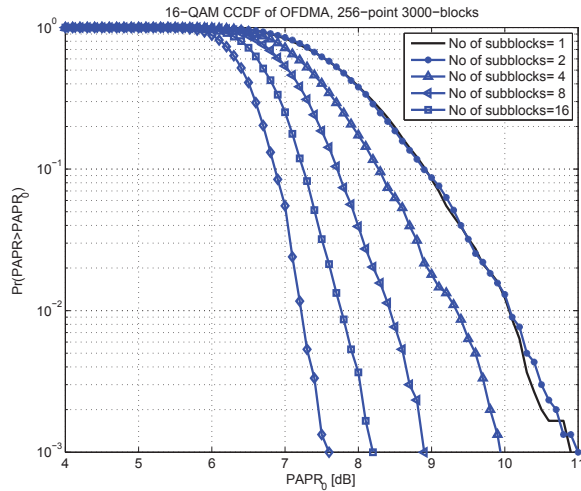


Figure 3: PAPR vs CCDF of Traditional Partial Transmit Sequence (T-PTS)

The simulation result for CCDFs of the PAPR of an OFDM PTS signal with 256 subcarriers for QPSK modulation with pseudo-random sub block partitioning shown in Figure 3. For 1 disjoint subsets (i.e. $M=1$), the PAPR is calculated is 10.9 (approx..) at CCDF of 10^{-3} , for 2 disjoint subsets (i.e. $M=2$), PAPR observed is 9.8 (approx..) at CCDF of 10^{-3} , for 4 disjoint subsets (i.e. $M=4$), PAPR is calculated is 7.8 (approx..) at CCDF of 10^{-3} . Similarly, for 16 disjoint subsets (i.e. $M=16$), PAPR is 7.6 (approx..) at CCDF of 10^{-3} . From the above simulation results it can be deduced that for more number of subsets the PAPR is less. Alternatively, PAPR reduction capability improves with increasing number of subcarriers [2].

4. IMPROVED HARMONY SEARCH PTS (IHS-PTS) OPTIMIZATION

In order to get the OFDM signals with the minimum PAPR, a suboptimal combinatorial method based on improved harmony search (IHS) is proposed to solve the optimization problem of PTS. IHS-PTS algorithm can get better PAPR performance.

The minimum PAPR for PTS method is relative to the problem:

Minimize

$$f(b) = \frac{\max [|x(b)|^2]}{E [|x(b)|^2]} \quad (6)$$

subject to

$$b \in \left\{ e^{j\phi_m} \right\}^M \quad (7)$$

where $\phi_m \in \left\{ \frac{2\pi k}{W} \mid k = 0, 1, \dots, W-1 \right\}$.

4.1 Harmony Search Algorithm

Harmony Search is a music-based metaheuristic optimization algorithm which is inspired by the observation of musician's improvisation process that aims to search for the perfect state of harmony in the music. The perfectly pleasing harmony is determined by the audio aesthetic standard. Similarly, in PTS algorithm aims to search best combination

of phase vectors that gives lowest PAPR as well as reduces complexity [8]. The harmony search Algorithm idealizes the improvisation process by a skilled musician. When a musician is improvising, a musician has three possible choices [4]:

- Play any famous piece of music (a series of pitches in harmony) exactly from the memory which corresponds to harmony memory.
- Play something similar to a known piece (thus adjusting the pitch slightly) which corresponds to pitch Adjusting.
- Compose new or random notes which corresponds to randomization.

State of perfect harmony is reached by adjusting above three parameters. Traditional Harmony Search Algorithm is :

Step 1: Initialization of all the parameters that include upper bound, lower bounds, Harmony memory accepting or consideration rate (HMCR), Pitch Adjustment Rate (PAR), distance bandwidth (bw), Harmony memory Size (HS) and total number of function evaluations (N)

Step 2: Population of harmony memory by possible set of solutions or harmonies randomly. It can be a matrix or vector as :

$$HM = \begin{bmatrix} x_1^1 & x_2^1 & \dots & x_N^1 \\ x_1^2 & x_2^2 & \dots & x_N^2 \\ \dots & \dots & \dots & \dots \\ x_1^{HMS} & x_2^{HMS} & \dots & x_N^{HMS} \end{bmatrix}$$

Step 3: Harmony Improvisation by determining a new vector by adjusting three parameters i.e. memory consideration, pitch adjustment and random selection. Memory Consideration determines whether the new vector will be generated by harmony memory values or randomly.

Step 4: If the new Harmony is better than the worst fit harmony in the then it will be replaced by the new harmony otherwise step 2 is repeated until total number of function evaluations is reached.

Various variants of harmony search algorithms are available in literature [1].

4.2 Improved Harmony Search Algorithm

Performance of any metaheuristic algorithm like harmony search depends upon two factors: exploration and exploitation. Optimal and balanced combination of these two contradicting factors determines the efficiency of the algorithm. Improved Harmony Search Algorithm attempts to enhance accuracy and convergence rate of harmony search by adjusting PAR and bw to be updated dynamically as [10]-

$$PAR(k) = PAR_{\min} + \left(\frac{PAR_{\max} - PAR_{\min}}{K} \right) k \quad (8)$$

Algorithm 1 Harmony Search Algorithm

Define fitness function $f(b), b = [b_1, b_2, \dots, b_m], m = 1, 2, \dots, M$
Define $HMCR, PAR, HMS$
Define Maximum number of iterations (K)
 $HM \leftarrow$ Generate Initial phase factor vectors.
while iterative number $\leq K$ **do**
 while $m < M$ **do**
 if $rand \in (0, 1) \leq HMCR$ **then**
 choose a value from HM form
 if $rand \in (0, 1) \leq PAR$ **then**
 adjust the value of m by:
 $b_{m_new} = b_{m_old} * rand\{+1, -1\}$
 end if
 else
 choose a value from the possible solution collections $\{+1, -1\}$
 $b_{m_new} = rand\{+1, -1\}$
 end if
 end while
 if $Fitness(b_{m_new}) \leq Max(Fitness(HM))$ **then**
 accept the new phase factor and replace the worst one in HM with it
 end if
end while
 $best =$ find the current best phase factor

$$bw(k) = bw_{\max} \exp\left(\frac{\ln(bw_{\min}/bw_{\max})}{K}\right) k \quad (9)$$

Where k and K represents the current number of improvisations and maximum number of improvisations respectively. As PAR and bw is initialized and fixed in traditional Harmony Search Algorithm it gives low performance as well as more number of iterations are needed for finding the optimal Solution. Adjusting PAR and bw in each improvisation step, gives better convergence rate leads to optimal solutions. These parameters are set initially high to find a diversity in search of solution vectors in the harmony memory so that all the possible combinations are explored to a greater level. On the other hand these parameters have low value for the exploitation of optimal solution in final stages. This gives suboptimal solution to enhance the performance of the traditional Harmony Search Algorithm.

4.3 Improved Harmony Search-PTS approach

The steps of proposed IHS-PTS algorithm are as follows:

- **Parameter initialization :**
The specified parameters are : harmony memory considering rate ($HMCR$) = 0.95; harmony memory size (HMS) i.e. number of solutions available in memory = 16; pitch adjustment rate (PAR) = 0.05; number of musical instruments i.e. number of each phase factor vector = M ; pitch range of each instrument i.e. value range of each phase factor = $\{+1, -1\}$ and stopping criterion K .
- **Harmony Memory (HM) initialization :**
Harmony memory is initialized with possible set of phase vectors .Each row in the matrix is a set of solution by evaluating the objective function between

lower and upper bound values which results in randomly populating the solutions for each structure i ($= 1, 2, \dots, HMS$), the objective function $f(x)$ is evaluated, which will take the value from the collection $\{+1, -1\}$.

- **New solution construction :**
Harmony improvisation is done by creating a new harmony vector as $x' = (x'_1, x'_2, \dots, x'_N)$ using Improved Harmony Search (IHS) Algorithm described in section 4.2 and updating the parameter values of PAR and BW in each iteration.
- **Update HM :**
Updation of the harmony vector is done by comparing the new harmony vector with the worst fit solution .If the new vector is better than the worst solution , then it is replaced by the new harmony vector.
- **Stopping criterion :**
Above Algorithm is repeated till the total number of function evaluations K is reached, otherwise step 2 is repeated. The search complexity in the IHS-PTS algorithm is equal to MK .

Algorithm 2 Proposed Improved harmony search based-PTS algorithm

Define the Objective function
Define $HMS, HMCR, K, PAR_{\min}, PAR_{\max}, bw_{\min}, bw_{\max}, PVB$ ($PVB =$ Possible value bound for x_i)
Initialize HM
for $x_i = 1 : HMS$ **do**
 Improvise new HM
 for iteration $\leq NI$ **do**
 for $x_i \leq no.ofvariable$ **do**
 $PAR(k) = PAR_{\min} + \left(\frac{PAR_{\max} - PAR_{\min}}{K}\right) k$
 $bw(k) = bw_{\max} \exp\left(\frac{\ln(bw_{\min}/bw_{\max})}{K}\right) k$
 for (all variable) **do**
 if $rand() \leq HMCR$ **then**
 $x'_i = x_i^j (j = 1, 2, \dots, HMS)$ (Choose value from HM)
 if $rand() \leq PAR$ **then**
 $x'_i = x'_i \pm rand() * bw$
 end if
 else
 (Choose a random value of variable)
 $x'_i = PVB_{lower} + rand() * (PVB_{upper} - PVB_{lower})$
 end if
 end for
 if new solution \leq worst solution **then**
 accept the new harmony and replace the worst in HM
 end if
 end for
end for
 $best =$ best current solution

5. SIMULATION RESULTS

To evaluate the performance of IHS-PTS algorithm for OFDM PAPR reduction, various simulations have been conducted.

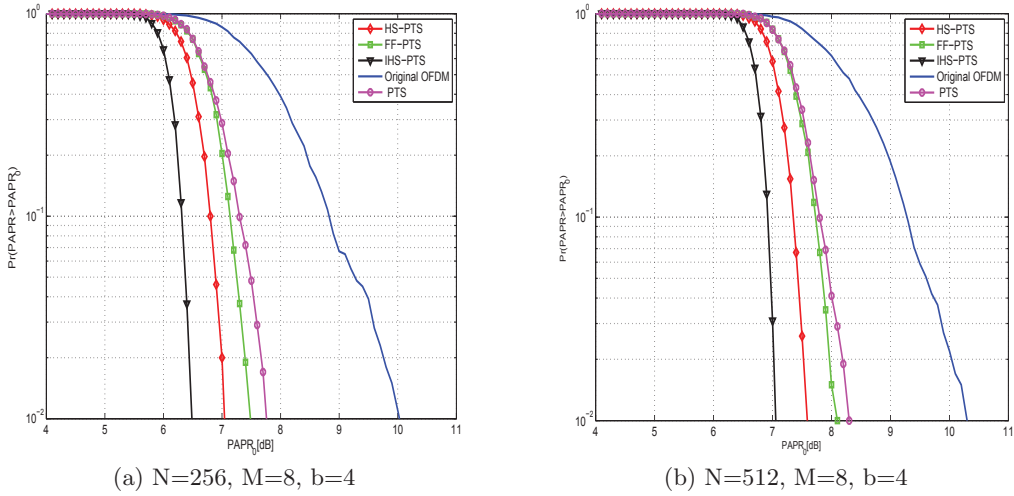


Figure 4: CCDF vs PAPR performance of 16-QAM OFDM PTS system with variation in subcarriers

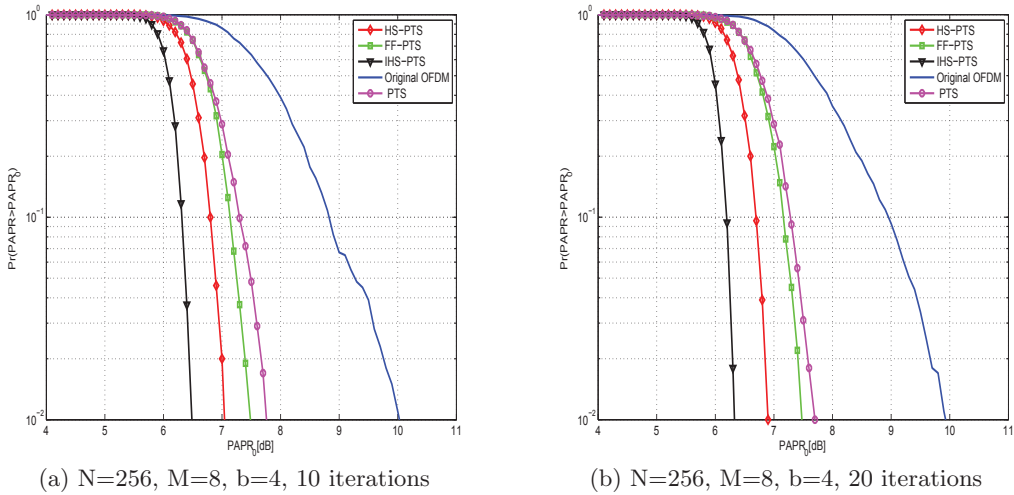


Figure 5: CCDF vs PAPR performance of 16-QAM OFDM PTS system with variation in iterations

The performance of IHS-PTS has been compared with HS-PTS and FF-PTS. For Firefly optimization algorithm, following parameters which are defined: Number of fireflies = 10, number of iterations = 10, absorption coefficient $\gamma = 1$, attractiveness of firefly $\beta = 0.2$ and randomness $\alpha = 0.5$. For Harmony Search Optimization based PTS (HS-PTS) the parameters are defined : harmony memory size (HMS) is set to 16, harmony memory consideration rate (HMCR) is 0.95, pitch adjustment rate (PAR) is 0.05 and the bandwidth of adjustment (bw) is 0.2. The total number of iterations K is 10. To improve the performance of algorithm, improved version of harmony search have been developed called Improved Harmony Search (IHS), in which PAR and bw is dynamically updated. When this algorithm is used with PTS (IHS-PTS), the PAPR performance is improved for OFDM signal. For simulation, the minimum and maximum value of PAR and bandwidth is defined. Here PAR_{min} and PAR_{max} are 0.3 and 0.9 respectively. bw_{min} and bw_{max} is 0.2 and 0.5 respectively. Both values are updated by using the equations

(8) and (9). When these optimizations is performed to optimize the phase factor in PTS method of PAPR reduction, the PAPR is reduced in each case.

In Figure 4, some results of the CCDF of the PAPR are simulated for the OFDM system with 256 and 512 subcarriers (N), in which $M=8$ sub-block employing original OFDM, T-PTS, FF-PTS, HS-PTS and IHS-PTS algorithm and the phase weight factor b , uniformly distributed random variable are used for PTS. In PTS, when $N= 256, M= 8$, then the PAPR of original OFDM signal and OFDM-PTS are 9.9 dB and 7.7 dB respectively. After applying HS-PTS, FF-PTS and IHS-PTS, PAPR are 7.1 dB, 7.5 dB and 6.5 dB respectively with CCDF of 10^{-2} . However with $N= 512$, the PAPR of original OFDM signal and OFDM-PTS are 10.3 dB and 8.3 dB approximately. The PAPR of OFDM signal after applying HS-PTS, FF-PTS and IHS-PTS, are 7.6 dB, 8.1 dB and 7.1 dB respectively with CCDF of 10^{-2} . So, it has been observed that PAPR values are dependent on number

Table 1: Comparison of computational complexity of different optimization algorithms at CCDF = 10^{-2} , Modulation format : 16-QAM

Methods	Computational Complexity	PAPR (db)
HS-PTS	$M * W * K = 8 * 2 * 10 = 160$	7.1
	$M * W * K = 8 * 2 * 20 = 320$	6.9
FF-PTS	$n^2 * K = 10^2 * 10 = 1000$	7.5
	$n^2 * K = 10^2 * 20 = 2000$	7.4
IHS-PTS	$M * W * K = 8 * 2 * 10 = 160$	6.5
	$M * W * K = 8 * 2 * 20 = 320$	6.4

of subcarriers used for OFDM generation and, the optimization algorithms based PTS gives the improvement in PAPR performance in each case. From above the descriptions, we can see that IHS-PTS is effective technique for reducing the PAPR of OFDM system.

Figure 5 compares the performance of the IHS-PTS technique as function of iterations. As the number of sub-blocks and the set of phase weighting factor are increased, the performance of the PAPR reduction becomes better. However, the processing time gets longer because of much iteration. When the number of iterations are 10, then PAPR of OFDM signal, after applying HS-PTS, FF-PTS and IHS-PTS are 7.1 dB, 7.5 dB and 6.5 dB respectively. When the number of iteration is 20, then PAPR of original OFDM signal after applying, HS-PTS, FF-PTS and IHS-PTS are 6.9 dB, 7.5 dB and 7.6.4 dB respectively. As the number of iterations are increased, the performance of the PAPR reduction becomes better. However, the processing time gets longer because of much iteration and the number of function evaluation or computation complexity is also increased. So it can be seen that the optimization based PTS schemes gives good PAPR performance.

The computational complexity is also analyzed and compared for each optimization method as shown in Table 1.

6. CONCLUSION

In this paper, we have applied a variant of harmony search algorithm i.e. improved harmony search based PTS algorithm (IHS-PTS) to search optimum combination of phase factor for OFDM signals. Compared to the existing PAPR reduction optimization methods, like Firefly algorithm, harmony Search algorithm etc., the IHS-PTS algorithm provides improved PAPR due to its simple structure and very few parameters to adjust for larger PTS subblocks at the same time. Simulation results of the IHS-PTS algorithm shows that it is an efficient and feasible method, which can provide better PAPR performance. Improved PAPR performance using efficient optimization techniques can be key to future wireless high speed communication systems.

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