Bio-Inspired Game Theory: The Case of Physarum Polycephalum

Andrew Schumann
University of Information Technology and Management
Sucharskiego 2
35-225 Rzeszow
Poland
andrew.schumann@gmail.com

Krzysztof Pancerz
University of Information Technology and Management
Sucharskiego 2
35-225 Rzeszow, Poland;
University of Management and Administration
Akademicka 4
22-400 Zamość, Poland
kpancerz@wszia.edu.pl

Andrew Adamatzky
Unconventional Computing Centre
UWE, Bristol, UK
andrew.adamatzky@uwe.ac.uk

Martin Grube
Institute of Plant Sciences,
University of Graz
Schubertstrasse 51
8010 Graz, Austria
martin.grube@uni-graz.at

ABSTRACT

In this paper, first, we show that the true slime mould (plasmodial stage of *Physarum polycephalum*) is a natural transition system which can be considered a biological model for concurrent games, i.e. it can simulate the game semantics in the form of concurrent games. Second, we extend the notion of concurrent games to context-based games and show that this new form of games is a game semantics that is more suitable for the implementation multi-agent games in the slime mould behavior. The notion of context-based games as strong extension of concurrent games is introduced for the first time. Games on the medium of one-cell organism are defined for the first time, too. In context-based games, we appeal to the following game-theoretic assumptions: (i) each game can be assumed infinite, because its rules can change; (ii) players can change their strategies and the set of actions is infinite for each player; (iii) resistance points for players are reduced to the payoffs if all actions are well-founded; (iv) for any game there is performative efficiency, when hybrid actions of players belong to the interval of expected modifications. Logic circuits on the medium of slime mould can be designed in the form of context-based games.

Categories and Subject Descriptors

D.2.2 [**Design Tools and Techniques**]: Object-oriented design methods; F.1.2 [**Models of computation**]: Parallelism and concurrency; I.2.1 [**Applications and Expert systems**]: Games

General Terms

Languages, Theory

Keywords

labelled transition system, concurrent game, reflexive game, Physarum polycephalum, Badhamia utricularis

1. INTRODUCTION

Physarum polycephalum belongs to the supergroup Amoebozoa, phylum Mycetozoa, class Myxogastria. At the active feeding stage, it has the form of plasmodium that moves by protoplasmic streaming and can switch its direction or even multiply in accordance with appropriate attractants (chemical signals which attract the organism) and repellents (chemical signals which repel it). This behavior is intelligent and can be controlled by different locations of chemical signals attracting and repelling the plasmodium. *Physarum polycephalum* can solve different tasks: maze-solving [9], solving the Steiner Problem [24], minimum-risk path finding [10], [11], solving the traffic optimization problem [27], associative learning [22], memorizing and anticipating repeated events [15], etc.

Hence, the plasmodium of *Physarum polycephalum* can be considered one of the biological models of natural computation [4]. The matter is that the plasmodium spread by networks can be programmable by our knowledge of chemotaxis (how the plasmodium responses to chemical signals). In standard electric devices, we deal with electrical signals, such as variations in voltage, current, frequency, phase, or duration, which serve to code information. We can assume that it is possible to design a biological device, too,

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where instead of electrical signals the calculation process is performed by using the plasmodium chemotaxis [4], [28]. So, in *Physarum Chip Project: Growing Computers From Slime Mould*, we are going to construct such an unconventional computer on programmable behavior of *Physarum polycephalum.* The main problem of this research goal is that the plasmodium is not a deterministic system and it can change its past decisions in looking for food. In other words, the plasmodium behavior is intelligent, but not algorithmic. This means that in designing the plasmodium chip we need mathematics beyond the foundation axiom [3], i.e. mathematics without conventional algorithms.

In this paper, we propose a context-based game theory as an example of new mathematics which could be applied in programming biological devices on plasmodia. The standard game theory is reduced to usual algorithmic mathematics and can be presented as multiplicative linear logic [2]. Mathematically, it is a very simple system. The context-based game theory proposed by us is constructed on transition (non-linear) systems, where agents, who can change their past decisions, move. So, we formulate the new mathematics (the so-called *non-well-founded mathematics*), which we are going to use in coding information on behavioral systems such as plasmodia, in the form of new game theory only to show abilities of this mathematics more explicitly. Our goal is to construct an abstract complete processor on plasmodia by using unconventional (co)algorithms of new mathematics.

The standard game theory as well as standard mathematics cannot be implemented by plasmodia as a kind of processors, because of behavioral freedom and contextuality in plasmodium decisions. Logic circuits on behavioral system need the non-well-founded mathematics which can be exemplified by context-based game theory. Thus, this new game theory is our intermediate result to express features of plasmodium processor. However, it can be interesting as such, too, as a new scientific branch called *bio-inspired game theory.*

2. TRANSITION SYSTEMS OF PLASMODIA AND HYBRID ACTIONS

Notice that the *Physarum polycephalum* motions are a kind of *natural transition systems*, $\langle States, Edges \rangle$, where $States$ is a set of states presented by attractants and $Edges \subseteq States \times States$ is a transition of plasmodium from one attractant to another [4], [19], [20]. The point is that the plasmodium looks for attractants, propagates protoplasmic tubes towards them, feeds on them and goes on. As a consequence, a transition system is built up.

According to experiments with the slime mould [4], we can define the following four basic forms of plasmodium transition (motion): (i) $Direct$, the *direction* which is a move from one place, where the plasmodium is located, towards another place, where there is a neighbor attractant; (ii) $Fuse$, the *fusion* of two plasmodia at the place, where they meet the same attractant; (iii) $Split$, the *splitting* of plasmodium from one active place into two active places, where two neighbor attractants with a similar power of intensity were located; (iv) $Repel$, the *repelling* of plasmodium or its *inaction.* On the basis of $Direct$, $Fuse$, $Split$, and $Repel$ we can construct the labelled transition system for *Physarum polycephalum* (called *Physarum* spatial logic), where actions of $Direct$ and $Repel$ are understood as simple and actions of $Fuse$ and $Split$ as composite: namely, actions of $Fuse$ is considered a composition $\alpha \& \beta$ of simple actions α and β and actions of $Split$ is considered a composition $\alpha + \beta$ of simple actions α and β (for more details, see [19]). However, we can define simple actions more complicatedly as logic gates AND, OR, NOT at once. Evidently, AND, NOT, in the way they are defined in [21], can be reduced to some actions of $Direct$, and OR to a combination of actions of $Direct$, $Fuse$, and $Split$. Also, we can define simple actions as logic gates ALL, NO, SOME, SOME-NOT of Aristotelian and pragmatic syllogistic. Thus, we can use different logics for defining simple actions of *Physarum* transition systems: spatial logic [19], Boolean logic [21], Aristotelian and non-Aristotelian syllogistics [18], etc. In this paper, we focus on actions as composite from logic gates AND, OR, NOT. All the transitions will be built on their compositions with n inputs.

Now, labelled transition systems have been used for defining the so-called *concurrent games*, a new semantics for games [1]. Traditionally, a play of the game is formalized as a sequence of moves. This way assumes the polarization of two-person games, when in each position there is only one player's turn to move. These sequential games can hold just on models of fragments of linear logic such as multiplicative [2] or multiplicative-exponential fragments [5]. In concurrent games, players can move concurrently. On the medium of *Physarum polycephalum* we can, first, define concurrent games and, second, extend the notion of concurrent games strongly and introduce the so-called *context-based games.*

While in concurrent games we deal with a finite set of actions, in context-based games we can move concurrently as well, but the set of actions is infinite. One of the simple versions of context-based games was introduced in [17], it is called reflexive games. In that version, we use cellular automata of payoffs to define reflexive games of human beings [16], [17]. The idea is that in reflexive games, on the one hand, we try to be unpredictable for other players and, on the other hand, to predict them. In these games, the set of actions is infinite. But this infiniteness is connected rather to non-well-founded properties of basic actions [3]. They cannot be regarded as atomic so that composite actions can be obtained over them inductively. In other words, it is possible to face a hybrid action which is singular, but it is not one of the basic simple actions. It is a hybrid of them. An example of this hybrid act for *Physarum polycephalum* was considered in [13]. In that paper, we performed the double-slit experiment for the slime mould and showed that just as in quantum mechanics we cannot approximate single photons (also, electrons, neutrons, etc.), in the experiments with *Physarum polycephalum* we cannot approximate single acts. In the current paper, we define context-based games for the first time. It is a generalization of concurrent games, on the one hand, and reflexive games, on the other hand. In our opinion, this form of games can be regarded as *fundamentals of bio-inspired game theory.* Also, this form of game is a simple example of non-well-founded mathematics that can be applied in programming the plasmodium processors or in programming processors on any other predictable behavioral systems.

According to our hypothesis [12], natural computations may be constructed on an infinite set of hybrid actions, which are built up on the set of basic simple actions that is always finite. We are going to design logic circuits for the unconventional computer on programmable behavior of *Physarum polycephalum* as a kind of context-based games, where hybrid actions for transitions are obtained on logic gates AND, OR, NOT. This approach in unconventional computing proposed by us is relatively new. In this approach, on the one hand, we consider the behavior of one-cell organisms as intelligent and we want to obtain the new game theory on the basis of biological experiments, where fundamentals are defined on the experiments with one-cell organisms. On the other hand, we design logic circuits by using non-well-founded mathematics illustrated by context-based games and these circuits can be implemented on the medium of behavior of different organisms from parasites [20] to human beings. The context-based game is a generalization of incomplete information game, concurrent game, repeated game, reflexive game and some other modern approaches to games.

3. CONCURRENT GAMES ON PHYSARUM POLYCEPHALUM

So, the *Physarum polycephalum* plasmodium can be interpreted as transition system $\mathcal{S} = \langle States, Edges \rangle$, where (i) $States$ is a set of states presented by attractants occupying by the plasmodium, (ii) $Edges \subseteq States \times States$ is the set of transitions presenting the plasmodium propagation from one state to another. States will be regarded as possible payoffs for *Physarum polycephalum.* By different localizations of attractants we can manage its motions differently. These localizations combined with different intensity of attractants are stimuli for *Physarum polycephalum* motions. We can interpret these stimuli as Boolean functions on payoffs. For example, we can define the following simplest logical gates:

- The *AND gate* (*Physarum conjunction*) (see Fig.1), the serial connection of contacts. Plasmodium P follows conjunction $A_1 \wedge A_2$ if and only if it reaches both attractants A_1 and A_2, i.e. the plasmodium is attracted to A_1, when it is placed in its region of influence, then it is is attracted by A_2 in the region of its influence. If we have deactivated at least one of the attractants A_1, A_2, the plasmodium cannot reach conjunction $A_1 \wedge A_2$, i.e. the latter is false. *Physarum polycephalum* conjunction has two inputs A_1, A_2 and one output $A_1 \wedge A_2$.
- The *OR gate* (*Physarum disjunction*) (see Fig.2), the parallel connection of contacts. Plasmodium P follows disjunction $A_1 \vee A_2$ if and only if one of the attractants A_1 or A_2 or both are occupied by the plasmodium. In case of deactivation of both attractant A_1 and attractant A_2 the plasmodium cannot start from the initial position. This means that $A_1 \vee A_2$ is false. *Physarum polycephalum* disjunction has two inputs A_1, A_2 and one output $A_1 \vee A_2$.
- The *NOT gate* (*Physarum negation*) (see Fig.3). Plasmodium P follows negation $\neg A_1$ if and only if its behavior is simulated by the repellent R before the attractant A_1, which avoids the plasmodium to be attracted by A_1. *Physarum polycephalum* negation has one input A_1 and one output $\neg A_1$.

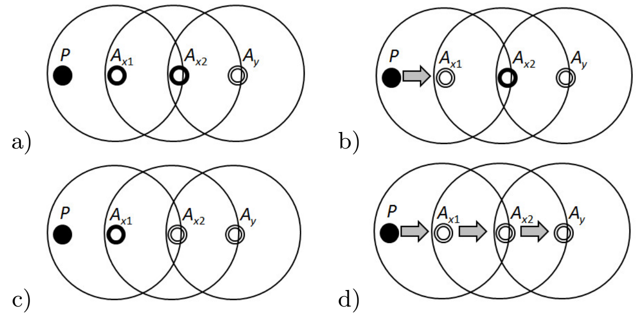

Figure 1: States of the AND gate for all input combinations of A_{x1} and A_{x2} with one output A_y: (a) both attractants A_{x1} and A_{x2} are deactivated; (b) A_{x1} is activated and A_{x2} is deactivated; (c) A_{x1} is deactivated and A_{x2} is activated; (d) both A_{x1} and A_{x2} are activated. The *Physarum polycephalum* plasmodium denoted by P begins to move from the left hand-side.

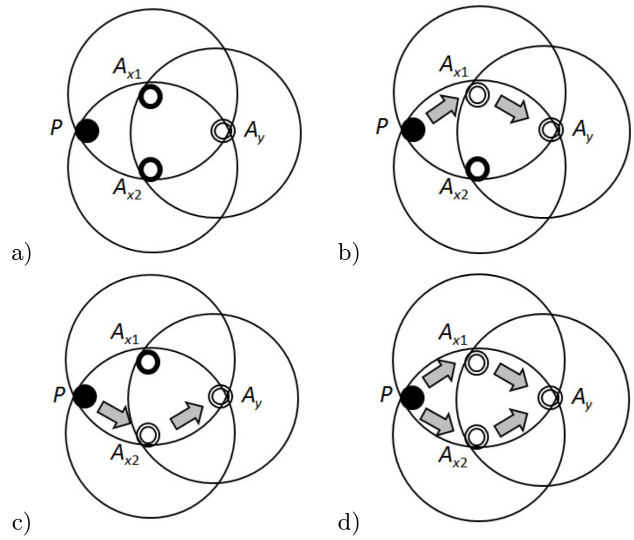

Figure 2: States of the OR gate for all input combinations of A_{x1} and A_{x2} with one output A_y: (a) both attractants A_{x1} and A_{x2} are deactivated; (b) A_{x1} is activated and A_{x2} is deactivated; (c) A_{x1} is deactivated and A_{x2} is activated; (d) both A_{x1} and A_{x2} are activated. The *Physarum polycephalum* plasmodium denoted by P begins to move from the left hand-side.

Plasmodia perform logical gates to realize Boolean functions on payoffs. Games constructed on reasoning on payoffs are called *reflexive* [16], [17]. Some of them can be simulated by means of *Physarum polycephalum* behaviors. If reflexive games are constructed by a finite set of actions defined as inductive compositions of logic gates presented above, then these games are called *concurrent*, where plasmodia are players:

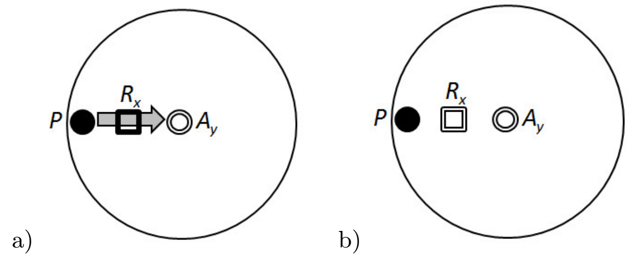

Figure 3: States of the NOT gate for all input combinations with one output A_y: (a) repellent R_x is deactivated; (b) R_x is activated.

Definition 1. *A (finite)* concurrent game on Physarum polycephalum *is a tuple* $\mathcal{G} = \langle States, Agt, Act_n, Mov^n, Tab^n, (\preceq_A)_{A \in Agt} \rangle$, *where*

- *States is a (finite) set of* states *presented by attractants occupying by the plasmodium;*
- $Agt = \{1, \ldots, k\}$ *is a finite set of* players *presented by different active zones of plasmodium;*
- Act_n *is a non-empty set of* actions *presented by logic gates or their inductive combinations with n inputs and one output, an element of* Act_n^{Agt} *is called a* move*;*
- $Mov^n \colon States^n \times Act_n^{Agt} \to 2^{Act} \setminus \{\emptyset\}$ *is a mapping indicating the* available *sets of actions to a given player in a given set of states,* $n > 0$ *is said to be a radius of plasmodium actions, a move* $m^n_{Agt} = (m^n_A)_{A \in Agt}$ *is legal at* $\langle s_1, \ldots, s_n \rangle$ *if* $m^n_A \in Mov^n(\mathbf{s}, A)$ *for all* $A \in Agt$*, where* $\mathbf{s} = \langle s_1, \ldots, s_n \rangle$*;*
- $Tab^n \colon States^n \times Act_n^{Agt} \to States$ *is the transition table which associates, with a given set of states and a given move of the players, the set of states resulting from that move;*
- *for each* $A \in Agt$, $\preceq_A$ *is a preorder (reflexive and transitive relation) over* $States^\omega$*, called the* preference relation *of player A, indicating the intensity of attractants; for each* $\pi, \pi' \in States^\omega$*, by* $\pi \preceq_A \pi'$ *we denote that* π' *is* at least as good as π *for A and when it is not* $\pi \preceq_A \pi'$*, we say that A* prefers π *over* π'*.*

Assume there is a game G and its states are formulated as some propositions, whose set is denoted by *Prop*.

Definition 2. *A concurrent game on* Physarum polycephalum $\mathcal{G}$ *is a* model $\mathcal{M}$ *for the game G if* $\mathcal{M} = \langle \mathcal{G}, \varphi \rangle$*, where* $\varphi \colon States \to \mathcal{P}(Prop)$ *is a labelling function such that labels the states in* $\mathcal{G}$ *by proposition symbols from the set Prop of the game G.*

Actions presented above can be expressed in our new object-oriented programming language created for *Physarum polycephalum* computing [21]. In this language, there are inbuilt sets of prototypes corresponding to the several high-level models used for describing behavior of *Physarum polycephalum*, e.g. ladder diagrams, transition systems, Petri nets.

Usually, for each player (plasmodium) named $1, \ldots, k$, where $k = |Agt|$, we have a separate space of attractants. However, the space can be joint, too. The point is that if two players (plasmodia) move to the same state, then their transitions from this state are the same. In order to avoid this property, we should design different moves of different players in different spaces.

The set of *outcomes* $Out_\mathcal{G}(s)$ of the concurrent game $\mathcal{G}$ from the state s is a set of all infinite paths $s_0 s_1 \ldots \in States^\omega$ such that $s_0 = s$ and for all $j > 0$, there exists a move $m \in \prod_{k=1}^{|Agt|} Mov(s_j, k)$ and $Tab(s_j, m) = s_{j+1}$. The *set of all outcomes* is as follows: $Out_\mathcal{G} = \bigcup_{s' \in States} Out_\mathcal{G}(s)$. The set of *histories* $Hist_\mathcal{G}(s)$ starting in s is a set of all finite paths $s_0 s_1 \ldots s_l$ such that $s_0 = s$ and there exists $\sigma \in Out_\mathcal{G}(s)$ which starts with $s_0 s_1 \ldots s_l$. The set $Hist_\mathcal{G} = \bigcup_{s' \in States} Hist_\mathcal{G}(s)$ is said to be a *set of all histories.*

A *strategy* of a player j in $\mathcal{G}$ is a mapping $strat_j \colon Hist_\mathcal{G} \to Act$ such that for any history $\sigma \in Hist_\mathcal{G}$ it is true that $strat_j(\sigma) \in Mov(last(\sigma), j)$, where $last(\sigma)$ denotes the last state in the finite path σ. In other words, a strategy $strat_j(\sigma)$ is the choice of legal action in the last state of the history σ which was observed by player j. If we have one space for all players and they have the same last state $last(\sigma)$, then for all of them the next legal action will be the same, too. So, in this case histories of players are unimportant for choosing the next action, i.e. since that $last(\sigma)$ their histories will be the same.

We have performed some experiments showing that there are cases when sets of strategies for players in the same space are always disjoint. Let us suppose that we have only two agents. The first is presented by a usual *Physarum polycephalum* plasmodium. The second by its modification called a *Badhamia utricularis* plasmodium (references on this new culture are contained in [14]). *Physarum polycephalum* grows definitely faster than *Badhamia utricularis* and overtakes more flakes at the same time than the latter (see Fig. 4). Only if the inoculum was "fatter" for *Badhamia utricularis*, it might grow faster. Moreover, if the invasive growth front of *Badhamia utricularis* is well nourished by oat it easily overgrows the opposing tube system of *Physarum polycephalum.* So, at the microscopic level we can find out that in most observations *Physarum polycephalum* could grow into branches of *Badhamia utricularis*, while *Badhamia utricularis* could grow over *Physarum polycephalum* strands. We can see that somehow *Physarum polycephalum* feeds on small branches of *Badhamia utricularis* (see Fig. 5). Thus, in case of *Physarum polycephalum* and *Badhamia utricularis* we observe a competition in the small branches. For them the sets of strategies are disjoint. They never meet the same states.

On the basis of competitions between *Physarum polycephalum* and *Badhamia utricularis*, we can study simplest biological forms of *zero-sum games.*

A *strategy for several players A* is defined as the following tuple $(strat_j)_{j \in A}$ of strategies for all players of A. The set of all outcomes if the players in A follow the strategy $strat_A = (strat_j)_{j \in A}$ is denoted by $Out_\mathcal{G}(s, strat_A)$. All possible outcomes if the players in A obey $strat_A$ is denoted

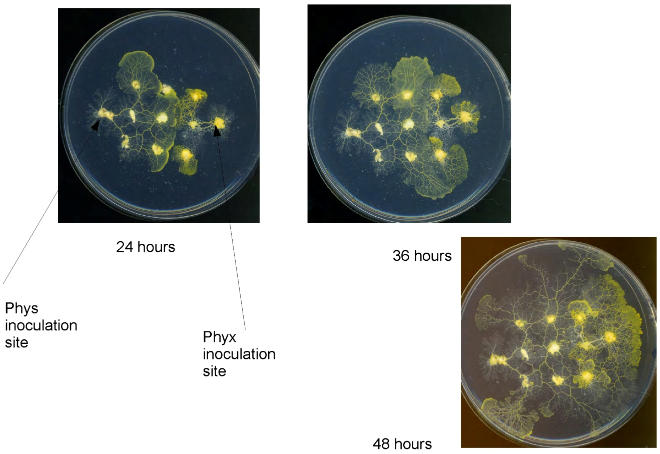

Figure 4: The experiment with two agents: fronts of growing pseudopodia of *Physarum polycephalum* (Phys for short) and *Badhamia utricularis* (PhyX for short).

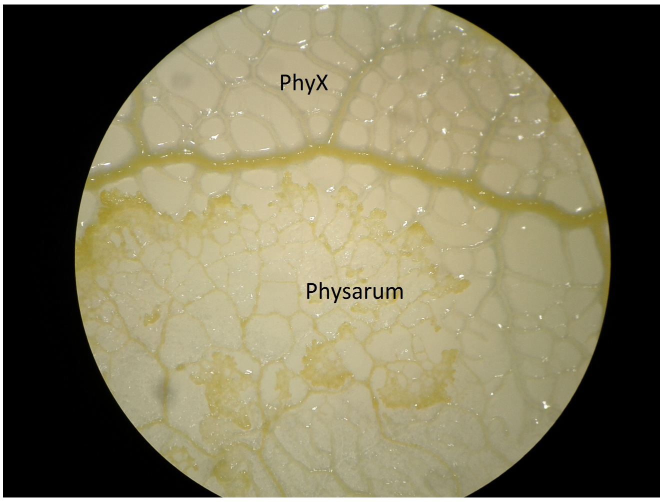

Figure 5: Experiment with two agents: *Physarum polycephalum* (Physarum for short) could grow into branches of *Badhamia utricularis* (PhyX for short).

by $Out_{\mathcal{G}}(strat_A) = \bigcup_{s \in States} Out_{\mathcal{G}}(s, strat_A)$. A strategy for Agt: $(strat_j)_{j \in Agt}$ is called a *strategy profile*.

We will say that a strategy $strat_A$ is *memoryless* for players of A at a state s if they choose their joint action based only on s as the last state of play. This holds if they meet s in a joint space, simultaneously. Notice that in a game with two players presented by *Physarum polycephalum* plasmodium 1 and *Badhamia utricularis* plasmodium 2 in a joint space, their strategies $strat_{Agt=\{1,2\}}$ cannot be memoryless at any time.

Let us take a move m_{Agt} and an action m' for some player B. Assume, the move n_{Agt} with $n_A = m_A$ when $A \neq B$ and $n_B = m'$ is denoted by $m_{Agt}[B \to m']$. Then a Nash equilibrium for concurrent games is defined as follows (for more details on equilibria in concurrent games, see [6], [7], [8]):

DEFINITION 3. *Let $\mathcal{G}$ be a concurrent game with preference relation $(\preceq_A)_{A \in Agt}$ and let s be a state of $\mathcal{G}$. A* Nash equilibrium *of $\mathcal{G}$ from s is a strategy profile $strat_{Agt}$ such that $Out(s, strat_{Agt}[B \to strat']) \preceq_B Out(s, strat_{Agt})$ for all players $B \in Agt$ and all strategies $strat'$ of B.*

Notice that concurrent games can be implemented in plasmodium behavior only with low accuracy, because concurrent games do not assume possibilities to change past decisions, i.e. the set of possible actions is always finite. Nevertheless, the set of actions for the plasmodium is infinite.

4. CONTEXT-BASED GAMES ON PHYSARUM POLYCEPHALUM

Let us consider the following thought experiment as counterexample showing that the set of actions for the plasmodium is infinite in principle. This means that concurrent games can be only approximated on the medium of slime mould, i.e. plasmodia follow concurrent games only uncertainly and probabilistically. Assume that the labelled transition system for the plasmodium consists just of one action presented by one neighbor attractant. The plasmodium is expected to propagate a protoplasmic tube towards this attractant. Now, let us place a barrier with one slit in front of the plasmodium. Because of this slit, the plasmodium can be propagated according to the shortest distance between two points and in this case the plasmodium does not pay attention on the barrier. However, sometimes the plasmodium can evaluate the same barrier as a repellent for any case and it gets round the barrier to reach the attractant according to the longest distance. So, even if the environment conditions change a little bit, the behavior changes, too. The plasmodium is very sensible to the environment.

Thus, simple actions of *Physarum* plasmodia cannot be regarded as atomic so that composite actions can be obtained over them inductively. In other words, it is always possible to face a hybrid action which is singular, but it is not one of the basic simple actions. It is a hybrid of them. In the labelled transition system with only one stimulus presented by one attractant, a passable barrier can be evaluated as a repellent 'for any case'. Therefore the transition system with only one stimulus and one passable barrier may have the following three simple actions: (i) pass trough, (ii) avoid from the left, (iii) avoid from the right. But in essence, we deal only with one stimulus and, therefore, with one action, although this action has the three modifications defined above. Simple actions which have modifications depending on the environment are called *hybrid*. The problem is that the set of actions in any labelled transition systems must consist of the so-called atomic actions – simple actions that have no modifications.

Let us show in examples, how we can extend definition 1 up to the case of context-based games, where actions can be hybrid.

Example 1.

Let $\mathcal{G} = \langle States, Agt, Act_n, Mov^n, Tab^n, (\preceq_A)_{A \in Agt}\rangle$ and $n = 9$, $Agt = \{A_1, A_2\}$, $States = \{0, 1\}$, $Act_n = \{\max, \min\}$. Assume that A_1 follows only max at all legal moves and A_2 follows only min at his legal moves, but the number of outputs is equal the number of inputs. This means that in transition system $\mathcal{S}$, $Edges_{A_i} = \{(\mathbf{s}, \mathbf{s}') \in States^9 \times States^9 \colon Tab^9(\mathbf{s}, m^9_{A_i}) = \mathbf{s}'\}$, where $i = 1, 2$, $m^9_{A_1} = \max$, and $m^9_{A_2} = \min$. This game can be illustrated in the cellular-automatic form with the neighborhood $|N| = 8$ [17]. Let us take cells belonging to the set $\mathbf{Z}^2$, therewith each of cell takes its value in *States*. Let transitions depend on local transition rule $\delta\colon States^9 \to States$ that transforms states of cells taking into account states of 8 neighbor cells. Each step of dynamics is fixed by discrete time $t = 0, 1, 2, \ldots$ At the moment t, the configuration of the whole system (or the global state) is given by the mapping x^t from $\mathbf{Z}^2$ into *States*, and the evolution is the sequence $x^0x^1x^2x^3\ldots$ defined as follows: $x^{t+1}(z) = \delta(x^t(z), x^t(z + \alpha_1), x^t(z + \alpha_2), \ldots, x^t(z + \alpha_8))$, where $\langle \alpha_1, \alpha_2, \ldots, \alpha_8\rangle$ are neighbors of z.

Let us suppose, at $t = 0$ we have the following states, where, given $\langle s_i, s_j\rangle$, s_i means a state for A_1 and s_j means a state for A_2:

$\langle 0,1\rangle$	$\langle 0,0\rangle$	$\langle 1,1\rangle$	$\langle 0,1\rangle$	$\langle 1,0\rangle$
$\langle 1,1\rangle$	$\langle 1,1\rangle$	$\langle 1,0\rangle$	$\langle 1,0\rangle$	$\langle 1,1\rangle$
$\langle 0,1\rangle$	$\langle 1,0\rangle$	$\langle 1,1\rangle$	$\langle 1,0\rangle$	$\langle 0,0\rangle$
$\langle 0,0\rangle$	$\langle 1,0\rangle$	$\langle 0,0\rangle$	$\langle 1,1\rangle$	$\langle 1,0\rangle$
$\langle 0,1\rangle$	$\langle 1,0\rangle$	$\langle 1,1\rangle$	$\langle 1,1\rangle$	$\langle 0,0\rangle$

Then at $t = 1$ we obtain the following states:

$\langle 1,0\rangle$	$\langle 1,0\rangle$	$\langle 1,0\rangle$	$\langle 1,0\rangle$	$\langle 1,0\rangle$
$\langle 1,0\rangle$	$\langle 1,0\rangle$	$\langle 1,0\rangle$	$\langle 1,0\rangle$	$\langle 1,0\rangle$
$\langle 1,0\rangle$	$\langle 1,0\rangle$	$\langle 1,0\rangle$	$\langle 1,0\rangle$	$\langle 1,0\rangle$
$\langle 1,0\rangle$	$\langle 1,0\rangle$	$\langle 1,0\rangle$	$\langle 1,0\rangle$	$\langle 1,0\rangle$
$\langle 1,0\rangle$	$\langle 1,0\rangle$	$\langle 1,0\rangle$	$\langle 1,0\rangle$	$\langle 1,0\rangle$

Hence, the first difference of context-based games from concurrent ones that is followed from example 1 is that we deal with the set of actions presented by logic gates or their inductive combinations which give n outputs for n inputs and we take into account time t.

Example 2.

Let us return to our thought experiment, where the plasmodium faces a barrier with one slit. For this experiment we can use the game $\mathcal{G} = \langle States, Agt, Act_n, Mov^n, Tab^n, (\preceq_A)_{A \in Agt}\rangle$, where $n = 3$, $Agt = \{A_1\}$, $States = \{0, 1\}$, $Act_n = \{\neg\}$. There are the following three modifications of one action to propagate towards one attractant: (i) the plasmodium can pass trough, (ii) it can avoid the barrier from the left, (iii) and the plasmodium can avoid the barrier from the right. The hybrid action of negation $\neg$ in $\mathcal{G}$ is defined as the transition rule which maps (a) the string $\langle 0, 1, 0\rangle$ into the string $\langle 0, 1, 0\rangle$ (it corresponds to passing through); (b) the string $\langle 1, 0, 0\rangle$ into the string $\langle 0, 1, 0\rangle$ (it corresponds to avoiding from the right); (c) the string $\langle 0, 0, 1\rangle$ into the string $\langle 0, 1, 0\rangle$ (it corresponds to avoiding from the left); (d) and it transforms all other strings to the string $\langle 0, 0, 0\rangle$.

Example 3.

Let $\mathcal{G} = \langle States_t, Agt, Act_{t,n}, Mov^n_t, Tab^n_t, (\preceq_A)_{A \in Agt}\rangle$, where

- $States_t$ means states for different time $t = 0, 1, \ldots$;
- Agt is a finite set of players;
- $Act_{t,n}$ is an infinite set of actions presented by logic gates or their inductive combinations with n inputs and n outputs, these actions can be applicable only to states of time t and give states of time $t + 1$
- Mov^n_t is a set of legal moves at time $t = 0, 1, \ldots$;
- $Tab^n_t\colon States^n_t \times Act^{Agt}_{t,n} \to States^n_{t+1}$ is the transition table which associates, with a given set of states and a given move of the players, the set of states resulting from that move;
- for each $A \in Agt$, $\preceq_A$ is a preorder over $States^\omega$, called the *preference relation* of player A.

Suppose that $n = 9$, $Agt = \{A_1, A_2\}$, $States_t = \{0, 1\}$, $Act_{t,9} = \{\max \Rightarrow \min, \min \Rightarrow \max\}$. Assume that A_1 follows the rule: max{the states of A_1 at t} $\Rightarrow$ min{the states of A_2 at t}, and A_2 follows the rule: min{the states of A_2 at t} $\Rightarrow$ max{the states of A_1 at t}. This means that in transition system $\mathcal{S}$, $Edges_{A_i} = \{(\mathbf{s}, \mathbf{s}') \in States^9_t \times States^9_{t+1}\colon Tab^9(\mathbf{s}, m^9_{A_i}) = \mathbf{s}'\}$, where $i = 1, 2$, $m^9_{A_1} =$ max{the states of A_1 at t} $\Rightarrow$ min{the states of A_2 at t}, and $m^9_{A_2} =$ min{the states of A_2 at t} $\Rightarrow$ max{the states of A_1 at t}.

In the cellular-automatic form, at $t = 0$:

$\langle 0,1\rangle$	$\langle 0,0\rangle$	$\langle 1,1\rangle$	$\langle 0,1\rangle$	$\langle 1,0\rangle$
$\langle 1,1\rangle$	$\langle 1,1\rangle$	$\langle 1,0\rangle$	$\langle 1,0\rangle$	$\langle 1,1\rangle$
$\langle 0,1\rangle$	$\langle 1,0\rangle$	$\langle 1,1\rangle$	$\langle 1,0\rangle$	$\langle 0,0\rangle$
$\langle 0,0\rangle$	$\langle 1,0\rangle$	$\langle 0,0\rangle$	$\langle 1,1\rangle$	$\langle 1,0\rangle$
$\langle 0,1\rangle$	$\langle 1,0\rangle$	$\langle 1,1\rangle$	$\langle 1,1\rangle$	$\langle 0,0\rangle$

At $t = 1$:

$\langle 1,0\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$
$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$
$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$
$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$
$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$	$\langle 0,1\rangle$

The second difference of context-based games from concurrent ones that is shown by example 2 is that we involve actions which are different for all $t = 0, 1, \ldots$ and are mutually dependent.

Let us consider an example, when $n = 1$ and $Agt = \{A_1, A_2\}$ in $\mathcal{G}$, to demonstrate that the actions of context-based games are hybrid, i.e. non-well-founded indeed, because they cannot be built up by means of atomic actions as their inductive compositions.

Assume that agent A_1 moves from the state s_1^t to the state s_1^{t+1}, $t = 0, 1, \ldots$ and agent A_2 moves from the state s_2^t to the state s_2^{t+1}, $t = 0, 1, \ldots$ by the following transition rules: $s_1^{t+1} = (s_1^t \Rightarrow s_2^t)$ and $s_2^{t+1} = (s_2^t \Rightarrow s_1^t)$. Then we obtain the following infinite streams: $\langle s_1^0 \Rightarrow s_2^0, s_1^1 \Rightarrow s_2^1, s_1^2 \Rightarrow s_2^2, \ldots\rangle$ and $\langle s_2^0 \Rightarrow s_1^0, s_2^1 \Rightarrow s_1^1, s_2^2 \Rightarrow s_1^2, \ldots\rangle$. Let us pay attention that the stream $\langle s_1^0 \Rightarrow s_2^0, s_1^1 \Rightarrow s_2^1, s_1^2 \Rightarrow s_2^2, \ldots\rangle$ (resp. $\langle s_2^0 \Rightarrow s_1^0, s_2^1 \Rightarrow s_1^1, s_2^2 \Rightarrow s_1^2, \ldots\rangle$) may be understood as an infinite propositional formula $(((s_1^0 \Rightarrow s_2^0) \Rightarrow s_2^1) \Rightarrow s_2^2) \Rightarrow \ldots$ (resp. $(((s_2^0 \Rightarrow s_1^0) \Rightarrow s_1^1) \Rightarrow s_1^2) \Rightarrow \ldots$). Both formulas are mutually dependent and they cannot be presented as linear sequence (inductive composition). Hence, we have a hybrid formula of radius 2 presented by two infinite mutually dependent propositional formulas. Also, we can show that any hybrid formula of radius $n = |Agt|$ can be formulated as n infinite mutually dependent propositional formulas. In concurrent games, there is no reflexion of players. They do not pay attention, which actions are involved in transitions by others. A hybrid action of radius $n = |Agt|$ means that each player of Agt coordinates his action with others by using their reasoning in the form he can foresee, maybe wrongly. Reflexive games are considered in [17], [18]. They are version of context-based games.

In the case of slime mould, hybrid actions are not results of predictions of others as in the case of human reflexive games. The matter is that the one plasmodium can follow some hybrid actions [13], since for $n \geq 2$ inputs there is an uncertainty, which logic gates with n inputs are involved in fact. Therefore we can consider all possible logic gates with n inputs. These possible gates cannot exceed the number n. So, maximally, we can have only n outputs.

DEFINITION 4. *A (finite)* context-based game on Physarum polycephalum *is a tuple* $\mathcal{G} = \langle States_t, Agt, Act_{t,n}, Mov_t^n, Tab_t^n, (\preceq_A)_{A\in Agt}\rangle$*, where*

- $States_t$ *is a (finite) set of* states *presented by attractants occupying by the plasmodium at time* $t = 0, 1, 2, \ldots$*;*
- $Agt = \{1, \ldots, k\}$ *is a finite set of* players *presented by different active zones of plasmodium;*
- $Act_{t,n}$ *is a non-empty set of* hybrid actions *with radius* n *at* $t = 0, 1, 2, \ldots$*, an element of* $Act_{t,n}^{Agt}$ *is called a* move *at time* $t = 0, 1, 2, \ldots$*;*
- $Mov_t^n\colon States_t^n \times Act_{t,n}^{Agt} \to 2^{Act} \setminus \{\emptyset\}$ *is a mapping indicating the* available *sets of actions to a given player in a given set of states,* $n > 0$ *is said to be a radius of plasmodium actions, a move* $m_{Agt}^n = (m_A^n)_{A\in Agt}$ *is legal at* $\langle s_1, \ldots, s_n\rangle$ *if* $m_A^n \in Mov^n(\mathbf{s}, A)$ *for all* $A \in Agt$*, where* $\mathbf{s} = \langle s_1, \ldots, s_n\rangle$*;*
- $Tab_t^n\colon States_t^n \times Act_{t,n}^{Agt} \to States_{t+1}^n$ *is the transition table which associates, with a given set of states at t and a given move of the players at t, the set of states at* $t+1$ *resulting from that move;*
- *for each* $A \in Agt$*,* $\preceq_A$ *is a preorder (reflexive and transitive relation) over* $States^\omega$*, called the* preference relation *of player A, indicating the intensity of attractants.*

All other notions such as outcomes, histories, and strategies are defined in the way of the previous section. We can prove a statement that for any concurrent game $\mathcal{G}$ on the medium of slime mould, there is an appropriate context-based game as a greatest fixed point for all uncertain modifications of $\mathcal{G}$ in experiments with plasmodia. This statement allows us to build logic circuits on the slime mould using context-based game notions.

5. CONCLUSIONS

We have demonstrated that the slime mould can be a model for concurrent games (definition 1) and context-based games (definition 4). In context-based games, players can move concurrently as well as in concurrent games, but the set of actions is infinite.

All definitions of this paper were verified in the experiments with *Physarum polycephalum* and *Badhamia utricularis*. All basic algorithms are implemented in the object-oriented language for simulations of plasmodia [21].

So, we are going to propose a bio-inspired experimental game theory on the medium of *Physarum polycephalum* and *Badhamia utricularis*. Some preliminary results allow us to claim that the slime mould can be an uncertain model for concurrent games and a logic circuit model for context-based games. In context-based games, players can move concurrently as well as in concurrent games, but simple actions are always hybrid. In our experiments, we share the following interpretations of basic entities: (i) attractants as payoffs; (ii) attractants occupied by the plasmodium as states of the game; (iii) active zones of plasmodium as players; (iv) logic gates for behaviors as moves (available actions) for the players; (v) propagation of the plasmodium as the transition table which associates, with a given set of states and a given move of the players, the set of states resulting from that move.

As a result, in context-based games, we are based on the following new game-theoretic assumptions:

1. each game can be assumed infinite, because its rules can change and these eventual changes are fixed by hybrid formulas;

2. players can change or modify their strategies and the set of actions is infinite for each player and consists of all possible modifications of simple actions;

3. resistance points for players are reduced to the payoffs if all actions are well-founded, i.e. there are no modifications for all simple actions;

4. for any game there is performative efficiency, when hybrid actions of players belong to the interval of expected modifications.

We suppose that these results can be practically applicable in programming biological behavioral systems such as plasmodia. Our future work is to design an (abstract) processor on plasmodia by using non-well-founded mathematics that has been illustrated by context-based games.

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6. REFERENCES

[1] S. Abramsky, P.-A. Mellies. Concurrent Games and Full Completeness. In *Proceedings of the 14th Symposium on Logic in Computer Science*, pages 431–442, 1999.

[2] S. Abramsky and R. Jagadeesan. Games and Full Completeness for Multiplicative Linear Logic. *Journal of Symbolic Logic*, 59(2):543–574, 1994.

[3] P. Aczel. *Non-Well-Founded Sets*, Stanford, 1988.

[4] A. Adamatzky. *Physarum Machines: Computers from Slime Mould* (World Scientific Series on Nonlinear Science, Series A), World Scientific Publishing Company, 2010.

[5] P. Baillot, V. Danos, T. Ehrhard, and L. Regnier. Believe it or not, AJM's games is a Model of Classical Linear Logic. In *Proceedings of LiCS'97*, pages 68–75, 1997.

[6] R. Brenguier. PRALINE: A Tool for Computing Nash Equilibria in Concurrent Games. In *CAV*, pages 890–895, 2013.

[7] P. Bouyer, R. Brenguier, N. Markey, M. Ummels. Concurrent Games with Ordered Objectives. In *FoSSaCS*, pages 301–315, 2012.

[8] P. Bouyer, R. Brenguier, N. Markey, M. Ummels. Nash Equilibria in Concurrent Games with Büchi Objectives. In *FSTTCS*, pages 375–386, 2011.

[9] T. Nakagaki, H. Yamada, A. Toth. Maze-solving by an amoeboid organism. *Nature* 407:470–470, 2000.

[10] T. Nakagaki, H. Yamada, and A. Tothm. Path finding by tube morphogenesis in an amoeboid organism. *Biophysical Chemistry*, 92:47–52, 2001.

[11] T. Nakagaki, M. Iima, T. Ueda, Y. Nishiura, T. Saigusa, A. Tero, R. Kobayashi, K. Showalter. Minimum-risk path finding by an adaptive amoeba network. *Physical Review Letters*, 99:68–104, 2007.

[12] A. Khrennikov, A. Schumann. p-Adic Physics, Non-well-Founded Reality and Unconventional Computing. *P-Adic Numbers, Ultrametric Analysis, and Applications*, 1(4):297–306, 2009.

[13] A. Khrennikov, A. Schumann. Quantum Non-Objectivity from Performativity of Quantum Phenomena. *Physica Scripta*, 2014 (accepted).

[14] H. Neubert, W. Nowotny, K. Baumann, H. Marx. *Die Myxomyceten Deutschlands und des angrenzenden Alpenraumes unter besonderer Berücksichtigung Österreichs*. Bd. 2, Karlheinz Baumann Verlag, Gomaringen, 1995.

[15] T. Saigusa, A. Tero, T. Nakagaki, Y. Kuramoto. Amoebae Anticipate Periodic Events. *Phys. Rev. Lett*, 100(1), 2008.

[16] A. Schumann. Reflexive Games and non-Archimedean Probabilities. *P-Adic Numbers, Ultrametric Analysis, and Applications*, 6(1):66–79, 2014.

[17] A. Schumann. Payoff Cellular Automata and Reflexive Games. *Journal of Cellular Automata*, 9(4):287–313, 2014.

[18] A. Schumann. Physarum Syllogistic L-Systems. In *FUTURE COMPUTING 2014, The Sixth International Conference on Future Computational Technologies and Applications*, pages 15–20, 2014.

[19] A. Schumann, A. Adamatzky. Physarum Spatial Logic. *New Mathematics and Natural Computation*, 7(3):483–498, 2011.

[20] A. Schumann, L. Akimova. Simulating of Schistosomatidae (Trematoda: Digenea) Behavior by Physarum Spatial Logic. In *Annals of Computer Science and Information Systems, Volume 1. Proceedings of the 2013 Federated Conference on Computer Science and Information Systems*. IEEE Xplore, pages 225–230, 2013.

[21] A. Schumann, K. Pancerz. Towards an Object-Oriented Programming Language for Physarum Polycephalum Computing: A Petri Net Model Approach, *Fundamenta Informaticae*, IOS Press, Amsterdam, 133(2-3):271–285, 2014.

[22] T. Shirakawa, Y.-P. Gunji, and Y. Miyake. An associative learning experiment using the plasmodium of *Physarum polycephalum. Nano Communication Networks*, 2:99–105, 2011.

[23] T. Sun, S. Tsuda, K.-P. Zauner, H. Morgan. Single cell imaging using electrical impedance tomography. In *NEMS '09: Proc. of the 2009 4th IEEE Int. Conf, on Nano/Micro Engineered and Molecular Systems*, pages 858–863, 2009.

[24] A. Tero, T. Nakagaki, K. Toyabe, K. Yumiki, R. Kobayashi. A Method Inspired by Physarum for Solving the Steiner Problem. *IJUC*, 6(2):109–123, 2010.

[25] S. Tsuda, M. Aono, and Y.P. Gunji. Robust and emergent Physarum-computing. *BioSystems*, 73:45–55 2004.

[26] S. Tsuda, J. Jones, A. Adamatzky, J. Mills, Routing Physarum with electrical flow/current. *Int. J. Nanotechnology and Molecular Comput.*, 3, 2011.

[27] Sh. Watanabe, A. Tero, A. Takamatsu, T. Nakagaki. Traffic optimization in railroad networks using an algorithm mimicking an amoeba-like organism, Physarum plasmodium. *Biosystems*, 105(3):225–232, 2011.

[28] T. Umedachi, M. Akiyama, A. Tero, A. Ishiguro. Simulation of a soft-bodied fluid-driven amoeboid robot that exploits thixotropic flow, *ICRA*, pages 5123–5128. 2011.