Whether and When to Share: Spectrum Sensing as An Evolutionary Game

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***Abstract*—Cooperative spectrum sensing is an efficient sensing scheme for cognitive radio networks (CRNs), in which secondary users (SUs) share sensing results with other SUs to improve the overall sensing performance. For a single SU, if the sensing results are shared early, there is more time for data transmission, which improves the throughput. In cases with multiple SUs sending early sensing results, it is more likely that the sensing results will be sent simultaneously over the same signaling channel. In this situation, the conflicts that occur affect both the sensing performance and throughput. For this situation, importance of when to share is taken into account, for which we have modeled spectrum sensing as an evolutionary game. The strategy set for each player in our game model contains not only whether to share its sensing results, but also when to share. The payoff of each player is defined based on the throughput, which considers the influence of the time spent both on sensing and sharing. We prove the existence of the evolutionarily stable strategy (ESS). In addition, we propose a practical algorithm for each secondary user to converge at the ESS. We conduct experiments on our GNU/USRP testbed. The experimental results verify for our model, including the convergence to the ESS.**

***Index Terms*—Cognitive radio networks (CRNs), spectrum sensing, game theory, USRP testbed.**

I. Introduction

Cognitive radio networks (CRNs) [1] enable *secondary users* (SUs) to utilize the licensed spectrum when *primary users* (PUs) are not using it. Spectrum sensing is the key phase to identifying the spectrum availability. The fundamental task of spectrum sensing is that: when PUs are using the licensed spectrum, each SU should be able to detect it, and should quit transmitting on the corresponding spectrum band. When PUs are not using the licensed spectrum, each SU should be able to identify the corresponding spectrum band as available. The objectives of SUs are to maximize the utilization of the available spectrum and to prevent interference with PUs.

The spectrum sensing performance is measured by two metrics: probability of detection, which denotes the probability of a SU detecting a PU when the spectrum is occupied by the PU; and probability of false alarm, which denotes the probability of a SU falsely declaring a PU as present, when it is actually not occupied. To ensure the spectrum sensing quality, adequate sample collection is required over a period of time for analysis by SUs. The time spent by the SU on spectrum sensing will reduce the time spent on data transmission. The performance of spectrum sensing for all SUs is improved through cooperative sensing, because each SU shares its sensing results with others, and decides whether a spectrum band can be accessed, based on multiple users' sensing results.

Numerous studies have applied the game theory on cooperative spectrum sensing, which determines the relative probability of an SU participating [2], [3]. The strategy set of each player is usually {contribute, not contribute}, because not all SUs willingly contribute to cooperative sensing. To contribute indicates that the SU needs to share its sensing results with other SUs. The cooperative sensing can save the overall cost in spectrum sensing among SUs. However, not all SUs are willing to participate, because they will benefit from the free ride from others' sensing results. When more SUs choose not to contribute, the sensing performance will be affected. Many works have been done on the decision of an SU to contribute its sensing results or not.

Besides whether a SU is willing to share its sensing results or not, it is also important for each SU to decide when to share. Intuitively, SUs are willing to share their sensing results early, which means that more time can be used for data transmission. However, conflicts might occur because two or more SUs may send out their sensing results together, since the sensing results are usually sent through a common signaling channel. To ensure the sensing performance, the conflicted SU needs to back off for a certain amount of time, and resend its sensing results later. This would lead to the increment in the time spent on spectrum sensing, which in turn results in a decrease in the time spent on data transmission. In a distributed system, without a base station or central controller, each SU has to decide whether to share, as well as when to share. Moreover, if this is coordinated by the communication among different SUs, it would cause more overhead. So, it is better to have each SU decide when to share, itself, based on its own observation.

In this paper, we consider a CRN, in which the licensed spectrum band is divided into multiple subbands and a signaling band. Each SU is assigned a subband for data transmission, and the signalling band is for sharing sensing results. Each SU can not only decide whether to contribute its sensing results by sharing with others, but also decide when to send out its sensing results if it chooses to contribute. We model the process as an evolutionary game, in which each SU aims at maximizing its throughput, while assuring the sensing performance. We propose replicator dynamics for each SU, and prove the existence of an *evolutionarily stable strategy* (ESS). We propose a distributed algorithm for each SU to

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evolutionarily reach the ESS.

The main contributions of our paper are:

- To the best of our knowledge, this is the first work that models spectrum sensing as an evolutionary game, in which the strategy set for each SU (player) is able to render decisions whether to share its sensing result, as well as, when to share,
- We prove the existence of ESS in our game model, and propose a distributed algorithm for each SU to converge.
- We construct a testbed consisting of five USRP N200s, and have the SUs run our distributed algorithm. Our model is testified, in terms of the convergence to the ESS.

II. Related Works

Many works are done on the cooperative model design of spectrum sensing [2], [3]. [2] proposes an evolutionary game model for spectrum sensing. In the model, each user has a probability of performing sensing. The strategy set for each player is to contribute or not to contribute. The payoff of each player is defined based on the throughput, which considers the time spent on spectrum sensing. Our model is different from the model in [2] in two aspects. One is that our strategy set considers both whether to contribute, and when to contribute. Another is that our payoff function considers the influence caused by conflicts, while sharing the sensing results. [3] studies spectrum sensing as a noncooperative game under the constraints of sensing performance and QoS. Their game model is decoupled into a lower-level uncoupled game, and a higher-level optimization problem. A distributed hierarchical iterative algorithm is proposed for their model.

Many existing literature focuses on improving the spectrum sensing performance [4]–[6]. [4] introduces a spatial diversity technique to reduce the error probability between SUs and the data fusion center. The error in their model is mainly caused by the fading on the reporting channel. [5] aims at solving the problem when sensing samples are not sufficient for precisely detecting available channels. They apply matrix completion and joint sparsity recovery to improve the sensing performance. Cooperative compressed spectrum sensing is studied in [6]. They propose the belief propagation, based on compressed spectrum sensing for the statistical prediction of spectrum availability, and build a probabilistic graph model. Our work focuses mainly on improving the sensing performance through choosing a better strategy during cooperative spectrum sensing.

III. System model & Problem Formulation

A. Network Model

We consider a set of SUs, or nodes, $S = \{S_i\}$ in a CRN. Each node is assumed to know the total number of nodes N ($N = |S|$), and is able to reach one another within one hop. The privileged band is divided into N subbands. There is one signaling band. Each user is assigned with one subband for data transmissions, and shares its sensing results on the signaling band, as shown in Fig. 1. We assume that each SU, or node, is equipped with two antennas. One antenna is used for spectrum sensing, sensing results sharing, and data transmission. The other antenna is used for listening to the signaling channel to overhear the sensing results shared by others, and sending back ACKs when the sensing results from others are received. The time-slotted system is used. During each time slot, the SU needs to first sense the PU's activity. Since the PU operates on the whole licensed spectrum band, its activity can be sensed by any SU. This means that each SU can choose to cooperate and share the sensing results over the signaling channel, as to ensure a high detection probability, and a low false alarm probability.

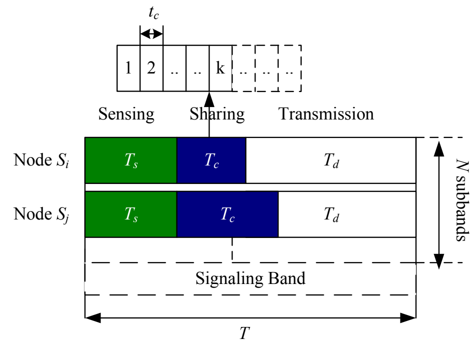

Fig. 1. Example of subbands and time slot division.

Each time slot T is divided into three parts: sensing phase T_s, sharing phase with a maximal length of T_c, and data transmission phase T_d, as shown in Fig. 1. The sensing phase is for each node to sense the channel independently. We assume that for each node, the time spent on independent sensing is static. The sharing phase is for each node to send its sensing results over the signaling channel. Suppose the minimal time required for sending the sensing results when there is no conflict is t_c. Then T_c is divided into $\lceil \frac{T_c}{t_c} \rceil$ sub slots. For a certain SU, it can choose whether to share its sensing results or not. If a node decides not to share its sensing results, its sharing time length would be 0. Additionally, if it chooses to cooperate with others, it needs to choose one sub slot of T_c to send the sensing results. The sensing results are confirmed to be received successfully through the ACKs. The sharing phase of a node ends as long as one ACK is received. Before that, the current SU keeps listening to the signaling channel for others' sensing results. The transmission phase is for data transmission. Therefore, the more time spent on spectrum sharing, the less time would be left for data transmission.

B. Objective & Constraints

First, we use P_{H_0} to denote the probability that the PU does not occupy the licensed spectrum band, which means that SUs can access and transmit on their subbands. Then $1 - P_{H_0}$ denotes the probability that the PU occupies the licensed channel, which means none of the subbands are available. We assume each SU uses an energy detector for spectrum sensing. Suppose the PU activity is a random process with mean 0 and variance σ_s^2. Suppose the additive Gaussian white noise is a circularly symmetric complex Gaussian with mean 0 and variance σ_w^2. For a single SU S_i, the probability of detection $p_d(S_i)$ and false alarm $p_f(S_i)$ can be calculated [7]:

$$p_d(S_i) = \frac{1}{2} erfc((\frac{\lambda}{\sigma_w^2} - \lambda - 1)\sqrt{\frac{K}{2(2\lambda+1)}}), \quad (1)$$

$$p_f(S_i) = \frac{1}{2} erfc(\sqrt{2\lambda+1} etf^{-1}(1 - 2\bar{P_D}) + \sqrt{\frac{K}{2}}\lambda), \quad (2)$$

where $erfc()$ denotes the complementary error function; $erf^{-1}()$ denotes the inverse function of the error function; K is the number of collected samples; $\bar{P_D}$ is the given target detection probability; λ denotes the received signal-to-noise ratio (SNR) of a PU under H_1, which equals to $h^2\sigma_s^2/\sigma_w^2$, and h is the gain of the channel from the PU's transmitter to the SU's receiver, which is assumed to be slow flat fading.

We assume that each SU overhears others' sensing results on the signaling channel. The channel availability is decided by both its own sensing results, and the others' collected sensing results. The data transmission starts after the sharing phase ends, and the current subband is identified as available. The fusion rule of deciding whether the channel is available or not can be different (AND, OR, Majority, etc.). Suppose we apply the OR rule here. For a SU S_i, its probability of detection and false alarm after the sensing results sharing phase would be:

$$P_d(S_i) = 1 - \prod_{k=1}^{N}(1 - p_d(S_i)A(S_i, S_k)), \quad (3)$$

$$P_f(S_i) = 1 - \prod_{k=1}^{N}(1 - p_f(S_i)A(S_i, S_k)), \quad (4)$$

where $A(S_i, S_k) = 1$ means that the sensing results of a SU S_k is received by a SU S_i before T_d starts; $A(S_i, S_k) = 0$ otherwise.

The expected throughput for a SU S_i under P_{H_0} is defined as [8]:

$$U(S_i) = P_{H_0}(1 - \tfrac{T_s+\delta(t_r)}{T})(1 - P_f(S_i))C_{H_0}(S_i) + (1 - P_{H_0}) (1 - \tfrac{T_s+\delta(t_r)}{T})(1 - P_d(S_i))C_{H_1}(S_i), \quad (5)$$

where $C_{H_0}(S_i)$ is the data rate of SU S_i under H_0, $C_{H_1}(S_i)$ is the data rate of SU S_i under H_1, and $\delta(t_r)$ denotes the time used to send the sensing results. This is due to the fact that conflicts may occur when more than one SU sends the sensing results together. Then they would need to backoff and resend the sensing results later. We use $\delta(t_r)$ to represent the total time spent on this sensing results sharing, and obviously $0 \le \delta(t_r) \le T_c$. Since $C_{H_1}(S_i)$ is much smaller than $C_{H_0}(S_i)$, due to the interference from the PU, the second term can be omitted. Therefore, the expected payoff for S_i can be approximated as:

$$U(S_i) = P_{H_0}(1 - \frac{T_s+\delta(t_r)}{T})(1 - P_f(S_i))C_{H_0}(S_i). \quad (6)$$

Algorithm 1 Evolutionary algorithm for S_i

$t_0 = 0$, $\forall(C, j)$, $p_{(C,j)}(S_i) = p_0$
$temp = 0$
while NOT an ESS **do**
  $\tilde{t} = t_0$
  while $\tilde{t} < t_0 + \tilde{T}$ **do**
    Choose (C, j) with probability $p_{(C,j)}$
    Calculate $U_{(C,j)}(S_i)$ using Eq. 8
    $\tilde{t} = \tilde{t} + T$
  $t_0 = t_0 + 1$
  Calculate $\bar{U}_{(C,j)}(S_i)$ and $\bar{U}(S_i)$
  $\forall(C, j)$, update $p_{(C,j),S_i}$ using Eq. 13
  if $temp * (\bar{U}_{(C,j)}(S_i) - \bar{U}(S_i)) < 0$ **then**
    $\mu = \mu/2$
  $temp = \bar{U}_{(C,j)}(S_i) - \bar{U}(S_i)$

The objective of each SU is to maximize its own $U(S_i)$, while satisfying the following constraints:

$$P_d(S_i) > \alpha \text{ and } P_f(S_i) < \beta, \quad (7)$$

where α and β are the corresponding required thresholds.

IV. Game Model

In this section, we first introduce the main concepts regarding the evolutionary game. Then we present our game model. Finally, we provide a practical algorithm for each SU.

A. Evolutionary Game

The key insight of evolutionary game theory is that many behaviors involve the interactions of multiple strategies of different players, and the success of any strategy depends on how it interacts with others. The analogous notion in evolutionary game theory is an ESS. We have the formal definition of an ESS, as follows [11]:

Definition 1. *A strategy $q*$ is an ESS if and only if, for any strategy $q \neq q*$ and all $\theta > 0$,*

$$U(q*, \theta q + (1-\theta)q*) > U(q, \theta q + (1-\theta)q*),$$

where $U(q*, \theta q + (1-\theta)q*)$ denotes the payoff of a player who adopts $q*$, while the θ portion of the others adopt q, and the remaining portion adopt $q*$.

From the definition, we can see that the strategy is an ESS, which tends to persist once it is adopted by most players. Due to dynamics in the spectrum availability in CRNs, there is not a static stable strategy for each user conducting spectrum sensing. Therefore, we apply the evolutionary game here to solve the problem.

B. Model Construction and Analysis

1) Strategy Set: The SUs are players in our game. Different from traditional works, the strategy set here is no longer limited to {contribute, not contribute}. Specifically, as introduced in Sec. III, each user needs to pick a sub slot from T_c to send its sensing results. Since each node tries to maximize

TABLE I
PROBABILITY OF DETECTION

	5	10	15	20	25
random	0.850	0.924	0.985	1.0	1.0
sorted	0.800	0.901	0.961	1.0	1.0

TABLE II
PROBABILITY OF FALSE ALARM

	5	10	15	20	25
random	0.030	0.053	0.025	0.016	0.011
sorted	0.044	0.034	0.020	0.025	0.015

its throughput, it would be more willing to increase the time spent on data transmission, which means the time spent on the sensing results sharing phase is less. Thus, intuitively, an SU tends to send its sensing results during the early sub slots of T_c, or may not even share its sensing results, in order to have more time for data transmission. However, if more and more SUs choose not to contribute, the sensing performance constraints in Eq. 7 cannot be satisfied. If many SUs choose the sub slots in the early part of T_c, more conflicts would happen in the signaling channel, and the interfered SUs would need to resend their sensing results. Then, the T_c is delayed and T_d is reduced, which results in the decrease of throughput.

Therefore, in our model, the strategy q for each SU needs to contain not only either C (share its sensing results to contribute) or D (deny to contribute), but also when to send out its sensing results over the signaling channel. We have the following definition of the strategy set:

Definition 2. *The strategy set of an SU is $\{(C,j)\}$, where $j \in \{0,1,...,\lceil\frac{T_c}{t_c}\rceil\}$. $j=0$ means the SU denies to share its sensing results. Otherwise, the SU sends its sensing results at the jth sub slot of T_c.*

2) Payoff: The payoff is defined based on the throughput of Eq. 6. For a secondary user S_i that adopts strategy (C,j), we have

$$\delta(t_r) = jt_c + \Delta,$$

where Δ is the time spent on backing off and resending the sensing results of S_i when conflicts happen. The value of Δ depends on the strategies chosen by others. To replace the $\delta(t_r)$ in Eq. 6, the payoff for S_i that adopts strategy (C,j) is:

$$U_{(C,j)}(S_i) = P_{H_0}(1 - \frac{T_s + jt_c + \Delta}{T})(1-P_f(S_i))C_{H_0}(S_i). \tag{8}$$

3) Analysis: Suppose the mixed strategy adopted by user S_i is $x(S_i)$. Since the starting point of T_c is the same for all SUs, the strategy set is homogenous for all SUs. Suppose during a time slot t, the probability of an SU S_i to adopt strategy (C,j) is: $p_{(C,j)}(S_i)$. The time evolutionary dynamics $\dot{p}_{(C,j)}(S_i)$ that determine $p_{(C,j)}(S_i)$ is:

$$\dot{p}_{(C,j)}(S_i) = [\bar{U}_{(C,j)}(S_i,-S_i) - \bar{U}_{x(S_i)}(S_i)]p_{(C,j)}(S_i), \tag{9}$$

where $\bar{U}_{(C,j)}(S_i,-S_i)$ is the average payoff for S_i playing pure strategy (C,j), and other SUs playing strategies other than S_i's strategy; $\bar{U}_{x(S_i)}(S_i)$ is the average payoff of user i using mixed strategy x_{S_i}. The intuition for these dynamics is that if S_i achieves a higher payoff using pure strategy (C,j), strategy (C,j) will be adopted more frequently. The growth rate is proportional to the excess of pure strategy (C,j) and the average payoff of the mixed strategy.

Next, we use $y_{(C,j)}$ to denote the proportion of nodes that adopt the pure strategy (C,j) at a given time t. The evolutionary dynamics $\dot{y}_{(C,j)}$ of $y_{(C,j)}$ is given by the following equation, according to the replicator dynamics:

$$\dot{y}_{(C,j)} = [\bar{U}_{(C,j)} - \bar{U}]y_{(C,j)}, \tag{10}$$

where $\bar{U}_{(C,j)}$ is the average payoff of players who use strategy (C,j), and $\bar{U}(x)$ is the average payoff of all players. The $\bar{U}_{(C,j)}$ depends on both of the populations that adopt (C,j). If more players adopt the same (C,j), then conflicts will occur, and the decrease in payoff will be shown in the replicator dynamics. In the following, we prove that starting from any y^*, the replicator dynamics converges to an ESS.

Theorem 1. *There exists an ESS to our game model. In specific, the replicator dynamics could converge to $y^*_{(C,0)} = \frac{1}{N}$, $y^*_{(C,k)} = 1/min\{N, \lceil\frac{T_c}{t_c}\rceil\}$, $k \in [1, min\{N, \lceil\frac{T_c}{t_c}\rceil\}]$ is a unique and consequent number for all users, or $y^*_{(C,0)} = \sigma, y^*_{(C,l)} = (1-\sigma)/min\{N, \lceil\frac{T_c}{t_c}\rceil\}$, l is a unique and consequent number from 1 to $min\{N, \lceil\frac{T_c}{t_c}\rceil\}$, and σ is the probability of choosing not to share sensing results.*

Proof. The first step is to prove the existence of an ESS. Since the maximal number of pure strategies for each SU is $1 + \lceil\frac{T_c}{t_c}\rceil$, the overall strategy set is closed. Since the probability of a certain SU S_i to adopt strategy $c \in \{(C,j)\}$ is $x_{c,S_i}(t)$, assume the backoff window size is doubled after each conflict for a single user during one time slot, and the initial backoff window size is t_c, then $\delta(t_c)$ is a linear function of t_c. Therefore, $\delta(t_c)$ will not affect $\frac{\partial^2 U(S_i)}{\partial S_i^2}$. From [3], we have $\frac{\partial^2 U(S_i)}{\partial S_i^2} > 0$. Therefore, an ESS exists.

Secondly, all players are treated equally. We use σ and $1-\sigma$ to first distinguish the probability of users that do not share their results, and users that share sensing results. From [2], we know that there exists three cases:

1) $\sigma = 0$: $T_s + \delta(t_c) = 0$;
2) $\sigma = 1$: all nodes choose to share their sensing results;
3) σ is the solution to the derivation of the payoff difference among users who choose to share and not to share [2].

When case 2 occurs, for any S_i that satisfies $y^*_{(C,k)} = 1/min\{N, \lceil\frac{T_c}{t_c}\rceil\}$, $k \in [1, min\{N, \lceil\frac{T_c}{t_c}\rceil\}]$ is a unique and consequent number for all users, S_i's strategy is (C, k_i). If S_i switches to another strategy (C, k_i'), there are two situations:

- No conflict happens:

$$\begin{aligned}\Delta U(S_i) = P_{H_0}(\alpha - \tfrac{T_c+\delta(t_c')}{T})(1-P_f(S_i))C_{H_0} \\ -P_{H_0}(\alpha - \tfrac{T_c+\delta(t_c)}{T})(1-P_f(S_i))C_{H_0},\end{aligned}$$

  $t_c' > t_c$ because $k \in [1, min\{N, \lceil\frac{T_c}{t_c}\rceil\}]$. Therefore, $\Delta U(S_i) < 0$, which causes a decrease in Eq. 10.

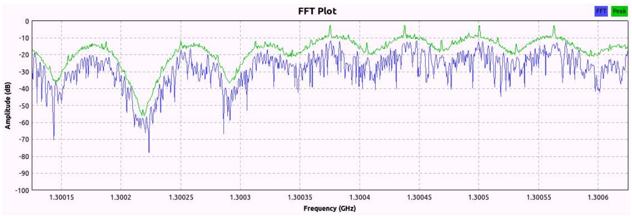

Fig. 2. Primary user sends at multiple bands.

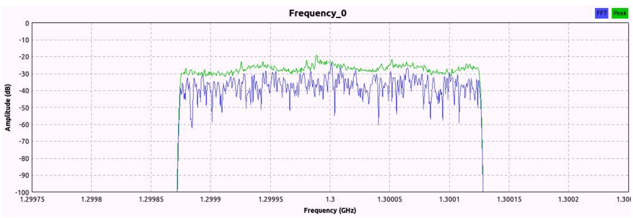

Fig. 3. Secondary user receives at 1.3GHz.

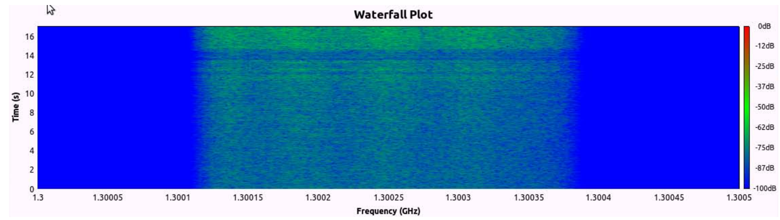

Fig. 4. Sensing results according time at 1.30025GHz.

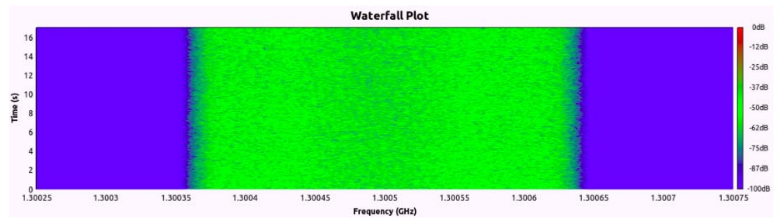

Fig. 5. Sensing results according time at 1.3005GHz.

- A conflict happens between SU S_i and SU $S_{i'}$ that chooses (C, k_i'). The new t_c for both S_i and S_i' would increase because of the backoff policy, which also causes a decrease in Eq. 10.

For case 3, the value of σ is solved in [2]. The part of y^* is similar to case 2. ■

C. Evolutionary Algorithm

The evolutionary dynamics for each player is in Eq. 9. To implement a distributed algorithm for each player, we need to define a practical way to calculate $\bar{U}(S_i)$. Therefore, we define a valid time window $\tilde{T}$. Only the payoff within $\tilde{T}$ will be counted to calculate the approximate values of $\bar{U}_{(C,j)}(S_i)$ and $\bar{U}(S_i)$, denoted as

$$\bar{U}_{(C,j)}(S_i) = \frac{\sum_{\tilde{t}=t_0}^{t_0+\tilde{T}} U_{(C,j)}(S_i) B_{(C,j)}(S_i)}{\sum_{\tilde{t}=t_0}^{t_0+\tilde{T}} B_{(C,j)}(S_i)}, \tag{11}$$

$$\bar{U}(S_i) = \lceil \frac{\tilde{T}}{T} \rceil \sum_{\tilde{t}=t_0}^{t_0+\tilde{T}} U(S_i, \tilde{t}), \tag{12}$$

where t_0 is the first time slot of a new time window $\tilde{T}$; $B_{(C,j)}(S_i)$ is the indicator function which is equal to 1 when S_i adopts (C, j) and is 0 otherwise; $U_{S_i}(\tilde{t})$ is the throughput of S_i during $\tilde{t}$; T denotes the default length of one time slot, as indicated before.

Therefore, the probability $p_{(C,j)}(S_i)$ of a user S_i to adopt the pure strategy (C, j) can be updated using Eq. 13. The value of the stepwise μ is not constant. To reduce the oscillation, μ would be divided by 2 if the value of the $\bar{U}_{(C,j)}(S_i) - \bar{U}(S_i)$ changes from positive to negative, or from negative to positive, during two adjacent time slots. The initial value of μ would be studied in our experiment.

$$\begin{aligned} p_{(C,j),S_i}((\tilde{t}+1)) &= p_{(C,j),S_i}(\tilde{t}) \\ +\mu(\bar{U}_{(C,j)}(S_i) &- \bar{U}(S_i)) p_{(C,j)}(S_i, \tilde{t}) \end{aligned} \tag{13}$$

The algorithm for each player to reach the ESS is in Algorithm 1. The player tries to converge to ESS within the loop from Step 3 to 14. In Step 4, the new starting time of calculating the average payoff is initialized. From Step 5 to 10, we calculate the average payoff only within the window size $\tilde{T}$. The starting point of the window moves forward by 1 in Step 9. At the end of $\tilde{T}$, each player uses the above equations to update the probability of choosing each strategy in Step 11. From Steps 12 and 14, the value of μ regarding to the stepwise is adjusted. Step 12 decides whether the player has passed the ESS. If it happens, the value of μ would be reduced by half, which means the stepwise is reduced. The process will end when reaching the ESS.

V. Experiment

A. Environment Settings

Our experiment consists of four USRN N200s. Three USRPs simulate three SUs, and each works on a subband. The remaining USRP simulates a PU. We place them at different positions. The distance between a SU and a PU ensures that the sensing results of a single SU are not sensitive enough to detect the signal from the PU. The PU occupies multiple bands at the same time, while each SU works on a single subband. As shown in Fig. 2, the PUs occupy wide bands. The received signals on each SU have different central frequencies (1.3GHz, 1.30025GHz, 1.3005GHz), as shown in Fig. 3 (we only show one figure here since the other two are similar). The green lines are the peak points.

We set the time slot length to 20s here (for better synchronization). The static sensing time is set to $5s$. The maximal length of T_c is 5s, which is divided into 5 sub slots. The window size for each SU to calculate the average throughput is 4 slots. The bandwidth of each SU is 50k bps. The gain at each receiver is set to 20. We generate an active sequence for the PU with P_{H_0} equal to 0. The thresholds for probability of detection and probability of false alarm are set to 0.9 and 0.1, respectively. Our experiment works as follows:

- The PU sends out signals while in its active slots.

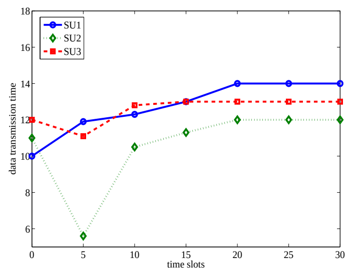

Fig. 6. Random initial values.

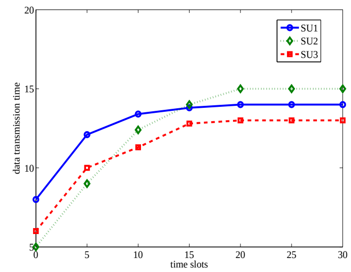

Fig. 7. Sorted initial values.

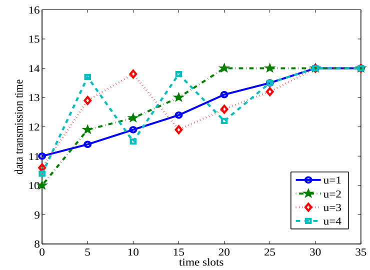

Fig. 8. Convergence under different u.

- SUs sense their own subband for 4s. We set the threshold as -60 dB, as to decide if the PU is active. The sensing results are sharing on a different subband with a central frequency of 1.30075 GHz.
- After the sharing phase ends, and if it is successful, we calculate how much time remains in the current time slot. If the sharing phase does not succeed, the time left is treated as 0. The payoff is denoted by the time left for data transmission in each time slot, instead of the real throughput. This is reasonable, based on the payoff definition in Eq. 8.

B. Experimental Results

1) Unreliability of single sensing: We set the PU to be active and to plot the detecting results of each SU, according to the time. The results are shown in Figs. 4 and 5. Due to space limitations, we only show two SUs' receiving results here. The blue parts indicate that no signal is detected, while the green parts indicate that a signal is detected. From the two figures, we can see that the sensing results by a single SU are unstable. Here, for the SU receiving at the central frequency 1.30025GHz, the blue and green parts are mixed, although the PU is active. If this node makes a decision based on its own sensing results, it is possible that it can mistake the unavailable band for an available one, and cause interference to the PU.

2) Performance versus different initial probabilities: Since the maximal number of sub slots in T_c is 5, the size of the strategy set is 6 for each SU, which is (C, j) and $0 \leq j \leq 5$. We generate two different situations for the initial probabilities of choosing each strategy. One is the random choice, which means each initial probability is equal to $1/6$. The second situation is that initial probabilities for 6 strategies are sorted. $(C, 0)$ has the largest initial probability to be chosen, while $(C, 5)$ has the minimal value. The results are shown in Figs. 6 and 7. We can see that under both settings, all three users converge to one pure strategy, and achieve a stable data transmission time, which indicates a stable payoff. We also testify to the sensing performance in Tables I and II. The probability of detection converges to 1. The probability of false alarm is low, initially. It converges to around 0.01.

3) Performance versus different settings of step size: We also evaluate the influence caused by different values of step size u. We set four different values for u, and calculate the time left for data transmission for SU 1 under the random settings of initial probabilities. The results are shown in Fig. 8. We can see that all four lines converge to the same point, eventually. This is because the value of u is adjusted (reduced by half) during the process. Also, from Fig. 8, we can see that when $u = 3$ or 4, the line has oscillation instead of a continual increase when $u = 1$ or 2. Among these four settings, $u = 2$ achieves the best result.

VI. Conclusion

In this paper, we consider both whether-to-share and when-to-share, regarding the cooperative spectrum sensing in CRNs. We build an evolutionary game model, in which each SU is treated as a player. We extend the strategy set for each SU, and define the payoff based on the time left for transmission. We prove the existence of the evolutionary stable strategy (ESS). Then, a practical algorithm is proposed for each SU to converge. In addition, we construct a testbed using 4 USRP N200s. One simulates the PU, and the other three simulate the SUs. We evaluate the performance under different settings. In addition, we study the influence of different values of step sizes on convergence to the ESS.

References

[1] I. F. Akyildiz, W. Y. Lee, M. C. Vuran, and S. Mohanty, "Next generation/dynamic spectrum access/cognitive radio wireless networks: A survey," *Computer Networks*, 2006.

[2] B. Wang, K. J. R. Liu, and T. C. Clancy, "Evolutionary cooperative spectrum sensing game: How to collaborate?" *IEEE Transactions on Communications*, 2010.

[3] W. Yuan, H. Leung, W. Cheng, S. Cheng, and B. Chen, "Participation in repeated cooperative spectrum sensing: A game-theoretic perspective," *IEEE Transactions on Wireless Communications*, 2012.

[4] W. Zhang and K. B. Letaief, "Cooperative spectrum sensing with transmit and relay diversity in cognitive networks," *IEEE Transactions on Wireless Communications*, 2008.

[5] J. Meng, W. Yin, H. Li, E. Hossain, and Z. Han, "Collaborative spectrum sensing from sparse observations in cognitive radio networks," *IEEE Journal on Selected Areas in Communications*, 2011.

[6] Z. Zhang, Z. Han, H. Li, D. Yang, and C. Pei, "Belief propagation based cooperative compressed spectrum sensing in wideband cognitive radio networks," *IEEE Transactions on Wireless Communications*, 2011.

[7] H. V. Poor, *An Introduction to Signal Detection and Estimation, 2nd ed.* New York: Springer-Verlag, 1994.

[8] Y. C. Liang, Y. Zeng, E. C. Y. Peh, and A. T. Hoang, "Sensing-throughput tradeoff for cognitive radio networks," *IEEE Transactions on Wireless Communications*, 2008.

[9] S. Lasaulce, M. Debbah, and E. Altman, "Methodologies for analyzing equilibria in wireless games," *IEEE Signal Process*, 2009.

[10] B. Wang, Z. Han, and K. J. R. Liu, "Distributed relay selection and power control for multiuser cooperative communication networks using stackelberg game," *IEEE Transactions on Mobile Computing*, 2009.

[11] J. M. Smith, *Evolution and the Theory of Games*. Cambridege University Press, 1982.