Analysis of the d-choices garbage collection algorithm with memory in flash-based SSDs

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ABSTRACT

Garbage collection algorithms have a profound impact on the performance and life span of flash-based solid state drives. Recently, the d-choices garbage collection algorithm was shown to provide an excellent tradeoff between simplicity and performance [21]. In this paper, we introduce the d-choices garbage collection algorithm with memory and analyze its write performance using both synthetic and real life workloads. The synthetic workloads consist of uniform random writes and the write amplification is analyzed by means of a mean field model. For the trace-based workloads we rely on simulation experiments and consider systems using either a single or a double write frontier. Apart from studying the impact of adding memory to the d-choices garbage collection algorithm, the paper also presents the first trace-based evidence that the double write frontier is very effective in reducing the write amplification in the presence of hot and cold data.

1. INTRODUCTION

Data on an SSD is organized in blocks each containing a fixed number of fixed sized pages. The page size is typically a few kilobytes and it is also the unit of data transfer with the SSD. The number of pages per block b is a power of 2 ranging from $b = 32$ to 256 in modern SSDs. The number of physical blocks N on the drive depends on the SSD storage capacity and often exceeds 100,000. At any point in time, a page is either in the valid, invalid or erase state. In order to write data on a page, it must be in the erase state. However, a page cannot be erased individually, instead erase operations must be performed on entire blocks. As such SSDs tend to write new data elsewhere and simply invalidate the data in the old location.

To support out-of-place writes the device maintains a mapping between the logical and physical page numbers, called the flash translation layer (FTL) [4, 6]. In a page-mapped FTL any logical page can be mapped to any physical page at the expense of requiring as many entries in the map as there are pages on the SSD. Many flash-based devices rely on a hybrid-mapped FTL (e.g., [10, 12, 9, 11]) as this reduces the size of the FTL map. However, some of these solutions were designed specifically for mobile embedded systems (e.g., MP3 and PDAs) and are not very suitable for the workloads with random writes and temporal locality characteristics encountered in general-purpose computing.

We focus on a page-mapped FTL and any new data is sequentially written to a special block, called the write frontier (WF). The main task of the garbage collection (GC) algorithm is to select a new WF whenever the current one becomes full and ideally it should select blocks with as few valid pages as possible (see Section 2.1 for more details). Recently, the d-choices GC algorithm was shown to provide an excellent tradeoff between simplicity and performance [21]. This GC algorithm was inspired by the well-known d-choices algorithm in load balancing [24, 15], where whenever a job needs to be dispatched d queues are chosen uniformly at random and the job is assigned to the shortest among the d chosen queues. The d-choices GC algorithm behaves similarly as it chooses d blocks uniformly at random and selects the one containing the fewest number of valid pages.

Often several of the d chosen blocks may in fact contain a small number of valid pages and one may wonder whether in general the system performance improves if we keep track of some of the best choices, instead of reselecting d blocks each time the GC algorithm is activated. Adding memory on top of the d choices should be beneficial, but it is less obvious if any gain can be achieved if we keep track of the block ids of the next c best choices and reselect only $d - c$ blocks, for some $c > 0$. The idea of combining memory with d-choices to recall some of the best choices of the previous selection has also been explored in a load balancing setting [16], where the main finding is that memory may increase the mean response time, but it does give rise to improved queue length asymptotics.

In this paper we study the write amplification of the d-choices garbage collection (GC) algorithm with memory by considering both synthetic and trace-based workloads. More specifically, the synthetic workload consists of uniform random writes, meaning all pages are accessed equally often and there is no temporal or spacial locality, and the write amplification is analyzed by extending the mean field model of [21]. Although this extension is not hard, the resulting set of ODEs that captures the evolution of the mean field model relies on the steady state probabilities of a Markov chain with $(b+1)^c$ states, where c represents the number of block ids stored by the d-choices GC algorithm with memory and

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b denotes the number of pages per block. Hence, for realistic values of b and c, e.g., $b = 64$ and $c = 10$, this ODE cannot be solved in a direct manner. Instead we show that the necessary steady state probabilities of these $(b+1)^c$-state Markov chains can be expressed through the steady state probabilities of b different Markov chains, each having $c+1$ states only.

For the trace-based workloads, we rely on simulation experiments and analyze the write amplification of solid state drives (SSDs) that either make use of a single or the double write frontier as introduced in [22]. As such the paper makes the following contributions:

1. We extend the mean field model of [21] to analyze the write amplification of the d-choices GC with memory under uniform random writes. We use lumpability properties to solve the resulting ODE in an efficient manner for realistic parameter values, validate the model by means of simulation experiments and present numerical results to demonstrate the added value of having memory.

2. We use trace-based simulation experiments to confirm that the main findings of the mean field model that relies on uniform random writes, also apply under real workloads. As real workloads contain both hot and cold data we consider SSDs that operate either using a single or a double write frontier. In the latter case adding more memory does not always reduce the write amplification, that is, there exists an optimal amount of memory.

3. The trace-based simulation experiments presented in this paper also demonstrate that the use of the double write frontier significantly reduces the write amplification compared to the single write frontier under real workloads. Hence, these results further demonstrate the power of the double write frontier introduced in [22], which only considered a simple Rosenblum model [20] for hot and cold data.

The paper is structured as follows. Section 2 discusses the system operation, introduces the d-choices GC algorithm with memory and lists some of the related literature. The mean field model is presented and validated in Section 3 and also includes various numerical examples. In Section 4 we look at the impact of using real-life workloads, while conclusions are drawn in Section 5.

2. PROBLEM DESCRIPTION

2.1 System operation

As indicated in the introduction, we focus on a page-mapped FTL where new data is sequentially written to a special block, called the write frontier (WF). Suppose at some point in time that the first k pages of the WF are in the valid/invalid state, while the remaining $b-k$ pages are in the erase state. The data of the next write operation is stored on page $k+1$ of the WF, while altering its state from erase to valid. The data in the old location is invalidated and the FTL mapping is updated. When the WF becomes full, meaning all its pages are in the valid/invalid state, the GC algorithm creates a new WF. It does this by selecting a block (in some algorithm dependent manner) and by erasing the entire block after copying any valid data that was left on the block to memory. Assume there were j valid pages in the selected block before the erase operation took place. After the erase operation the j valid pages are written back to the selected block using the first j pages and the next $b-j$ externally requested write operations can make use of the last $b-j$ pages of the newly created WF.

Note, whenever the GC algorithm selects a block containing j valid pages, the SSD needs to perform one erase and b write operations in order to process $b-j$ externally requested write requests (i.e., requests issued by the operating system). The ratio of the total number of writes to the total number of externally requested writes is defined as the write amplification (WA). If we denote p_j as the probability that the GC algorithm selects a block with j valid pages, it can be defined as

$$WA = \frac{b}{b - \sum_{j=0}^{b} j p_j}.$$

The write amplification is an important performance measure because it directly impacts the speed of the drive (erase and write operations are much slower than read operations [3]) as well as the lifespan of the SSD (as each block can only be erased a limited number of times [7]).

The above discussion applies to an SSD that makes use of a single WF. The double WF was introduced in [22] and makes use of 2 WFs: the WFI used for internally requested writes and the WFE used for externally requested writes. Whenever the GC algorithm is activated it attempts to create a WFE with all of its pages in the erase state. It does this by first selecting a block (in some specific manner), where we denote the number of valid pages in the selected block as j. If the WFI has j or more pages in the erase state, the GC algorithm copies the j valid pages of the selected block to the WFI (changing the state of j of its pages from erase to valid) and erases the selected block, which becomes the new WFE with all of its pages in the erase state. If the WFI does not contain enough pages in the erase state, only some of the valid pages of the selected block are copied to the WFI, while the remaining pages are copied to memory and placed back on the selected block after the erase operation. In the latter case the newly selected block becomes the new WFI and the GC algorithm is immediately activated again in order to create a new WFE. For the double WF the write amplification is still defined as the ratio of the total number of writes over the total number of externally requested writes (see [22] for more details).

SSDs also rely on over-provisioning, meaning a drive with N physical blocks is perceived by the operating system as having a storage capacity of $U < N$ blocks. The spare factor S_f is defined as $1 - U/N$ and ranges from 0.02 to 0.2 in real life SSDs. The advantage of over-provisioning is that a fraction S_f of all the pages is guaranteed to be in the invalid/erase state, which increases the odds for the GC algorithm to locate a block with a small number of valid pages (which in turn reduces the WA). In fact, if the SSD has been operational for a while and the TRIM command is not supported (as in older SSDs) or used, the fraction of pages in the invalid/erase state is exactly equal to S_f. This is due to the fact that while the pages corresponding to a file that is deleted by the file system may change their state from valid to invalid, such a change does not occur because file delete information is not by default passed down to the SSD, unless explicitly done so by invoking the TRIM command. In this paper, as in all prior studies, we assume that the TRIM command is not supported or used. Also, the trace data used in Section 4 does not contain any information on the TRIM command either and consists of reads and writes only.

2.2 Garbage collection algorithms and related work

The write amplification (and the distribution of the number of valid pages per block) of the following garbage collection algorithms was analyzed in a number of earlier studies:

- The FIFO GC algorithm [19, 14, 25, 5] selects the blocks in a cyclic order.
- The Greedy GC algorithm [2, 5] selects the block containing the fewest number of valid pages among all the blocks.
- The d-choices GC algorithm [21, 13, 22] selects the block with the fewest number of valid pages out of a set of d randomly chosen blocks.
- The Windowed GC algorithm [8] maintains a window containing the w least recently selected blocks and selects the block with the fewest number of valid pages in the current window.

Most of the above studies consider uniform random writes and focus on the case where the number of blocks N tends to infinity. Under uniform random writes the Greedy algorithm is optimal, while the FIFO algorithm often achieves the highest write amplification. Both the d-choices and the Windowed algorithm provide a trade-off between the simplicity of FIFO and the performance of the Greedy algorithm (note, the Windowed algorithm corresponds to the FIFO and Greedy GC algorithm when the window size equals 1 and N, respectively). The d-choices GC algorithm however provides a much better trade-off than the Windowed algorithm as small values of d suffice, e.g., $d = 10$, to achieve a write amplification close to that of the Greedy algorithm (see [21] for details).

Far fewer results are available in case of non-uniform random writes, in this case some of the data, termed the hot data, is accessed more often, while the remaining data is termed the cold data. The performance of the FIFO and Greedy algorithm in the presence of hot and cold data was analyzed in [5] for a system using a single write frontier. These results indicated that the write amplification worsens significantly as the hot data gets hotter, especially for the FIFO GC algorithm. Similar results were provided in [22] for the d-choices GC algorithm, where additionally the concept of the double write frontier was introduced. With the double write frontier the write amplification was shown to decrease as the hot data gets hotter and the Greedy algorithm was shown to be no longer optimal, instead there exists an optimal value for d. A simple Rosenblum data model for the hot and cold data was used in [5, 22], though some trace-based results for FIFO were also provided in [5]. Under the Rosenblum model a fraction f of the logical address space corresponds to *hot* data and the remaining fraction to *cold* data. The fraction of write operations to the hot data is denoted as r. Note, in the Rosenblum model there is no spacial locality in the workload and the temporal locality does not vary over time.

2.3 The d-choices GC algorithm with memory

In this paper we introduce and analyze the d-choices GC algorithm with memory under both uniform random writes (using a mean field model) and trace-based workloads (using simulation). The d-choices GC algorithm with memory operates as follows: it stores the id of $c \geq 1$ blocks at all times, these blocks are initially selected at random and their corresponding blocks are called the c *stored* blocks. Whenever the d-choices GC algorithm with memory is activated, it chooses d blocks uniformly at random. Next, the GC algorithm selects the block with the least number of valid pages among the d chosen and the c stored blocks. Finally, the c ids of the blocks with least number of valid pages among the remaining $d + c - 1$ blocks become the c stored block ids. In other words, the d-choices GC algorithm with memory stores the id of the 2nd to $(c+1)$st best blocks for possible future use as they might also contains a limited number of valid pages.

3. UNIFORM RANDOM WRITES

3.1 Mean field model

In this section we generalize the mean field model introduced in [21] to study the impact of memory. When $c = 0$ the model introduced in this section reduces to the one in [21] for the d-choices GC algorithm. Under a uniform random workload, meaning all pages are accessed equally often and there is no temporal or spacial locality, it is not hard to prove that the single and double write frontier achieve the same write amplification. As such we can focus on the single write frontier case only.

We observe the system just prior to the time epochs where the GC algorithm is executed and let $X_n^N(t) \in S = \{0, 1, \ldots, b\}$, for $n = 1, \ldots, N$, be the number of valid pages on block number n at time t for an SSD with N physical blocks. Let $M^N(t) = (M_0^N(t), \ldots, M_b^N(t))$ be the occupancy measure of $X_n^N(t)$, that is,

$$M_i^N(t) = \sum_{n=1}^{N} 1[X_n^N(t) = i],$$

for $i = 0, \ldots, b$ and where $1[A] = 1$ if A is true and 0 otherwise. Further, let $j_i(t)$ denote the number of valid pages in the i-th stored block at time t, for $i = 1, \ldots, c$, and let $J^N(t)$ represent the string $j_1(t)j_2(t)\ldots j_c(t)$ and denote $f(J^N(t)) = \min_{i=1}^{c} j_i(t)$. Clearly, in case of uniform random writes $\{(M^N(t), J^N(t)), t \geq 1\}$ is a Markov chain on the state space $\Delta^N \times \mathcal{S}_c$, where $\Delta^N = \{\vec{m} \in \mathbb{R}^{b+1} | m_i N \in \{0, 1, \ldots, N\}, \sum_{i=0}^{b} m_i = 1, \sum_{i=0}^{b} i m_i = \rho b\}$ and $\mathcal{S}_c = \{J | J = j_1 j_2 \ldots j_c, j_i \in \{0, \ldots, b\}, i = 1, \ldots, c\}$, as the total fraction of valid pages equals $\rho = 1 - S_f$ at all times. As this chain cannot be analyzed directly for realistic values of N (e.g., $N = 50,000$), we focus on the system behavior as N tends to infinity using a mean field model, the accuracy of which is validated via simulation in Section 3.3.

Define $f_i^N(\vec{m}, j)$, for $i = 0, \ldots, b$, as the drift of the fraction of blocks with i valid pages provided that the least number of valid pages in one of the stored blocks is j and the occupancy measure is given by $\vec{m}$:

$$f_i^N(\vec{m}, j) = E[M^N(t+1) - M^N(t) | M^N(t) = \vec{m}, f(J^N(t)) = j].$$

Further, let $f_i(\vec{m},j) = \lim_{N\to\infty} N f_i^N(\vec{m},j)$. In order to determine the drift $f_i(\vec{m},j)$, define $p_i(\vec{m},j)$ as the probability that the d-choices GC algorithm with memory selects a block with i valid pages provided that $f(J(t)) = j$ and $M(t) = \vec{m}$. These probabilities can be expressed as

$$p_i(\vec{m},j) = \left(\sum_{\ell=i}^{b} m_\ell\right)^d - \left(\sum_{\ell=i+1}^{b} m_\ell\right)^d, \quad (1)$$

for $i < j$, as all of the d chosen blocks should contain at least i valid pages, but not all should contain more than i valid pages. Further,

$$p_j(\vec{m},j) = \left(\sum_{\ell=j}^{b} m_\ell\right)^d, \quad (2)$$

while for $i > j$ we clearly have $p_i(\vec{m},j) = 0$ as at least one of the stored blocks contains only j valid pages.

The reasoning in [21, Section 5.1] to determine the drift $f_i(\vec{m})$ for the d-choices GC algorithm without memory, can now be repeated to express $f_i(\vec{m},j)$ if we replace the probabilities $p_i(\vec{m})$ in [21] by the probabilities $p_i(\vec{m},j)$ and by noting that $p_i(\vec{m},j) = 0$ for $i > j$. Hence based on [21], for $i < b$, we may conclude

$$f_i(\vec{m},j) = \left(\sum_{\ell=1}^{b} p_{b-\ell}(\vec{m},j)\ell\right) \frac{(i+1)m_{i+1} - i m_i}{b\rho} - p_i(\vec{m},j), \quad (3)$$

while

$$f_b(\vec{m},j) = 1 - p_b(\vec{m},j) - \left(\sum_{\ell=1}^{b} p_{b-\ell}(\vec{m},j)\ell\right) \frac{b m_b}{b\rho}. \quad (4)$$

Intuitively, the expression for $f_i(\vec{m},j)$, with $i < b$, can be understood as follows. First, the sum $\sum_{\ell=1}^{b} p_{b-\ell}(\vec{m},j)\ell$ represents the mean number of external write requests that are performed in between two executions of the GC algorithm (given $M(t) = \vec{m}$ and $f(J(t)) = j$). Further, $i m_i/(b\rho)$ is the probability that such an external request updates a valid page on a block containing i valid pages, due to the uniform random writes and the fact that on average each block holds $b\rho$ valid pages. Hence, with probability $i m_i/(b\rho)$ we lose a block containing i valid, while with probability $(i+1)m_{i+1}/(b\rho)$ such a block is created by an external write request. Finally, we also lose a block containing i valid pages in between two executions of the GC algorithm, if the GC algorithm selects such a block (which happens with probability $p_i(\vec{m},j)$ if $M(t) = \vec{m}$ and $f(J(t)) = j$). The expression for $f_b(\vec{m},j)$ can be understood similarly by noting that a extra block with b valid pages is created (in the WF) unless the GC algorithm selects a block containing b valid pages.

To define the ODE that captures the evolution of the mean field model, note that $\mathcal{S}_c$ is the state space of $J(t)$. If we assume that $M(t) = \vec{m}$ for $t \geq 0$ for some $\vec{m} \in \Delta$ fixed, then $J(t)$ forms a Markov chain with state space $\mathcal{S}_c$. Denote its transition matrix of size $(b+1)^c$ as $K(\vec{m})$, as it depends on the occupancy measure $\vec{m}$. It is not hard to see that $K(\vec{m})$ contains a single recurrent class (for $d > 1$) that consists of all the strings $J = j_1 j_2 \ldots j_c$ for which $m_{j_i} > 0$ for $i = 1,\ldots,c$. Denote $\pi(\vec{m}) = (\pi_J(\vec{m}))_{J\in\mathcal{S}_c}$ as the invariant vector of $K(\vec{m})$ (with the entries of the possible transient states set to zero). Next, we define the set of ODEs that captures the evolution of the mean field model as

$$\frac{d\vec{\mu}(t)}{dt} = \vec{F}(\vec{\mu}(t)), \quad (5)$$

with

$$\vec{F}(\vec{m}) = \sum_{j=0}^{b} \left(\sum_{J\in\mathcal{S}_c, f(J)=j} \pi_J(\vec{m})\right) \vec{f}(\vec{m},j).$$

Hence, the ODE is expressed via the drifts $\vec{f}(\vec{m},j) = (f_0(\vec{m},j),\ldots,f_b(\vec{m},j))$ and the probabilities $\pi_j(\vec{m})$ defined as $\pi_j(\vec{m}) = \sum_{J\in\mathcal{S}_c, f(J)=j} \pi_J(\vec{m})$, for $j = 0,\ldots,b$.

Finally, define $\bar{M}^N(\tau)$ as the re-scaled process such that $\bar{M}^N(t) = M^N(\lfloor tN \rfloor)$. As in [21], it is not hard to verify that the following theorem holds due to Corollary 1 in [1]:

THEOREM 1. *If $M^N(0) \to \vec{m}$ in probability as N tends to infinity, then $\sup_{0\leq t\leq T} ||\bar{M}^N(t) - \vec{\mu}(t)|| \to 0$ in probability, where $\vec{\mu}(t)$ is the unique solution of the ODE* (5) *with $\vec{\mu}(0) = \vec{m}$.*

Hence, for N large we can approximate $M^N(t)$ by $\vec{\mu}(t/N)$ for any finite t. Ideally we also wish to show that the convergence extends to the stationary regime, as we are interested in the steady state behavior for finite large N (which boils down to showing that the order of the limits in t and N can be exchanged). Corollary 2 in [1] shows that the convergence extends to the stationary regime provided that the ODE has a global attractor. Unfortunately proving the existence of a global attractor for the set of ODEs in (5) appears hard, except for some specific cases (e.g., $c = 0$ and $b = 2$), but numerical experiments suggest that such an attractor exists.

To determine the write amplification WA, we use Euler's method to find a fixed point $\vec{\eta} = (\eta_0,\ldots,\eta_b)$ of (5) and compute the write amplification as

$$WA = \frac{b}{b - \sum_{j=0}^{b} \pi_j(\vec{\eta}) \sum_{i=0}^{j} i p_i(\vec{\eta},j)}.$$

Note Euler's method is an iterative method and when applied to (5) this implies that we need to determine the steady state probabilities of the transition matrix $K(\vec{m})$ for some $\vec{m}$ during each step (while the number of steps can be as large as a few thousand). As $K(\vec{m})$ is of size $(b+1)^c$, we cannot simply construct $K(\vec{m})$ and solve the corresponding linear system to determine the necessary steady state probabilities $\pi_j(\vec{m})$ for realistic values of b and c, e.g., $b = 64$ and $c = 10$. In the next subsection we show how we can reduce the problem of computing the probabilities $\pi_j(\vec{m})$, which is required during each step of Euler's method, to the computation of the steady state probabilities of b small Markov chains (each having $c+1$ states).

3.2 Computation of $\pi_j(\vec{m})$

To determine the probabilities $\pi_j(\vec{m})$, let $j \in \{0,\ldots,b-1\}$ be *fixed* and denote $A_{j,k} \subset \mathcal{S}_c$, for $k = 0,\ldots,c$, as the subset of $\mathcal{S}_c$ containing all the strings $J \in \mathcal{S}_c$ such that exactly k elements of J are larger than j. Clearly $A_{j,0},\ldots,A_{j,c}$ is a partition of $\mathcal{S}_c$ and we show that the Markov chain with transition matrix $K(\vec{m})$ can be lumped with respect to this partition. In other words, the number of stored blocks with more than j valid pages forms a $(c+1)$-state Markov chain with state space $\{0,\ldots,c\}$ for any $j \in \{0,\ldots,b-1\}$

fixed. Denote its transition matrix as $P^{(j)}$ and its transition probabilities as $p^{(j)}_{i,i'}$ with $i, i' \in \{0, \dots, c\}$, then

$$p^{(j)}_{i,i-k}(\vec{m}) = \begin{cases} B^{d,\sum_{\ell=0}^{j} m_\ell}_{k+1} & i < c; -1 \le k < i, \\ B^{d,\sum_{\ell=0}^{j} m_\ell}_{k+1} & i = c; 1 \le k < c, \\ \sum_{s=0}^{1} B^{d,\sum_{\ell=0}^{j} m_\ell}_{s} & i = c; k = 0, \\ 1 - \sum_{s=0}^{i} B^{d,\sum_{\ell=0}^{j} m_\ell}_{s} & k = i, \\ 0 & otherwise \end{cases}$$

where $B^{n,p}_j = \binom{n}{j} p^j (1-p)^{n-j}$. Indeed, if at least one stored block contains at most j valid pages (i.e., if $i < c$), the number of stored blocks with more than j valid pages increases by one during a single transition if none of the d chosen blocks contains at most j valid pages. Similarly, if $i < c$, the state remains the same if exactly one of the d chosen blocks contains at most j valid pages and it decreases by k (for $k < i$) if $k+1$ of the chosen blocks contain at most j valid pages. The expressions for the cases with $i = c$ and $k < c$ can be understood similarly. Finally, a transition to state 0 occurs as soon as more than i of the chosen d blocks contain at most j valid pages.

Denote the steady state probability vector of the $(c+1)$-state Markov chain with transition matrix $P^{(j)}$ as $(\theta^{(j)}_0(\vec{m}), \dots, \theta^{(j)}_c(\vec{m}))$. These vectors can be computed easily (in fact they can even be expressed explicitly in a recursive manner as this chain is skip-free in one direction) and they obey the following equalities:

$$\sum_{\ell=0}^{j} \pi_\ell(\vec{m}) = \sum_{J \in \mathcal{S}_c, f(J) \le j} \pi_J(\vec{m}) = 1 - \theta^{(j)}_c(\vec{m}),$$

as in any state $i < c$ there is at least one stored block with at most j valid pages. If we compute the steady state probabilities $\theta^{(j)}_c(\vec{m})$, for $j = 0, \dots, b-1$, by solving b Markov chains with $c+1$ states each, the above equality allows us to express the probabilities $\pi_j(\vec{m})$ as

$$\pi_j(\vec{m}) = \theta^{(j-1)}_c(\vec{m}) - \theta^{(j)}_c(\vec{m}),$$

for $j = 1, \dots, b-1$, while $\pi_0(\vec{m}) = 1 - \theta^{(0)}_c(\vec{m})$ and $\pi_b(\vec{m}) = \theta^{(b-1)}_c(\vec{m})$. To conclude, we can compute the required probabilities $\pi_j(\vec{m})$, for $j = 0, \dots, b$, by solving b Markov chains of size $c+1$ only, instead of trying to solve the $(b+1)^c$-state Markov chain characterized by $K(\vec{m})$. This allows us to generate numerical results in a matter of seconds with very low memory requirements even for systems with $b = 64$ pages per block that rely on $c = 50$ memory locations.

3.3 Model validation

To validate the mean field model for uniform random writes we conducted simulation experiments that basically simulate the Markov chain $\{(M^N(t), J^N(t)), t \ge 1\}$, where $M^N_i(t)$ is the number of blocks containing i valid pages, for $i = 0, \dots, b$, and $J(t)$ is the string representing the number of valid pages in the c stored blocks just prior to the time that the GC algorithm is executed for the t-th time. The only difference with the mean field model is therefore the system size N, which was set equal to $50{,}000$ in the simulation setup. For $b = 64$ and 4 KB pages this corresponds to an SSD with a total storage capacity of 12.8 GB.

b	S_f	d	c	ODE	simul. (95% conf.)	nruns
64	0.08	5	2	6.2461	6.2468 ±0.0006	100
64	0.12	6	24	4.2408	4.2405 ±0.0005	50
64	0.17	8	8	3.0596	3.0595 ±0.0003	25
32	0.07	6	5	6.4146	6.4147 ±0.0007	100
32	0.11	20	3	4.2113	4.2114 ±0.0006	50
32	0.16	15	19	3.0668	3.0664 ±0.0004	25
16	0.06	10	1	6.1340	6.1346 ±0.0010	100
16	0.10	4	10	4.5355	4.5344 ±0.0011	50
16	0.15	2	3	3.9448	3.9447 ±0.0017	25

Table 1: Write amplification: ODE-based results and simulation experiments for a system with $N = 50{,}000$ blocks for various parameter settings. Relative errors are less than 0.05%.

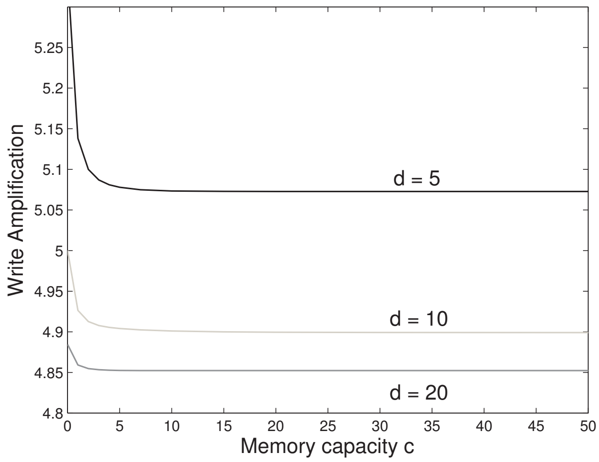


Figure 1: Impact of memory on the write amplification of the d-choices GC algorithm with memory for $b = 64$ and $S_f = 0.1$.

The number of runs used in the simulation setup to compute the 95% confidence intervals varied between 25 and 100 depending on the spare factor S_f (see Table 1). Each run had a length of $250{,}000$ and a warm-up period of $1/3$th of the total length. The results in Table 1 show a good agreement between the model and the simulation results with relative errors below 0.05% for some arbitrary combinations of b, c, d and S_f.

3.4 Numerical results

In this section we rely on the mean field model to look at the impact of introducing memory on the write amplification under uniform random writes. In the first experiment we increase c while all the other parameters, including d, remain fixed. The write amplification should reduce as c increases and one may expect that the write amplification approaches the write amplification of the greedy algorithm as c becomes large, because setting $c = N$ corresponds to the greedy GC algorithm. This might in fact be the case if we were to define c as a function of N and let N tend to infinity. We should however keep in mind that in our setup c remains fixed as N tends to infinity and therefore the write amplification does not necessarily reduce to the one of the greedy GC algorithm as c tends to infinity.

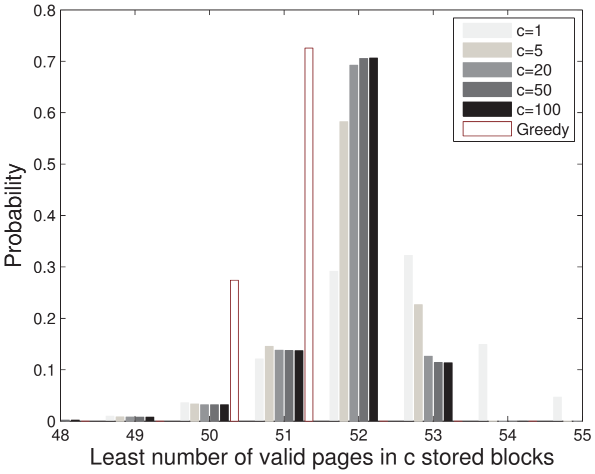
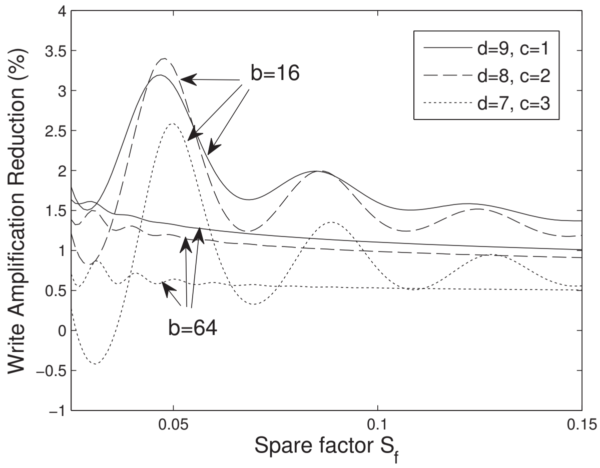


Figure 2: Comparson of the distribution of $\pi_j(\vec{\eta})$, the least number of valid pages in memory, for $b = 64$, $d = 10$ and $S_f = 0.1$ and various c values with the distribution of the greedy GC algorithm.

The results in Figure 1, where $b = 64$, $S_f = 0.1$ and $d = 5, 10$ and 20, confirm that the write amplification decreases as c increases, but the rate of decrease very quickly diminishes as c increases and seems to stagnate rather quickly. The write amplification of the greedy algorithm equals 4.8213 for $b = 64$ and $S_f = 0.1$, which seems to suggest that the write amplification does not converge to the one of the greedy algorithm as c tends to infinity (with d fixed). Figure 2 seems to confirm this as it suggests that the distribution of the least number of valid pages in a stored block converges to a distribution that is clearly different from the distribution of the number of valid pages in a block selected by the greedy algorithm (computed via [2]). In conclusion, adding a very limited amount of memory to reduce the write amplification is quite effective, e.g., $c \leq 5$, but the additional benefit of adding larger amounts of memory is far less significant. Similar results were obtained for other parameter settings.

In the second experiment we intend to see whether it is best to reselect d new blocks uniformly at random each time the GC algorithm is executed or whether to reselect only $d - c$ of the blocks and to store the ids of the c best blocks in memory. In other words, we fix b, S_f and $c + d$, while we increase c. Figure 3 depicts the relative gain of the d-choices algorithm with memory when $c > 0$ compared to the case with $c = 0$. The top figure shows that setting $c = 1$ is optimal for $c + d = 10$, except for $b = 16$ and spare factors around 0.05. The gains compared to setting $d = 10$ are quite limited and typically lie between 1 and 3 percent. More substantial gains are observed when we perform a similar experiment using trace-based data in Section 4.2. For $c+d = 20$ we observe similar results, meaning moderate gains are obtained if we do not always reselect d new blocks uniformly at random. The oscillations observed in the curves in Figure 3 are related to the fact that the write amplification of the d-choices GC algorithm approaches the amplification of the greedy GC algorithm as d tends to infinity and this latter write amplification is not a smooth function of S_f. The oscillations also become more pronounced as d increases.



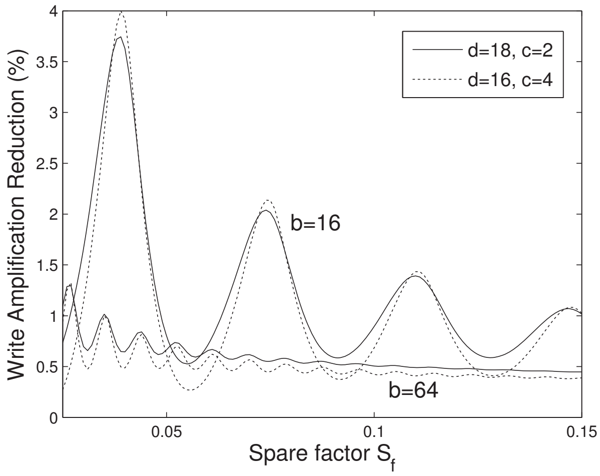


Figure 3: Reduction in the write amplification (%) of the d-choices algorithm with memory when $c > 0$ compared to the case with $c = 0$, for $c + d = 10$ (top) and $c + d = 20$ (bottom).

4. TRACE-BASED WORKLOADS

4.1 Simulation setup

To conduct trace-based simulations we have relied on a number of real-world I/O traces:

- **rsrch0** [18]: an I/O trace collected at a server supporting research projects at Microsoft Research.
- **prxy0** [18]: an I/O trace containing requests of a Firewall/web proxy server at Microsoft Research.
- **online** [23]: an I/O trace of a coursework management workload on Moodle at a university.
- **webmail** [23]: an I/O trace of webmail traffic on a university department mail server.

In order to use these traces in an SSD setting with 4 KB pages the traces were processed as follows. We first aligned the offset of each request to a multiple of 4 KB (all the offsets in the traces are multiples of 512 bytes). Requests with sizes above 4 KB (if present) were subsequently split into

I/O trace	%Writes	#Requests	%LBA_RO
rsrch0 [18]	88.87	3,253,639	19.02
prxy0 [18]	96.36	22,136,692	19.53
online [23]	73.88	5,700,499	64.87
webmail [23]	81.86	7,795,815	55.19

Table 2: Data set statistics. %Writes: percentage of writes, #Requests: number of request and %LBA_RO: the size of the LBA space that is only read.

I/O trace	Data Locality			
	20%	40%	60%	80%
rsrch0 [18]	0.01	0.09	7.15	19.17
prxy0 [18]	0.06	0.12	0.79	5.20
online [23]	1.30	7.76	14.30	30.79
webmail [23]	0.10	0.44	5.23	16.16

Table 3: Data set statistics: LBA locality.

S_f	$c = 0$	$c = 1$ (gain)	$c = 2$ (gain)	$c = 3$ (gain)
rsrch0 trace				
0.14	2.843	2.790 (1.86%)	2.803 (1.38%)	2.834 (0.32%)
0.10	3.739	3.649 (2.40%)	3.663 (2.03%)	3.709 (0.82%)
0.06	5.601	5.446 (2.77%)	5.470 (2.35%)	5.550 (0.92%)
prxy0 trace				
0.14	3.330	3.316 (0.42%)	3.325 (0.14%)	3.343 (-0.40%)
0.10	4.258	4.227 (0.48%)	4.249 (0.22%)	4.275 (-0.41%)
0.06	6.105	6.062 (0.70%)	6.086 (0.32%)	6.138 (-0.54%)
online trace				
0.14	1.513	1.387 (8.30%)	1.394 (7.90%)	1.441 (4.78%)
0.10	1.907	1.753 (8.09%)	1.772 (7.06%)	1.850 (3.00%)
0.06	2.830	2.600 (8.12%)	2.655 (6.19%)	2.809 (0.76%)
webmail trace				
0.14	1.859	1.781 (4.20%)	1.795 (3.46%)	1.836 (1.25%)
0.10	2.414	2.307 (4.43%)	2.327 (3.59%)	2.387 (1.12%)
0.06	3.600	3.412 (5.24%)	3.452 (4.11%)	3.554 (1.27%)

Table 4: Impact of memory on the write amplification using trace based simulations with $b = 64$ and $c + d = 10$: Single write frontier.

several (sequential) requests such that all requests have a size of at most 4 KB. Some statistics on the trace files after this processing was done are listed in Table 2 and Table 3. Table 2 lists the percentage of the requests that are write requests (%Writes), it lists the number of requests (#Requests), and the percentage of the accessed logical block address (LBA) space that is only accessed by read requests (%LBA_RO). The data locality, presented in Table 3, is expressed as follows: the y% most frequently accessed pages represent v% percent of the total accessed LBA space (including both writes and reads) if v appears in the column labeled y%, e.g., for the webmail trace the 60% most frequently accessed pages represent about 5.23% percent of the total accessed LBA space. Table 2 also shows that all these traces are write dominant, although in some cases (i.e., the webmail and online trace) a fairly large portion of the accessed LBA space is accessed only by read requests.

The SSD used in the trace-driven simulation experiments is composted of $U = b\lfloor x/b \rfloor$ logical pages, where x is equal to the number of logical pages accessed during the trace. The number of physical blocks N is determined by U by means of the spare factor S_f, i.e., $U = N(1 - S_f)$. In other words, a fraction $(1 - S_f)$ of the pages is in the valid state at all times during the simulation, while all of the logical pages are accessed at least once during the simulation (except for at most $b - 1$ pages), but data is only written to some of the pages. The data is initially placed in an unfragmented manner on the drive such that the first U physical blocks contain all the valid pages and the remaining $N - U$ blocks contain only pages in the erase state.

The initial state of the drive has, in some cases, an important impact on the results in this type of simulation setup. To understand this it is important to note that in some traces up to 50% of the logical address space is accessed by read operations only. If b pages that are only read during the simulation happen to reside on the same physical block at the start of the simulation, they will still reside there at the end of the simulation. Thus, if a substantial fraction of the blocks is full of valid data that is only read, say a fraction F, the spare space (of size S_f) is fragmented over a smaller part of the physical address space and the *effective* spare factor is more like $S_f/(1 - F)$; hence we expect to see lower values for the write amplification as F increases. As the initialization method used in our experiments starts with unfragmented data located at the first U blocks of the disk, we will observe lower values for the write amplification for larger %LBA_RO values, i.e., for the online and webmail trace.

Finally, to make the simulation runs sufficiently large we also adopted the *replay* method used in prior SSD work [13, 17]. By replaying a trace, we simply mean that the I/O pattern of the trace is repeated a number of times without change such that the overall trace length exceeds $50,000,000$ requests. This implies that pages are updated several times during a single run, unless the page is only read during the original trace. The results in Figure 4 are based on 5 to 12 runs (depending on the trace) such that the 95% confidence intervals are sufficiently small. The results in Tables 4 and 5 are based on 10 runs.

4.2 Numerical results

We start by repeating one of the experiments of Section 3.4 where we fixed $b = 64$ and $c + d = 10$ and compared the write amplification for different values of c. The results for $S_f = 0.06, 0.10$ and 0.14 in case of a single write frontier are presented in Table 4, while Table 5 contains the results for the double write frontier as introduced in [22]. A first observation is that the write amplification is reduced quite drastically if we replace the single by the double write frontier. This shows that the double write frontier not only significantly reduces the write amplification in case of a simple Rosenblum workload model as demonstrated in [22] (which contains no spacial locality and the temporal locality does not vary over time), but is equally effective in case of real life workloads.

A second observation is that the largest reduction in the write amplification, in case of the single write frontier, is obtained when $c = 1$ (when compared to setting $c = 0$). A similar observation was made in Figure 3 for $c+d = 10$ under uniform random workloads, where the gain was between 1% and 1.5% for $b = 64$. For real life workloads the gain is much more pronounced, with gains up to 8%. For the double write frontier the gains are even more substantial (up to 13%) and setting $c = 2$ is best in some cases, though the additional

S_f	$c = 0$	$c = 1$ (gain)	$c = 2$ (gain)	$c = 3$ (gain)
		rsrch0 trace		
0.14	1.785	1.690 (5.32%)	1.696 (5.00%)	1.723 (3.48%)
0.10	2.095	1.929 (7.90%)	1.942 (7.31%)	1.993 (4.85%)
0.06	2.826	2.538 (10.2%)	2.578 (8.78%)	2.725 (3.60%)
		prxy0 trace		
0.14	1.363	1.232 (9.59%)	1.228 (9.88%)	1.258 (7.68%)
0.10	1.623	1.439 (11.3%)	1.454 (10.5%)	1.551 (4.45%)
0.06	2.273	2.064 (9.20%)	2.187 (3.80%)	2.443(-7.47%)
		online trace		
0.14	1.429	1.267 (11.4%)	1.259 (11.9%)	1.301 (8.97%)
0.10	1.761	1.554 (11.7%)	1.556 (11.6%)	1.640 (6.88%)
0.06	2.506	2.237 (10.7%)	2.305 (8.01%)	2.497 (0.35%)
		webmail trace		
0.14	1.430	1.273 (11.0%)	1.259 (12.0%)	1.284 (10.2%)
0.10	1.729	1.507 (12.8%)	1.501 (13.2%)	1.580 (8.60%)
0.06	2.441	2.158 (11.6%)	2.243 (8.09%)	2.479 (-1.57%)

Table 5: Impact of memory on the write amplification using trace based simulations with $b = 64$ and $c + d = 10$: Double write frontier.

gain compared to having $c = 1$ is rather limited in such cases. Given the above results, we focus on SSDs that rely on the double write frontier in the remainder of this section.

The greedy GC algorithm is no longer optimal in case of non-uniform workloads [22]. In fact, the results in [22] showed that under the Rosenblum model, there exists an optimal parameter d for the d-choices GC algorithm (i.e., a choice for d that minimizes the write amplification). We have also analyzed the existence of an optimal value for d for the d-choices GC algorithm with memory for each of the four traces discussed in Section 4.1 and for various choices of c. We only present the results for the rsrch0 and webmail trace, as the results for the prxy0 and online trace are quite similar.

Figure 4 plots the write amplification of the rsrch0 and webmail trace as a function of the number of choices d for various c values with $S_f = 0.1$ and $b = 64$. Looking at the rsrch0 results, we see that for c fixed the curves appear convex and there exists an optimal value for d, located at $56, 37, 34$ and 37 for $c = 0, 1, 2$ and 3, respectively. Further, the lowest write amplification is realized by setting $c = 1$ and $d = 37$, meaning the optimal d-choices GC algorithm without memory is outperformed by the one with memory, though the gain is very limited. For the webmail trace we observe similar results for $c = 0, 2, 4$ and 6, but the optimal c and d value is larger (i.e., the optimal d is between 100 and 150) and the differences are even less substantial.

These results may seem to indicate that having memory is of little value after all as the optimal d with no memory performs quite similar to the optimal c and d combination. One should however keep in mind that the optimal d value is clearly case specific and hard to determine in practice (and may change over time). A more practical approach may therefore exist in using a fixed d, e.g., $d = 10$ or 20. In such a case adding a limited amount of memory can make a significant difference as shown before. Another thing to note is that having some memory often allows the use of a smaller value of d without increasing the write amplification.

Figure 4 also illustrates that adding more memory, i.e., increasing c, while keeping d fixed, does not always result in a reduction of the write amplification. This is in contrast to the results presented in Figure 1 which corresponded to the

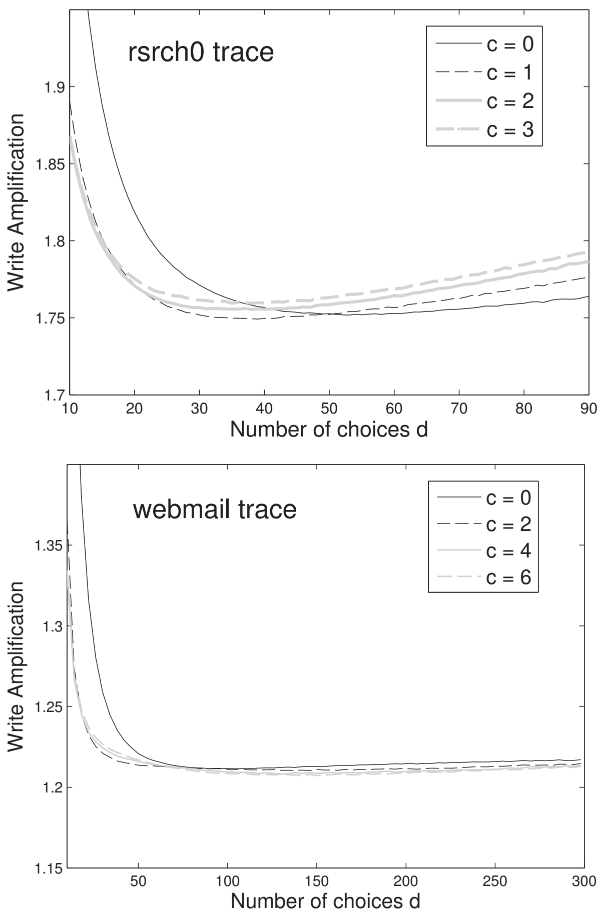


Figure 4: Write amplification as a function of d for various c values and $S_f = 0.1$ and $b = 64$ for the rsrch0 and webmail trace, respectively.

synthetic uniform random writes model (under which the performance of the single and double write frontier is identical). In other words, in the presence of hot and cold data there exists an optimal amount of memory when trying to minimize the write amplification by means of the d-choices GC algorithm with memory. Determining this value in practice is again hard and therefore using a limited fixed amount of memory, e.g., $c = 1$ or 2 for $d = 10$, seems like a good compromise given the results in this section.

5. CONCLUSIONS

In this paper we introduced and analyzed the write amplification of the d-choices GC algorithm with memory using both synthetic and trace-based workloads. The synthetic workloads consisted of uniform random writes and the resulting write amplification was analyzed via a mean field model, where the main challenge existed in determining the fixed point of the resulting set of ODEs in an efficient manner. The trace-based workloads were analyzed by simulation and considered both SSDs using a single and a double write frontier.

Although there exists an optimal combination of c, the number stored memory ids, and d, the number of random choices, in the presence of hot and cold data, these optimal combinations are hard to determine and very case specific. As such a more practical approach exists in working with a fixed c and d such that one obtains an overall good performance. In such a case selecting a small c value can reduce the write amplification by 10% or more in case of real workloads compared to having no memory at all. Further, adding memory to the d-choices GC algorithm allows for smaller d values without increasing the write amplification. Finally, the trace-based simulation results also demonstrated the effectiveness of the double write frontier in reducing the write amplification in case of real workloads.

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